SMITHSONIAN

CONTRIBUTIONS TO KNOWLEDGE.

VOL. XIX.

EVERY MAN IS A VALUABLE MEMBER OF SOCIETY, WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN.—SMITHSON.

CITY OF WASHINGTON:
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ADVERTISEMENT.

THIS volume forms the nineteenth of a series, composed of original memoirs on different branches of knowledge, published at the expense, and under the direction, of the Smithsonian Institution. The publication of this series forms part of a general plan adopted for carrying into effect the benevolent intentions of JAMES SMITHSON, Esq., of England. This gentleman left his property in trust to the United States of America, to found, at Washington, an institution which should bear his own name, and have for its objects the "*increase* and *diffusion* of knowledge among men." This trust was accepted by the Government of the United States, and an Act of Congress was passed August 10, 1846, constituting the President and the other principal executive officers of the general government, the Chief Justice of the Supreme Court, the Mayor of Washington, and such other persons as they might elect honorary members, an establishment under the name of the "SMITHSONIAN INSTITUTION FOR THE INCREASE AND DIFFUSION OF KNOWLEDGE AMONG MEN." The members and honorary members of this establishment are to hold stated and special meetings for the supervision of the affairs of the Institution, and for the advice and instruction of a Board of Regents, to whom the financial and other affairs are intrusted.

The Board of Regents consists of three members *ex officio* of the establishment, namely, the Vice-President of the United States, the Chief Justice of the Supreme Court, and the Mayor of Washington, together with twelve other members, three of whom are appointed by the Senate from its own body, three by the House of Representatives from its members, and six persons appointed by a joint resolution of both houses. To this Board is given the power of electing a Secretary and other officers, for conducting the active operations of the Institution.

To carry into effect the purposes of the testator, the plan of organization should evidently embrace two objects: one, the increase of knowledge by the addition of new truths to the existing stock; the other, the diffusion of knowledge, thus increased, among men. No restriction is made in favor of any kind of knowledge; and, hence, each branch is entitled to, and should receive, a share of attention.

The Act of Congress, establishing the Institution, directs, as a part of the plan of organization, the formation of a Library, a Museum, and a Gallery of Art, together with provisions for physical research and popular lectures, while it leaves to the Regents the power of adopting such other parts of an organization as they may deem best suited to promote the objects of the bequest.

After much deliberation, the Regents resolved to divide the annual income into two parts—one part to be devoted to the increase and diffusion of knowledge by means of original research and publications—the other part of the income to be applied in accordance with the requirements of the Act of Congress, to the gradual formation of a Library, a Museum, and a Gallery of Art.

The following are the details of the parts of the general plan of organization provisionally adopted at the meeting of the Regents, Dec. 8, 1847.

DETAILS OF THE FIRST PART OF THE PLAN.

I. To increase Knowledge.—*It is proposed to stimulate research, by offering rewards for original memoirs on all subjects of investigation.*

1. The memoirs thus obtained, to be published in a series of volumes, in a quarto form, and entitled "Smithsonian Contributions to Knowledge."

2. No memoir, on subjects of physical science, to be accepted for publication, which does not furnish a positive addition to human knowledge, resting on original research; and all unverified speculations to be rejected.

3. Each memoir presented to the Institution, to be submitted for examination to a commission of persons of reputation for learning in the branch to which the memoir pertains; and to be accepted for publication only in case the report of this commission is favorable.

4. The commission to be chosen by the officers of the Institution, and the name of the author, as far as practicable, concealed, unless a favorable decision be made.

5. The volumes of the memoirs to be exchanged for the Transactions of literary and scientific societies, and copies to be given to all the colleges, and principal libraries, in this country. One part of the remaining copies may be offered for sale; and the other carefully preserved, to form complete sets of the work, to supply the demand from new institutions.

6. An abstract, or popular account, of the contents of these memoirs to be given to the public, through the annual report of the Regents to Congress.

II. To increase Knowledge.—*It is also proposed to appropriate a portion of the income, annually, to special objects of research, under the direction of suitable persons.*

1. The objects, and the amount appropriated, to be recommended by counsellors of the Institution.

2. Appropriations in different years to different objects; so that, in course of time, each branch of knowledge may receive a share.

3. The results obtained from these appropriations to be published, with the memoirs before mentioned, in the volumes of the Smithsonian Contributions to Knowledge.

4. Examples of objects for which appropriations may be made:—

(1.) System of extended meteorological observations for solving the problem of American storms.

(2.) Explorations in descriptive natural history, and geological, mathematical, and topographical surveys, to collect material for the formation of a Physical Atlas of the United States.

(3.) Solution of experimental problems, such as a new determination of the weight of the earth, of the velocity of electricity, and of light; chemical analyses of soils and plants; collection and publication of articles of science, accumulated in the offices of Government.

(4.) Institution of statistical inquiries with reference to physical, moral, and political subjects.

(5.) Historical researches, and accurate surveys of places celebrated in American history.

(6.) Ethnological researches, particularly with reference to the different races of men in North America; also explorations, and accurate surveys, of the mounds and other remains of the ancient people of our country.

I. To diffuse Knowledge.—*It is proposed to publish a series of reports, giving an account of the new discoveries in science, and of the changes made from year to year in all branches of knowledge not strictly professional.*

1. Some of these reports may be published annually, others at longer intervals, as the income of the Institution or the changes in the branches of knowledge may indicate.

2. The reports are to be prepared by collaborators, eminent in the different branches of knowledge.

3. Each collaborator to be furnished with the journals and publications, domestic and foreign, necessary to the compilation of his report; to be paid a certain sum for his labors, and to be named on the title-page of the report.

4. The reports to be published in separate parts, so that persons interested in a particular branch, can procure the parts relating to it, without purchasing the whole.

5. These reports may be presented to Congress, for partial distribution, the remaining copies to be given to literary and scientific institutions, and sold to individuals for a moderate price.

The following are some of the subjects which may be embraced in the reports:—

I. PHYSICAL CLASS.

1. Physics, including astronomy, natural philosophy, chemistry, and meteorology.
2. Natural history, including botany, zoology, geology, &c
3. Agriculture.
4. Application of science to arts.

II. MORAL AND POLITICAL CLASS.

5. Ethnology, including particular history, comparative philology, antiquities, &c.
6. Statistics and political economy.
7. Mental and moral philosophy.
8. A survey of the political events of the world; penal reform, &c.

III. LITERATURE AND THE FINE ARTS.

9. Modern literature.
10. The fine arts, and their application to the useful arts.
11. Bibliography.
12. Obituary notices of distinguished individuals.

II. TO DIFFUSE KNOWLEDGE.—*It is proposed to publish occasionally separate treatises on subjects of general interest.*

1. These treatises may occasionally consist of valuable memoirs translated from foreign languages, or of articles prepared under the direction of the Institution, or procured by offering premiums for the best exposition of a given subject.

2. The treatises to be submitted to a commission of competent judges, previous to their publication.

DETAILS OF THE SECOND PART OF THE PLAN OF ORGANIZATION.

This part contemplates the formation of a Library, a Museum, and a Gallery of Art.

1. To carry out the plan before described, a library will be required, consisting, 1st, of a complete collection of the transactions and proceedings of all the learned societies of the world; 2d, of the more important current periodical publications, and other works necessary in preparing the periodical reports.

2. The Institution should make special collections, particularly of objects to verify its own publications. Also a collection of instruments of research in all branches of experimental science.

3. With reference to the collection of books, other than those mentioned above, catalogues of all the different libraries in the United States should be procured, in order that the valuable books first purchased may be such as are not to be found elsewhere in the United States.

4. Also catalogues of memoirs, and of books in foreign libraries, and other materials, should be collected, for rendering the Institution a centre of bibliographical knowledge, whence the student may be directed to any work which he may require.

5. It is believed that the collections in natural history will increase by donation, as rapidly as the income of the Institution can make provision for their reception; and, therefore, it will seldom be necessary to purchase any article of this kind.

6. Attempts should be made to procure for the gallery of art, casts of the most celebrated articles of ancient and modern sculpture.

7. The arts may be encouraged by providing a room, free of expense, for the exhibition of the objects of the Art-Union, and other similar societies.

8. A small appropriation should annually be made for models of antiquity, such as those of the remains of ancient temples, &c.

9. The Secretary and his assistants, during the session of Congress, will be required to illustrate new discoveries in science, and to exhibit new objects of art; distinguished individuals should also be invited to give lectures on subjects of general interest.

In accordance with the rules adopted in the programme of organization, each memoir in this volume has been favorably reported on by a Commission appointed

for its examination. It is however impossible, in most cases, to verify the statements of an author; and, therefore, neither the Commission nor the Institution can be responsible for more than the general character of a memoir.

The following rules have been adopted for the distribution of the quarto volumes of the Smithsonian Contributions:—

1. They are to be presented to all learned societies which publish Transactions, and give copies of these, in exchange, to the Institution.

2. Also, to all foreign libraries of the first class, provided they give in exchange their catalogues or other publications, or an equivalent from their duplicate volumes.

3. To all the colleges in actual operation in this country, provided they furnish, in return, meteorological observations, catalogues of their libraries and of their students, and all other publications issued by them relative to their organization and history.

4. To all States and Territories, provided there be given, in return, copies of all documents published under their authority.

5. To all incorporated public libraries in this country, not included in any of the foregoing classes, now containing more than 10,000 volumes; and to smaller libraries, where a whole State or large district would be otherwise unsupplied.

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SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
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AN

INVESTIGATION

OF THE

ORBIT OF URANUS,

WITH GENERAL TABLES OF ITS MOTION.

BY

SIMON NEWCOMB,

PROFESSOR OF MATHEMATICS, UNITED STATES NAVY.

[ACCEPTED FOR PUBLICATION, FEBRUARY, 1873.]

ADVERTISEMENT.

In the investigation of the Orbit of Uranus which forms the subject of the accompanying memoir, as well as in that of the Orbit of Neptune previously published in the Smithsonian Contributions, a large amount of arithmetical computation has been required, especially in the reduction and comparison of observations. The cost of this, in accordance with the spirit of the Institution in advancing science, has been defrayed from the income of the Smithson fund.

As required by the rules of the Institution, the accompanying memoir was referred to competent authority for examination, and the persons selected for this purpose were Professor J. H. C. Coffin, of the Nautical Almanac Office, and Professor Asaph Hall, of the Naval Observatory.

JOSEPH HENRY,
Secretary S. I.

Washington, 1873.

PHILADELPHIA:
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PREFACE.

The present work was undertaken as far back as the year 1859. But the labor devoted to it at first amounted to little more than tentative efforts to obtain numerical data of sufficient accuracy, and to decide upon a satisfactory method of computing the general perturbations of the planet. The elements of Neptune employed in the earlier computations were found to deviate too widely from the truth to be used in computing the perturbations of Uranus with the first order of accuracy, and it became necessary to correct them. This was done during the years 1864 and 1865, and the investigation was printed by the Smithsonian Institution in the latter year. It was then found that the adopted elements of Uranus also differed too widely from the truth to serve as the basis of the work, and they were provisionally corrected by a series of heliocentric longitudes derived from observations extending from 1781 to 1861. Finally it was found that the adopted method of computing the perturbations, that of the "variation of elements," though not deserving of the disfavor into which it has fallen of late years, was practically inapplicable to the computation of the most difficult terms, namely, those of the second order with respect to the disturbing forces. Indeed, it appeared to the author that the only method of computing those terms which was at the same time general, practicable, and fully developed, was that of Hansen. But, were this method adopted, all that had previously been done would have been useless, even for the purpose of comparison and verification, owing to the expression of the coordinates in terms of a disturbed mean anomaly. It appeared to the author that, although this form of theory led to expressions having fewer terms than the other, it was not without its relative disadvantages. Other considerations being equal, he conceived that astronomers generally would greatly prefer to see the perturbations expressed directly in terms of the time, owing to the ease with which the results of different investigators could then be compared, and with which corrections to the theory may be introduced.

Under these circumstances the method described in the first chapter of the present paper was worked out. The question how much it contains that is essentially new is one that the author has never closely examined: it is, however, certain

that the mode of considering the subject is well known, being that employed by La Place, Herschel, De Pontécoulant, Encke, and perhaps others. The method of forming the required derivations of the perturbative function from the analytical development of that quantity, he has not seen elsewhere.

With these improved elements and methods the work was recommenced in 1868. The earlier investigations being merely provisional, it has not been deemed necessary to present them in the present work. Some of the results, corrected for errors of the older elements, are, however, given for the purpose of comparison.

Although this investigation has absorbed the greater part of the author's leisure for more than five years, it is only through the aid of the Smithsonian Institution and Nautical Almanac that he has been enabled to bring it to a conclusion within that time. At an early stage of the work Professor Henry responded favorably to a request for aid by the employment of computers; it was, however, not found practicable to use such aid until the perturbations had been completed, and the provisional theory concluded. Then, the comparison of theory and observation, and the construction of the tables, involved a large amount of mechanical computation, and on this part of the work a number of persons have been employed by the Institution at various times, among whom may be mentioned Professor F. W. Bardwell, of the University of Kansas, and Dr. C. L. F. Kampf, late of the Observatory of Leiden. Every part of the work has, however, been done under the author's immediate direction, and, as nearly as possible, in the same way as if he had done it himself, a result which, in one or two cases, has been attained only by the expenditure of an amount of labor approximating that saved by the employment of the computer.

In presenting the steps of the investigation, the end has been kept constantly in view to render as easy as possible the detection and correction of any error, or the introduction of any alteration in the elements or other data. It is, of course, impossible to present the steps of the computation with any approach to fulness without far transcending the limits of the printed work: The results given are, therefore, those which it was supposed would be most useful to the future investigator of the same subject. There is reason to believe that the original computations will ultimately become the property of the National Academy of Sciences, so that they may always be referred to for the clearing up of any difficulty in the printed text.

The author's acknowledgments are due to Professor J. H. C. Coffin, Superintendent of the Nautical Almanac, and Mr. E. J. Loomis, of the Nautical Almanac Office, for reading the proof sheets of the last twelve tables during the absence of the former abroad.

WASHINGTON, July 31, 1873.

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ERRATA.

Pages 100 to 105. In computing the latitude from the provisional theory the values of the secular terms of $\delta\eta$ and δk on page 97 have been interchanged. The provisional latitude, therefore, requires the correction

$$-0''.53\ T \sin \text{v} + 0''.53\ T \cos \text{v}$$

where

$$\text{v} = g + 12^\circ\ 45' + 2e \sin g.$$

This correction is not applied in the subsequent investigation. Its effect would have been to change the value of b deduced on p. 176 by probably $0''.2$ or $0''.3$. The effect on the other elements of latitude would have been much smaller, and therefore unimportant.

Page 122, line 15. Add: the corrections in the sixth column being omitted.

Page 151. Add foot-note: In forming these comparisons the corrections to the heliocentric longitude in the sixth column of the provisional ephemeris, pages 100 to 105, are not applied.

Page 159. Equation 7. In this equation the coefficients of $\delta\lambda$ and $\delta\rho$ have been multiplied by $\frac{1}{6}$, instead of $\frac{2}{3}$, the factor of δl. The effect of this error enters into all the subsequent results, but in the comparisons of theory and observation it is corrected.

Page 184. The element here represented by $\varkappa$ (kappa) is the same which, in the preceding chapters, has by mistake been represented by k, and which is defined on p. 24. The k of Chapter VIII is, therefore, not the same with that of preceding chapters.

ON THE ORBIT OF URANUS.

INTRODUCTION.

THE connection of the planet Uranus with the most brilliant astronomical achievement of the century lends a peculiar interest to its theory. The researches of Adams and Le Verrier showed that the observed motions of that planet were represented, at least approximately, by the action of a theoretical planet having the longitude of Neptune. Peirce showed that the action of Neptune itself accounted for these motions within the limits of possible error of the observations used by Le Verrier. It remains to be seen whether the agreement between theory and observation still subsists when the comparatively few observations used by those investigators are reduced with the more refined data now at our disposal, and when the great mass of additional observations made both before and since the date of Le Verrier's researches are included.

The circumstances connected with the discovery of Neptune have been so exhaustively recounted by a number of authors that it would be difficult to add anything not already familiar to astronomers without transcending our present limits. I shall therefore confine myself to such an account of previous researches on the theory of Uranus as may give an idea of their nature and extent, and facilitate their comparison with the methods and results of the present investigation.

The perturbations used by Bouvard in his tables are those of the Mécanique Céleste. Although not affected with any striking error, the numerical methods adopted in their computation are necessarily too rough to allow of much interest attaching to their comparison with the results of the more recent researches.

It is essential to a clear understanding of subsequent researches that we classify the methods which have been or may be adopted in the computation of the general perturbations of the planets. This computation comprises two distinct operations: (1) the development of the disturbing forces, or some quantities of which these forces are functions; (2) the integration of the equations of motion under the influence of these forces. In each of these operations three methods have been employed.

In developing the perturbative function, we have first the purely analytic method used by the great geometers of the last century. In this method this function is developed in powers of the eccentricities and mutual inclination of the orbits of the two planets, and the numerical coefficients are found by substituting the values of the elements in these expressions. It is only applicable when the eccentricities

and mutual inclination are small, and has for that reason fallen, of late, into a certain disrepute. The extended tables published by Le Verrier[1] have, however, added so much to its facility for use that it is not wholly unworthy of attention.

At the other extreme stands the purely mechanical method, in which special values of the disturbing force are computed for many combinations of the mean anomalies of the two planets, and the values of the coefficients in the general expression for the force thence deduced.

Between these two stands what I conceive we may designate as the Cauchy-Hansen method, in which the development is made mechanically with respect to the one planet, but the eccentric anomaly of the other is retained as an undetermined quantity. The germ of this method is found in several papers, by Cauchy, in the earlier volumes of the *Comptes Rendus* of the French Academy, which have since been combined into a homogeneous memoir by Puiseux.[2] The object had in view by these authors is only the computation of inequalities of long period. But Hansen has taken up the essential principle of the method, first, in his prize memoir on the perturbations of comets, crowned by the French Academy of Sciences, about 1848, and afterwards in his "*Auseinandersetzung einer zweckmässigen Methode zur Berechnung der Störungen der kleinen Planeten*,"[3] and applied it to the general development of perturbations.

Among the three methods of integration, the first in point of analytical elegance and generality, but the last in order of convenience in use, is that of the variation of elements, a method with which the name of La Grange is inseparably associated.

In the second the direct integration of the differential equations which express the perturbations of longitude, latitude, and radius vector is effected by special devices.

In the first of these methods the problem is presented in this form: The equations of motion being completely integrated for the action of the principal forces only, how must the arbitrary constants of integration vary in order that the same expressions may represent the motion of the planet under the influence of the disturbing forces? In the second method, the same thing being presupposed, the question is, what expressions must be added to the integrals of undisturbed motion in order that the sum may represent the integrals of the disturbed motion?

The third is Hansen's method, in which the co-ordinates are partly expressed in terms of a certain function of the time known as the disturbed mean anomaly, determined by the condition that the true longitude in the disturbed orbit shall be the same function of the disturbed time that the longitude in the elliptic orbit is of the simple time.

Although the last two methods have a great advantage over the first in the computation of the periodic perturbations, I conceive the first to be best adapted to the computation of the secular variations, and perhaps, of terms of very long period in the mean longitude and the elements of the orbit.

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[1] *Annales de l'Observatoire Impérial de Paris.* Tome I.

[2] *Annales de l'Observatoire Impérial de Paris.* Tome VII.

[3] *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften.* Band V. VI, VII.

In his researches on the motion of Uranus, the first thing done by Le Verrier was to recompute the perturbations by Jupiter and Saturn. It will sufficiently describe his method of doing this to say that in the developments he used the purely mechanical method for the action of Saturn, and the algebraic development of the perturbative function for the action of Jupiter, while in the integration he used the method of the variation of elements. After completing the perturbations of the first order he made the earliest attempt at a complete determination of those of the second order. Beginning with the terms of this order which arise from the secular variations of the elements, he determines them by recomputing the terms of the first order for the epoch 2300, and assuming that the general term will then be given by interpolating between the two terms thus found, supposing them to increase uniformly with the time. This proceeding has the sanction of such high authority that it is worth while to call attention to its want of rigor. The differential coefficient of each element being given in the form

$$\frac{da}{dt} = k \cos bt,$$

k being a function of the elements, the perturbation of the first order will be

$$\delta a = \frac{k}{b} \sin bt.$$

When we take into account the variation of k, and suppose it of the form $k_0 + k't$, the process is equivalent to supposing that in this case

$$\delta a = \frac{k_0 + k't}{b} \sin bt,$$

whereas it really contains the additional term,

$$\frac{k'}{b^2} \cos bt,$$

which appears to be neglected in the process in question. It will be seen that the neglected coefficient is equal to the secular variation of the term during the time that its argument requires to increase by an amount equal to the unit radius. It is therefore the more important the longer the period of the inequality.

To obtain the periodic terms of the second order Le Verrier begins by determining the ten principal terms of the perturbations of the elements of Saturn produced by Jupiter. Next he takes up the terms in the mean longitude of Uranus which depend on the square of the mass of Saturn. The only sensible terms he finds are

$$-1''.17 \sin(\zeta' - 3\zeta) - 0''.35 \cos(\zeta' - 3\zeta)$$
$$+ 0''.43 \sin(\zeta'' - 4\zeta' + 4\zeta) - 0''.21 \cos(\zeta'' - 4\zeta' + 4\zeta),$$

ζ, ζ', and ζ'' being the mean anomalies of Uranus, Saturn, and Jupiter, respectively. The terms depending on the product of the masses of Jupiter and Saturn are then taken up. Fifteen arguments are found the coefficients of which vary from a small fraction of a second to one or two seconds, while a single one of long period amounts to 32″.

When the method of variation of elements is used, it is necessary not only to determine these variations to quantities of the second order, but, in the transforma-

tion of the perturbations of the elements into perturbations of the co-ordinates, to carry this transformation to terms of the second order also. This Le Verrier avoids by showing that the terms of the lowest order with respect to the eccentricities thus introduced are destroyed by certain terms in the perturbations of the elements, so that it is only necessary to omit both classes of terms. These terms are of that fictitious class which disappear of themselves by a simple change of elements. When, instead of the eccentricity and longitude of the perihelion, we take h and k, which represent the products of the eccentricity into the sine and cosine of this longitude respectively, these terms disappear of themselves both from the perturbations of the elements and of the co-ordinates. It is not likely that any of the neglected terms of this class exceed $0''.1$.

As soon as the elements of Neptune were known, the nature of its general action on Uranus became of interest. This subject was taken up by Prof. Peirce, whose results are found in the Proceedings of the American Academy of Arts and Sciences, Vol. I, pp. 334–337. This paper is accompanied with a comparison of his theory of Uranus with observations, to which similar comparisons of the theories of Adams and Le Verrier are added. This comparative exhibit is of sufficient interest to be given here. The numbers given are probably excesses of computed over observed longitudes.

Residual Differences between the Theoretical and Observed Longitudes of Uranus, from the Theories of Peirce, Le Verrier, and Adams.

Year.	From Le Verrier's best orbit of Uranus from the modern observations without any external planet.	From Le Verrier's original theory with his best orbit of hypothetical planet, of which the mass is $\frac{1}{9322}$.	From Adams's original theory with his second hypothetical planet of which mass is $\frac{1}{6666}$.	From Peirce's theory of Neptune adopting for its mass		
				That of Struve from his own observations of the satellite $\frac{1}{14496}$.	That deduced by Peirce from Bond's & Lassel's observations combined $\frac{1}{18780}$.	That deduced by Peirce from Bond's observations of Lassel's satellite $\frac{1}{19840}$.
	$''$	$''$	$''$	$''$	$''$	$''$
1690	+ 289.0	— 19.9	+ 50.0	— 124.7	+ 13.0	+ 0.8
1715	+ 279 6	+ 5.5	— 6.6	— 99.6	+ 10.0	+ 8.7
1756	+ 230 9	— 4.0	— 4.0	— 102.4	— 12.7	+ 4.0
1769	+ 123 3	+ 3.7	+ 1.8	— 67.0	— 16.0	— 6.0
1782	+ 20 5	+ 2 3	0.0	— 18.3	— 5.6	— 3.0
1787	+ 2.0	— 1.2	— 0.2	— 4.7	— 1 2	— 0.5
1792	— 7.8	+ 0 3	— 1.1	+ 1.6	+ 0 5	+ 0.3
1797	— 6.7	— 1.0	— 0.5	+ 3 3	+ 0.8	+ 0.3
1803	— 3.4	+ 0.8	+ 1.6	+ 3.2	+ 1.2	+ 0.8
1808	+ 3.8	+ 0.8	0.0	— 1 3	— 0.6	— 0.4
1813	+ 4.5	— 0 9	— 1.0	— 2.3	+ 1.1	— 0.3
1819	+ 3.8	+ 0.4	— 2 2	+ 0.9	+ 0.7	+ 1.0
1824	— 7.6	— 5 4	+ 1.7	— 1 6	— 1.9	— 2.0
1829	— 7.8	— 2.2	+ 2.0	+ 2 5	+ 1.3	+ 0.8
1835	— 4 5	— 0.8	— 1.2	+ 3.9	+ 2 4	+ 2.0
1840	+ 0.7	+ 2.2	+ 1.3	— 1.3	— 1.3	— 1.1
1845	+ 6.5	— 0.3	.....	— 2.8	— 1 2	— 0.9

In this paper Professor Peirce presents the results of a complete computation of the general perturbations of Uranus by Neptune in longitude and radius vector,

but without any details whatever of the investigation, or any statement of the methods employed. The minuteness of the residuals in the last column of the preceding table shows that employing these perturbations by Neptune, and those of Le Verrier by Jupiter and Saturn, we had a theory of Uranus from which quite accurate tables might have been constructed. But this never seems to have been done. The ephemeris of Uranus in the American Nautical Almanac was intended to be founded on this theory, but the proper definitive elements do not seem to have been adopted in the computations, as the ephemeris does not correspond with the theory.

Although twenty-five years have elapsed since the epoch of these researches, I am not aware of any published work of importance on the theory of Uranus during the interval. Mr. T. H. Safford has, however, made a very extended investigation of the subject, but has published nothing more than a brief general description of his work, which may be found in the Monthly Notices of the Royal Astronomical Society, Vol. 22. Like Professor Peirce, he took Le Verrier's perturbations by Jupiter and Saturn, but, instead of using general perturbations by Neptune, he computed the effect of the action of this planet by mechanical quadratures for the whole period of the observations of Uranus, and thus corrected the elements and the mass of Neptune from modern observations alone. The mass in question deduced was

$$\frac{1}{20039}$$

Mr. Safford does not give the representation of the modern observations, but presents the following comparison of the ancient ones, alongside which we place for comparison the corresponding numbers of Peirce's theory and those of the present investigation.

EXCESS OF OBSERVATION OVER THEORY.

Date.	No. of obs.	Safford.	Peirce.	Newcomb.
1690	1	+ 5″.0	— 0″.8	— 11″
1715	3	— 4.2	— 8.7	— 8
1750	2	— 1 2	.....	+ 2.9
1753	1	— 0.2	.....	
1756	1	— 0.9	— 4.0	
1764	1	+ 0.4		
1769	8	+ 4.5	+ 6.0	— 1.4

CHAPTER I.

METHOD OF DETERMINING THE PERTURBATIONS OF LONGITUDE, RADIUS VECTOR, AND LATITUDE OF A PLANET BY DIRECT INTEGRATION.

LET us conceive a plane determined by the condition that it shall pass through the sun and contain the tangent to the orbit of a planet at any moment. If the planet were acted on by the sun alone, the position of this plane would be invariable, but, under the influence of the disturbing forces of the other planets, it is subject, at each instant, to a motion of rotation around the radius vector of the planet. We may regard this as the instantaneous plane of the planet's orbit. The disturbing and the disturbed planet will each have its own instantaneous plane.

Let us now put:—

v, the longitude of a planet counted from a determinate point in the instantaneous plane of its orbit.

v, its distance from the node of intersection of its own orbit with that of another planet.

γ, the mutual inclination of the two orbits.

σ, $\sin \frac{1}{2} \gamma$.

r, the radius vector of the planet.

ρ, its logarithm.

μ, the attractive force of the sun upon unit of matter at unit distance.

a, the mean distance corresponding to the observed mean motion of the planet, determined by the condition

$$a^3 = \frac{\mu (1 + m)}{n^2},$$

m and n being as usual the mass and mean motion.

a_0, the value of a corrected for the constants introduced by the perturbations, so that, as in the elliptic motion, we have

$$\rho = \log a + f(l, e, \varpi),$$

we shall have in the disturbed motion

$$\rho = \log a_0 + f(l, e, \varpi) + \text{periodic terms only.}$$

a_1, the mean distance of an outer planet, whether it be a disturbing or disturbed planet.

$\mathfrak{v}$, the logarithm of a.

α, the ratio of two mean distances, taken less than unity.

R, the perturbative function.

h, the coefficient of any term of $\frac{a_1}{m'}R$, so that we have

$$R = \Sigma \frac{m'h}{a_1} \cos N$$

m' being here the mass of the disturbing planet.

λ, the mean distance of the planet from the node, or the mean value of v.

ω, the distance of the perihelion from the node.

g, the mean anomaly.

l, the mean longitude, or the mean value of v.

ψ, the angle of eccentricity so that $e = \sin \psi$.

r_0, the radius of the planet in the undisturbed ellipse.

r_1, the quotient of r_0 divided by the mean distance, which is a function of the eccentricity and mean anomaly only.

T, the time after the epoch 1850, Jan. 0, Greenwich mean noon, counted in Julian centuries.

ν, the integrating factors of the periodic terms, or the ratio $\frac{n}{N}$, N being the change of the angle in unit of time.

u, the eccentric anomaly, and, in the tables, the argument of latitude.

We have for the value of R

$$R = \frac{m'}{\sqrt{r^2 - 2rr'(\cos \mathrm{v} \cos \mathrm{v}' + \sin \mathrm{v} \sin \mathrm{v}' \cos \gamma) + r'^2}} - \frac{m'r}{r'^2}(\cos \mathrm{v} \cos \mathrm{v}' + \sin \mathrm{v} \sin \mathrm{v}' \cos \gamma)$$

or, if we suppose r replaced by its value in ρ, namely

$$r = c^{\rho}$$

we shall have

$$R = m'f(\mathrm{v}, \mathrm{v}', \rho, \rho', \gamma).$$

With this value of R it is well known that the differential equations for the longitude and radius vector of a planet are

$$\left.\begin{aligned} r\frac{d^2r}{dt^2} - r^2\frac{dv^2}{dt^2} + \frac{\mu(1+m)}{r} &= \mu\frac{\partial R}{\partial \rho}; \\ r^2\frac{d^2v}{dt^2} + 2r\frac{dr}{dt}\frac{dv}{dt} &= \mu\frac{\partial R}{\partial \mathrm{v}}. \end{aligned}\right\} \quad (1)$$

If we multiply the first of these equations by $2\frac{d\rho}{dt}$ and the second by $2\frac{dv}{dt}$ and add them together, putting, for brevity,

$$\frac{\partial R}{\partial \rho}\frac{d\rho}{dt} + \frac{\partial R}{\partial \mathrm{v}}\frac{dv}{dt} = D'_t R, \quad (2)$$

and then integrate, we shall have

$$\frac{dr^2}{dt^2} + r^2\frac{dv^2}{dt^2} - \frac{2\mu(1+m)}{r} = 2\mu\left(C + \int D'_t R dt\right)$$

C being the arbitrary constant added to the integral. Adding this equation to the first of equations (1) we have

$$\frac{1}{2}\frac{d^2(r^2)}{dt^2}-\frac{\mu(1+m)}{r}=\mu\left(2C+2\int D'_t Rdt+\frac{\partial R}{\partial\rho}\right) \qquad (3)$$

Let us now represent by r_0 that elliptic value of r which satisfies the equation

$$\frac{1}{2}\frac{d^2(r_0^2)}{dt^2}-\frac{\mu(1+m)}{r_0}=2\mu C.$$

Subtracting this equation from the last we have

$$\frac{1}{2}\frac{d^2(r^2-r_0^2)}{dt^2}-\mu(1+m)\left(\frac{1}{r}-\frac{1}{r_0}\right)=\mu\left(2\int D'_t R\,dt+\frac{\partial R}{\partial\rho}\right).$$

in which no constant is to be added to the integral, and both sides of the equation are of the order of the disturbing forces. As there is a decided advantage in taking the logarithm of the radius vector as the variable instead of r itself, we substitute for the latter its value

$$r=c^\rho,\quad r_0=c^{\rho_0}$$

and put

$$\delta\rho=\rho-\rho_0.$$

Then

$$r^2=c^{2\rho_0+2\delta\rho}=r_0^2\,c^{2\delta\rho}=r_0^2\left(1+2\delta\rho+\frac{2^2}{1.2}\delta\rho^2+\text{etc.}\right)$$

$$\tfrac{1}{2}(r^2-r_0^2)=r_0^2\delta\rho+r_0^2\delta\rho^2+\text{etc.}$$

$$\frac{1}{r}-\frac{1}{r_0}=-\frac{\delta\rho}{r_0}+\frac{\delta\rho^2}{2r_0}+\text{etc.}$$

Substituting these values in the above equation, carrying the development only to terms of the second order, and transposing those terms to the right hand side of the equation, and putting $\mu'=\mu(1+m)$, we find

$$\frac{d^2(r_0^2\delta\rho)}{dt^2}+\frac{\mu'}{r_0^3}(r_0^2\delta\rho)=\mu\left(2\int D'_t Rdt+\frac{\partial R}{\partial\rho}\right)-\frac{d^2(r_0^2\delta\rho^2)}{dt^2}+\frac{\mu\delta\rho^2}{2r_0}, \qquad (4)$$

an equation which gives the perturbations of radius vector.

The general mode of solving this equation by successive approximation is familiar. The principles on which the successive approximations are made being the same, we shall begin by assuming that we have obtained first approximations to the values of δv, $\delta v'$, $\delta\rho$, $\delta\rho'$, $\delta\gamma$, and that from these we wish to pass to a second approximation. We must first carry this approximation into the functions of R in the second member of (4). To effect this we must show how, from the development of R in terms of the elements and the time, we may form its successive derivatives with respect to the quantities which enter into it. R, while originally a function of v, v', ρ, ρ', and γ, is, in its developed form, a function of λ, λ', ω, ω', e, e', ϖ, ϖ' and γ, the development being effected by substituting for the first set of quantities their values in terms of the second. The substitution is as follows:

$$\begin{aligned} v&=\lambda+Fg,\\ v'&=\lambda'+Fg',\\ \rho&=\varpi+\phi g,\\ \rho'&=\varpi'+\phi g', \end{aligned} \qquad (5)$$

Fg being the equation of the centre, and ϕg the part of ρ depending on the eccentricity in the elliptic motion. It follows that if we express the developed expression for R as a function of λ, λ', g, g', ϖ, ϖ', which we may do by putting

$$\omega = \lambda - g, \quad \omega' = \lambda' - g';$$
$$a = c^{\varpi}, \qquad a' = c^{\varpi'};$$

we shall have by successive differentiation

$$\begin{aligned}
\frac{\partial R}{\partial \lambda} &= \frac{\partial R}{\partial \mathrm{v}} \frac{\partial \mathrm{v}}{\partial \lambda} = \frac{\partial R}{\partial \mathrm{v}} \\
\frac{\partial^2 R}{\partial \lambda^2} &= \frac{\partial^2 R}{\partial \mathrm{v}^2} \frac{\partial \mathrm{v}}{\partial \lambda} = \frac{\partial^2 R}{\partial \mathrm{v}^2} \\
\frac{\partial R}{\partial \varpi} &= \frac{\partial R}{\partial \rho} \frac{\partial \rho}{\partial \varpi} = \frac{\partial R}{\partial \rho} \\
\frac{\partial^2 R}{\partial \varpi^2} &= \frac{\partial^2 R}{\partial \rho^2} \frac{\partial \rho}{\partial \varpi} = \frac{\partial^2 R}{\partial \rho^2} \\
&\text{etc.} \quad \text{etc.} \quad \text{etc.}
\end{aligned} \tag{6}$$

and in general

$$\frac{\partial^{m+n+m'+n'} R}{\partial \lambda^m \, \partial \varpi^n \, \partial \lambda'^{m'} \, \partial \varpi'^{n'}} = \frac{\partial^{m+n+m'+n'} R}{\partial \mathrm{v}^m \, \partial \rho^n \, \partial \mathrm{v}'^{m'} \, \partial \rho'^{n'}}.$$

Thus, by expressing the developed R in the above form, we may find the derivative of any order with respect to v, v$'$, ρ and ρ', by taking the corresponding derivative with respect to λ, λ', ϖ and ϖ'.

The developed R is usually expressed in the form

$$R = \Sigma \frac{m'h}{a_1} \cos (i'\lambda' + i\lambda + j'\omega' + j\omega)$$

a_1 being the mean distance of the outer planet, whether disturbing or disturbed, and h a function of e, e', α, and γ. Substituting for ω its value in g, this equation will become

$$R = \Sigma \frac{m'h}{a_1} \cos \Big((i' + j') \, \lambda' + (i + j) \, \lambda - j'g' - jg \Big).$$

Putting for brevity

$$N = i'\lambda' + i\lambda + j'\omega' + j\omega,$$

the formulæ (6) give

$$\begin{aligned}
\frac{\partial R}{\partial \mathrm{v}} &= -\Sigma \frac{m'h}{a_1} (i + j) \sin N \\
\frac{\partial^2 R}{\partial \mathrm{v}^2} &= -\Sigma \frac{m'h}{a_1} (i + j)^2 \cos N \\
\frac{\partial R}{\partial \rho} &= \Sigma \, m' \frac{\partial \frac{h}{a_1}}{\partial \varpi} \cos N
\end{aligned} \tag{7}$$

and in general

$$\frac{\partial^{u+u'+n+n'} R}{\partial \mathrm{v}^u \, \partial \mathrm{v}'^{u'} \, \partial \rho^n \, \partial \rho'^{n'}} = \Sigma \pm m' (i + j)^u (i' + j')^{u'} \frac{\partial^{n+n'} \frac{h}{a_1}}{\partial \varpi^n \, \partial \varpi'^{n'}} \begin{matrix} \cos \\ \sin \end{matrix} N$$

The formation of the derivatives in the second member of this equation demands attention. In the analytic development of the perturbative function each value of h is composed of a series of terms each of the form

$$E \times A,$$

E being a function of the eccentricities and mutual inclination, and A a function of α of the form

$$(0)\,\alpha^{s-\frac{1}{2}} b_s^{(i)} + (1)\,\alpha^{s+\frac{1}{2}} \frac{\partial b_s^{(i)}}{\partial \alpha} + (2)\,\alpha^{s+\frac{3}{2}} \frac{\partial^2 b_s^{(i)}}{\partial \alpha^2} + \text{etc.} + \alpha^{\frac{2s+2n-1}{2}} \frac{\partial^n b_s^{(i)}}{\partial \alpha^n}, \qquad (8)$$

(0), (1), etc., being numerical coefficients connected with the coefficients $V^{(i)}$ tabulated by Le Verrier, in Tome I of his Annales de l'Observatoire, by the relation

$$(n) = \frac{V_n^{(i)}}{1.2.3\ldots n},$$

and $b_s^{(i)}$ being, as usual, the coefficient of $\cos i\phi$ in the development of

$$(1 - 2\alpha \cos \phi + \alpha^2)^{-s}$$

in multiples of $\cos \phi$, and $n - 1$ the sum of the exponents of the eccentricities in E.

It would have been much more convenient if in effecting this development the derivatives of $b_s^{(i)}$ had been taken with respect to $\mathfrak{v}$ instead of α. In fact the derivative $\frac{\partial^n b_s^{(i)}}{\partial \alpha^n}$ when expressed in terms of the derivatives with respect to $\mathfrak{v}$ is of the form

$$\alpha^n \frac{\partial^n b_s^{(i)}}{\partial \alpha^n} = n_1 \frac{\partial b_s^{(i)}}{\partial \mathfrak{v}} + n_2 \frac{\partial^2 b_s^{(i)}}{\partial \mathfrak{v}^2} + \text{etc.} + n_n \frac{\partial^n b_s^{(i)}}{\partial \mathfrak{v}^n}$$

Therefore, when expressed in terms of the derivatives with respect to $\mathfrak{v}$, A will be of the form

$$\alpha^{s-\frac{1}{2}} \Big((0)'\, b_s^{(i)} + (1)' \frac{\partial b_s^{(i)}}{\partial \mathfrak{v}} + (2)' \frac{\partial^2 b_s^{(i)}}{\partial \mathfrak{v}^2} + \text{etc.} \Big),$$

from which the derivatives $\frac{\partial A}{\partial \mathfrak{v}}, \frac{\partial^2 A}{\partial \mathfrak{v}^2}$, etc., may be found with great facility.

As in the actual developments of R which we possess, the values of A are given in the form (8), we must find the expression for the first two derivatives of its several terms with respect to $\mathfrak{v}$, which we easily do by the application of the symbolic formulæ

$$D_{\mathfrak{v}} = \alpha D_\alpha$$
$$D^2_{\mathfrak{v}} = \alpha (D_\alpha + \alpha D^2_\alpha).$$

Beginning with the case of $s = \frac{1}{2}$, we have

$$\frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \mathfrak{v}} = \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha}$$

$$\frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \mathfrak{v}^2} = \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + \alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2}$$

$$\frac{\partial\left(\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial\alpha^n}\right)}{\partial v} = n\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial\alpha^n} + \alpha^{n+1}\frac{\partial^{n+1} b_{\frac{1}{2}}^{(i)}}{\partial\alpha^{n+1}}$$

$$\frac{\partial^2\left(\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial\alpha^n}\right)}{\partial v^2} = n^2\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial\alpha^n} + (2n+1)\,\alpha^{n+1}\frac{\partial^{n+1} b_{\frac{1}{2}}^{(i)}}{\partial\alpha^{n+1}} + \alpha^{n+2}\frac{\partial^{n+2} b_{\frac{1}{2}}^{(i)}}{\partial\alpha^{n+2}}$$

$$= n\,\frac{\partial\left(\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial\alpha^n}\right)}{\partial v} + \frac{\partial\left(\alpha^{n+1}\frac{\partial^{n+1} b_{\frac{1}{2}}^{(i)}}{\partial\alpha^{n+1}}\right)}{\partial v}$$

consequently we have for the derivatives of A from formulæ (8)

$$\frac{\partial A}{\partial v} = (0)\,\alpha\frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial\alpha} + (1)\left(\alpha\frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial\alpha} + \alpha^2\frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial\alpha^2}\right) + (2)\left(2\alpha^2\frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial\alpha^2} + \alpha^3\frac{\partial^3 b_{\frac{1}{2}}^{(i)}}{\partial\alpha^3}\right) + \text{etc.}$$

$$\frac{\partial^2 A}{\partial v^2} = (0)\left(\alpha\frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial\alpha} + \alpha^2\frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial\alpha^2}\right) + (1)\left(\alpha\frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial\alpha} + 3\alpha^2\frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial\alpha^2} + \alpha^3\frac{\partial^3 b_{\frac{1}{2}}^{(i)}}{\partial\alpha^3}\right) + \text{etc.} \tag{9}$$

The derivatives of A being formed in this way, those of h are immediately deduced from the equations

$$\frac{\partial h}{\partial v} = \Sigma E\,\frac{\partial A}{\partial v},$$

$$\frac{\partial^2 h}{\partial v^2} = \Sigma E\,\frac{\partial^2 A}{\partial v^2}.$$

When s is equal to $\frac{3}{2}$, A is of the form

$$\alpha\left\{(0')\,b_{\frac{3}{2}}^{(i)} + (1)'\,\alpha\frac{\partial b_{\frac{3}{2}}^{(i)}}{\partial\alpha} + (2)'\,\alpha^3\frac{\partial b_{\frac{3}{2}}^{(i)}}{\partial\alpha^2} + \text{etc.}\right\}$$

The quantity within parentheses is of the same form with A, in the case of $s=\frac{1}{2}$. If we represent it by A' we shall have

$$\frac{\partial A}{\partial v} = \alpha\left(\frac{\partial A'}{\partial v} + A'\right) = \alpha\frac{\partial A'}{\partial v} + A$$

$$\frac{\partial^2 A}{\partial v^3} = A + 2\alpha\frac{\partial A'}{\partial v} + \alpha\frac{\partial^2 A'}{\partial v^2}$$

A' being the same form with A, the derivatives $\frac{\partial A'}{\partial v}$ and $\frac{\partial^2 A'}{\partial v^2}$ will be of the form (9), substituting $\frac{3}{2}$ for the index $\frac{1}{2}$, and $(0)'$, $(1)'$, etc., for (0), (1), etc.

In the case of $s=\frac{5}{2}$ the derivatives are obtained in the same way, which is too simple to need elucidation.

We have now to pass from the derivatives of h to those of $\frac{mh}{a_1}$, the coefficients of the perturbative function. The form of these derivatives will depend not on whether the planet is disturbing or disturbed, but on whether it is an outer or

inner one. Let us then suppose for the present, that a and $\mathfrak{v}$ refer to the inner planet, and put $\mathfrak{v}_1$ for the logarithm of the mean distance of the outer one. We then have for the derivatives relatively to $\mathfrak{v}$

$$\frac{\partial^n \dfrac{h}{a_1}}{\partial \mathfrak{v}^n} = \frac{1}{a_1}\frac{\partial^n h}{\partial \mathfrak{v}^n},$$

and for the first derivative relatively to $\mathfrak{v}$, using the symbolic notation,

$$\frac{\partial \dfrac{h}{a_1}}{\partial \mathfrak{v}_1} = \frac{1}{a_1}(D_{a_1} - 1)\, h.$$

The symbols in the second member being distributive, we have by successive differentiation

$$\frac{\partial^n \dfrac{h}{a_1}}{\partial \mathfrak{v}_1^{\,n}} = \frac{1}{a_1}(D_{\mathfrak{v}_1} - 1)^n h.$$

The quantity h is a function of α, the ratio of the mean distances or of $C^{\mathfrak{v} - \mathfrak{v}_1}$, C being the neperian base. Hence

$$D_{\mathfrak{v}_1} h = - D_{\mathfrak{v}} h,$$

which substituted in the last equation gives

$$\frac{\partial^n \dfrac{h}{a_1}}{\partial \mathfrak{v}_1^{\,n}} = \frac{(-1)^n}{a_1}(D_{\mathfrak{v}} + 1)^n h. \tag{10}$$

This formula gives for the first two derivatives

$$\frac{\partial \dfrac{h}{a_1}}{\partial \mathfrak{v}_1} = -\frac{1}{a_1}\left(h + \frac{\partial h}{\partial \mathfrak{v}}\right)$$

$$\frac{\partial^2 \dfrac{h}{a_1}}{\partial \mathfrak{v}_1^{\,2}} = \frac{1}{a_1}\left(h + 2\frac{\partial h}{\partial \mathfrak{v}} + \frac{\partial^2 h}{\partial \mathfrak{v}^2}\right).$$

Substituting in the general formulæ (7) these expressions for the derivatives relatively to $\mathfrak{v}$ and $\mathfrak{v}_1$ we have expressions for the derivatives of R relatively to v, v′, ρ, ρ', it being understood, however, that all the quantities are expressed in functions of the elements of elliptic motion.

In order to compute the perturbations of the second order we must carry R and such of its derivatives as enter into the differential equations (1) to quantities of the first order with respect to the perturbations. Let us then represent by v_0, v_0', ρ_0, ρ'_0, γ_0, the elliptic values of v, v′, ρ, ρ', and γ, which we have assumed in the first approximation to the perturbations, and by δv, $\delta v'$, etc., the quantities to be

added to v_0, v_0', etc., to make the true values of v, v', etc., whether perturbations or corrections of the elements. We shall then have

$$\begin{aligned}
\delta R &= \frac{\partial R_0}{\partial v_0}\delta v + \frac{\partial R_0}{\partial v'_0}\delta v' + \frac{\partial R_0}{\partial \rho_0}\delta\rho + \frac{\partial R_0}{\partial \rho'_0}\delta\rho' + \frac{\partial R_0}{\partial \gamma_0}\delta\gamma \\
\delta\frac{\partial R}{\partial v} &= \frac{\partial^2 R_0}{\partial v_0^2}\delta v + \frac{\partial^2 R_0}{\partial v_0 \partial v'_0}\delta v' + \frac{\partial^2 R_0}{\partial v_0 \partial \rho_0}\delta\rho + \frac{\partial^2 R_0}{\partial v_0 \partial \rho'_0}\delta\rho' + \frac{\partial^2 R_0}{\partial v_0 \partial \gamma_0}\delta\gamma \qquad (11) \\
\delta\frac{\partial R}{\partial \rho} &= \frac{\partial^2 R_0}{\partial \rho_0 \partial v_0}\delta v + \frac{\partial^2 R_0}{\partial \rho_0 \partial v'_0}\delta v' + \frac{\partial^2 R_0}{\partial \rho^2_0}\delta\rho + \frac{\partial^2 R_0}{\partial \rho_0 \partial \rho'_0}\delta\rho' + \frac{\partial^2 R_0}{\partial \rho_0 \delta \gamma_0}\delta\gamma
\end{aligned}$$

The value of $D'_t R$ may be found either by equation (2), or by differentiating with respect to the time as introduced by the co-ordinates of the disturbed planet. When quantities of the first order only are considered the latter operation is very simple, but it is different when terms of the second order come in, because the true longitude of the planet is then expressed in terms not only of its own mean longitude, but also of the mean longitude of all the disturbing planets. The result can still be obtained in the same way by separating all the mean longitudes introduced by the co-ordinates of the disturbed planet from those introduced by the co-ordinates of the other until after the differentiation relatively to t'.

Let us now resume the equation (4), representing its second member by μQ, so that it becomes

$$\frac{d^2(r_0^2\delta\rho)}{dt^2} + \frac{\mu(1+m)}{r_0^3}r_0^2\delta\rho = \mu Q \qquad (12)$$

where

$$Q = 2\int D'_t R dt + \frac{\partial R}{\partial \rho} - \frac{1}{\mu}\frac{d^2(r_0^2\delta\rho^2)}{dt^2} + \tfrac{1}{2}\frac{\delta\rho^2}{r_0}$$

By the operations already given Q has become a known function of the time.

It is well known that the integration of (12) may be effected by finding two values of $r_0^2\delta\rho$ which satisfy this equation when the second member is neglected, or, in other words, by finding two variables x and y which satisfy the equations

$$\frac{d^2x}{dt^2} + \frac{\mu(1+m)}{r_0^3}x = 0,$$

$$\frac{d^2y}{dt^2} + \frac{\mu(1+m)}{r_0^3}y = 0,$$

when the required integral is

$$r_0^2\delta\rho = \frac{\mu}{x\dfrac{dy}{dt} - y\dfrac{dx}{dt}}\left\{ y\int xQdt - x\int yQdt \right\}.$$

The above differential equations are satisfied by the rectangular co-ordinates of the planet in its assumed elliptic orbit. The position of the axes of co-ordinates being arbitrary we shall take the line of apsides for the axis of X, the perihelion being on the positive side. If we put

$$e_0 = \sin\psi,$$

we have

$$x\frac{dy}{dt} - y\frac{dx}{dt} = \sqrt{a\mu(1+m)}\cos\psi = \frac{\mu(1+m)\cos\psi}{an}$$

Let us, for convenience, replace x and y by two other variables ξ and η connected with them by the equations

$$x = a\xi,$$
$$y = a\eta \cos\psi.$$

ξ and η are then functions of the eccentricity and mean anomaly only, and may be developed according to the multiples of the latter. Substituting the last three expressions in the preceding value of $r_0^2\delta\rho$ it becomes

$$r_0^2\delta\rho = \frac{a^3n}{1+m}\left\{ \eta \int \xi Q dt - \xi \int \eta Q dt \right\}.$$

If we put r_1^2 for the value of r_0 when the mean distance of the planet is put equal to unity, so that r_1, like ξ and η contains only the eccentricity and mean anomaly, we shall have

$$\delta\rho = \frac{nr_1^{-2}}{1+m}\left\{ \eta \int \xi a Q dt - \xi \int \eta a Q dt \right\} \qquad (13)$$

We must now express ξ and η in terms of the time, or of the mean anomaly. Putting for the present u for the eccentric and v for the true anomaly, we have, by the theory of the elliptic motion,

$$x = r\cos v = a(\cos u - e),$$
$$y = r\sin v = a\cos\psi \sin u,$$

from which follow

$$\xi = \cos u - e,$$
$$\eta = \sin u.$$

As ξ and η are to be expressed in the form

$$\xi = \tfrac{1}{2}\Sigma p_i \cos ig,$$
$$\eta = \tfrac{1}{2}\Sigma q_i \sin ig,$$

the finite integrals extending to all values of i from $-\infty$ to $+\infty$, we shall deduce general expressions from p_i and q_i arranged according to the power of the eccentricity. Since

$$u = g + e\sin u,$$

we have by Lagrange's theorem

$$\cos u = \cos g - e\sin^2 g - \frac{e^2}{2!}\frac{\partial \sin^3 g}{\partial g} - \frac{e^3}{3!}\frac{\partial^2 \sin^4 g}{\partial g^2} - \text{etc.};$$

or

$$\cos u = -\sum_{n=0}^{n=\infty} \frac{e^n}{n!}\frac{\partial^{n-1}\sin^{n+1} g}{\partial g^{n-1}}$$

using the notation

$$n! = 1.2.3 \ldots\ldots n = \Gamma(n+1).$$

We then have

$$0! = 1! = 1.$$

Substituting in the general term of the above series for sin g its value in imaginary exponential functions

$$2 \sin g = \sqrt{-1}\,(c^{-g\sqrt{-1}} - c^{g\sqrt{-1}})$$

we find by the binomial theorem, using the notation of combinations,

$$\overset{s}{\underset{n}{C}} = \frac{n\,(n-1)\ldots\ldots(n-s+1)}{1.2.3\ldots s} = \frac{n!}{s!\,(n-s)!}$$

$$2^{n+1}\sin^{n+1} g = (\sqrt{-1})^{n+1}\left\{c^{-(n+1)g\sqrt{-1}} - \overset{1}{\underset{n+1}{C}} c^{-(n-1)g\sqrt{-1}} + \overset{2}{\underset{n+1}{C}} c^{-(n-3)g\sqrt{-1}} - \ldots\right.$$

$$\left. + (-1)^n \overset{1}{\underset{n+1}{C}} c^{(n-1)g\sqrt{-1}} + (-1)^{n+1} c^{(n+1)g\sqrt{-1}}\right\}$$

Differentiating $n-1$ times with respect to g, and putting together the first and last terms, the one after the first, and that before the last, and so on, we find

$$-2^{n+1}\frac{\partial^{n-1}\sin^{n+1}g}{\partial g^{n-1}} = (n+1)^{n-1}(c^{(n+1)g\sqrt{-1}} + c^{-(n+1)g\sqrt{-1}})$$

$$- \overset{1}{\underset{n+1}{C}}(n-1)^{n-1}(c^{(n-1)g\sqrt{-1}} + c^{-(n-1)g\sqrt{-1}}) + \text{etc.}$$

Substituting for the exponentials their values in circular functions, and dividing by 2^{n+1} we have

$$\frac{\partial^{n-1}\sin^{n+1}g}{\partial g^{n-1}} = -\frac{1}{2^n}\left\{(n+1)^{n-1}\cos(n+1)g - \overset{1}{\underset{n+1}{C}}(n-1)^{n-1}\cos(n-1)g\right.$$

$$\left. + \overset{2}{\underset{n+1}{C}}(n-3)^{n-1}\cos(n-3)g - \text{etc.}\right\}$$

the series terminating at the last positive coefficient of g. Substituting this last value in the general term of the series which gives cos u, we have

$$\cos u = \sum_{n=0}^{n=\infty}\frac{e^n}{n!\,2^n}\left\{(n+1)^{n-1}\cos(n+1)g - \overset{1}{\underset{n+1}{C}}(n-1)^{n-1}\cos(n-1)g + \text{etc.}\right\}$$

Let us now substitute for n another variable i, putting in the first term of the last factor $i = n+1$, in the second $i = n-1$, in the third $i = n-3$, etc. The limits of finite integration with respect to i will then be

in the first term,	$+1$ to $+\infty$,
in the second term,	-1 to $+\infty$,
in the third term,	-3 to $+\infty$,
etc.	etc.

But all the coefficients of g will then be i, and the formula supposes the factor of cos $i\,g$ to vanish whenever i is zero or negative; whence, those elements of the finite integral in which i is negative must be omitted, and all the terms must be taken between the limits $+1$ and $+\infty$. Making the proposed substitution we have

$$\cos u = \sum_{i=1}^{i=\infty} \left\{ \frac{i^{i-2}}{(i-1)!\,2^{i-1}} e^{i-1} - \frac{i^i}{(i+1)!\,2^{i+1}} \overset{1}{\underset{i+2}{C}} e^{i+1} + \frac{i^{i+2}}{(i+3)!\,2^{i+3}} \overset{2}{\underset{i+4}{C}} e^{i+3} - \text{etc.} \right\} \cos ig$$

$$= \sum_{i=1}^{i=\infty} \frac{i^{i-2} e^{i-1}}{(i-1)!\,2^{i-1}} \left\{ 1 - \frac{i^2 e^2}{2^2\, i(i+1)} \overset{1}{\underset{i+2}{C}} + \frac{i^4 e^4}{2^4 i(i+1)(i+2)(i+3)} \overset{2}{\underset{i+4}{C}} - \text{etc.} \right\} \cos ig$$

We have, therefore, for all values of i different from zero

$$p_i = p_{-i} = \frac{i^{i-2} e^{i-1}}{(i-1)!\,2^{i-1}} \left\{ 1 - \overset{1}{\underset{i+2}{C}} \frac{i\, e^2}{2^2 (i+1)} + \overset{2}{\underset{i+4}{C}} \frac{i^3 e^4}{2^4 (i+1)(i+2)(i+3)} - \text{etc.} \right\} \quad (14)$$

To obtain the value of p_0 we remark that the only constant term in $\cos u$ arises from the term $-e \sin^2 g$; its value is therefore $-\frac{1}{2} e$. The constant term in $\xi = \cos u - e$ is therefore $\frac{3}{2} e$, whence

$$p_0 = -3e. \quad (15)$$

The values of q_i may be obtained in a similar way by developing $\sin u$ by La Grange's theorem. But the development is rather more complex, and it is easier to derive them from p_i. Let us take up the equations

$$\xi = \cos u - e$$
$$\eta = \sin u$$
$$u - e \sin u = g$$

Considering u, like ξ and η, as a function of the independent variables e and g, we have by differentiation

$$\frac{\partial u}{\partial e} - \frac{\partial (e \sin u)}{\partial e} = 0$$

$$\therefore \frac{\partial u}{\partial e} = \frac{\partial (e\eta)}{\partial e} = \frac{\sin u}{1 - e \cos u} \quad (a)$$

$$\frac{\partial u}{\partial g} = \frac{1}{1 - e \cos u}$$

$$\therefore \frac{\partial u}{\partial e} = \sin u \frac{\partial u}{\partial g} = -\frac{\partial \xi}{\partial u}\frac{\partial u}{\partial g} = -\frac{\partial \xi}{\partial g} \quad (b)$$

Comparing (a) and (b)

$$\frac{\partial \xi}{\partial g} = -\frac{\partial (e\eta)}{\partial e}$$

Putting in this equation for ξ and η their developed values this equation becomes

$$\Sigma\, i p_i \sin ig = \Sigma \frac{\partial (e q_i)}{\partial e} \sin ig$$

which gives by equating the coefficients of $\sin ig$

$$q_i = \frac{i}{e} \int p_i\, de. \quad (16)$$

The following are special values of p_i and q_i, developed to the sixth power of the eccentricities, as derived from the preceding formulæ:

$$p_0 = -3e$$
$$p_1 = 1 - \frac{3}{8}e^2 + \frac{5}{192}e^4 - \frac{7}{9216}e^6$$
$$p_2 = \frac{1}{2}e - \frac{1}{3}e^3 + \frac{1}{16}e^5$$
$$p_3 = \frac{3}{8}e^2 - \frac{45}{128}e^4 + \frac{567}{5120}e^6$$
$$p_4 = \frac{1}{3}e^3 - \frac{6}{15}e^5$$
$$p_5 = \frac{125}{384}e^4 - \frac{4375}{9216}e^6$$
$$p_6 = \frac{27}{80}e^5$$
$$p_7 = \frac{16807}{46080}e^6$$
$$(16)'$$
$$q_1 = 1 - \frac{1}{8}e^2 + \frac{1}{192}e^4 - \frac{1}{9216}e^6$$
$$q_2 = \frac{1}{2}e - \frac{1}{6}e^3 + \frac{1}{48}e^5$$
$$q_3 = \frac{3}{8}e^2 - \frac{27}{128}e^4 + \frac{243}{5120}e^6$$
$$q_4 = \frac{1}{3}e^3 - \frac{4}{15}e^5$$
$$q_5 = \frac{125}{384}e^4 - \frac{3125}{9216}e^6$$
$$q_6 = \frac{27}{80}e^5$$
$$q_7 = \frac{16807}{46080}e^6$$

Having the developed ξ and η in terms of time, let us resume the equation (13). As only purely linear operations are performed on Q in this equation, it follows that if we represent its several parts by Q_1, Q_2, etc., and by $\delta\rho_1$, $\delta\rho_2$, etc., the values $\delta\rho$ obtained by putting $Q = Q_1$, $Q = Q_2$, etc., we shall have

$$\delta\rho = \delta\rho_1 + \delta\rho_2 + \text{etc.}$$

We have, therefore, only to find the separate values of $r_0^2\delta\rho$ corresponding to the different terms of Q, and to take their sum. Let us then represent, as before, by

$$\frac{m'}{a_1} h \cos(i'\lambda' + i\lambda + j'\omega' + j\omega)$$

any one term of R.

We then have, considering only terms of the first order with respect to the disturbing forces,

$$D'_t R = -\frac{m'ihn}{a_1} \sin N, \tag{17}$$

$$\int D'_t R = \frac{m'ih\nu}{a_1} \cos N;$$

$$\frac{\partial R}{\partial \rho} = m' \frac{\partial \frac{h}{a_1}}{\partial v} \cos N;$$

where we put for brevity,

$$\nu = \frac{n}{i'n' + in}$$

$$N = i'\lambda' + i\lambda + j'\omega' + j\omega.$$

Let us represent by Q_0 the terms in Q which are of the first order with respect to the disturbing forces, so that we have

$$Q_0 = 2\int D'_t R_0 + \frac{\partial R_0}{\partial \rho_0}.$$

The general term in R will then give rise in Q_0 to the term

$$m' \left[\frac{2ih\nu}{a_1} + \frac{\partial \frac{h}{a_1}}{\partial v} \right] \cos N.$$

In the case of the action of an outer on an inner planet this expression becomes

$$\frac{m'}{a_1}\left(2i\nu h + \frac{\partial h}{\partial v}\right) \cos N;$$

while in the contrary case it is

$$\frac{m'}{a_1}\left(2i\nu h - h - \frac{\partial h}{\partial v'}\right) \cos N,$$

both derivatives being taken with respect to the logarithm of the mean distance of the inner planet.

In the integration it will be more convenient to substitute for λ' and λ the mean longitudes counted from the perihelion of the disturbed planet. If we put

$$\lambda = g + \omega$$

$$\lambda' = l' + \omega$$

the angle N will become,

$$i'l' + ig + j'\omega' + (i + i' + j)\,\omega.$$

Since corresponding to each set of values of i' and i there are several values of j' and j, it will be convenient in the numerical computation to combine these different terms into a single one, because after forming the derivatives of R there is no need that ω, ω' and the other elements should appear in an analytical form. If we put

k for the coefficient of $\frac{m'}{a_1}$ cos N in the preceding general term of Q_0, this term will become

$$Q_0 = \frac{m'}{a_1} k \cos [j'\omega' + (i' + i + j)\omega] \cos [i'l' + ig]$$
$$- \frac{m'}{a_1} k \sin [j'\omega' + (i' + i + j)\omega] \sin [i'l' + ig]$$

If we put

$$k_c = \Sigma\, k \cos [j'\omega' + (i' + i + j)\omega],$$
$$k_s = \Sigma\, k \sin [j'\omega' + (i' + i + j)\omega],$$

the sign Σ being extended so as to include all values of j and j' which correspond to the given values of i and i', we shall have for the general terms of Q_0

$$\frac{m'}{a_1} \left\{ k_c \cos (i'l' + ig) + k_s \sin (i'l' + ig) \right\},$$

or, when we represent the angle $i'l' + ig$ by N_1

$$Q_0 = \frac{m'}{a_1} \left\{ k_c \cos N_1 + k_s \sin N_1 \right\}.$$

This we are to combine with the values of ξ and η

$$\xi = \tfrac{1}{2} \Sigma\, p_i \cos ig,^*$$
$$\eta = \tfrac{1}{2} \Sigma\, q_j \sin jg,$$

in the general integral formula (13). If we substitute them in this formula, and represent by μ the coefficient of t in the value of N we shall have to integrate differentials of the form

$$\begin{matrix}\sin\\ \cos\end{matrix} \left(N_1 \pm ig\right).$$

in which the coefficient of the time t in the angle is $\mu + in$. Let us represent by ν_i the integrating factor

$$\frac{n}{\mu + in}.$$

The formula (13) will become by these substitutions, which, though a little complex, offer no difficulty,

$$\delta\rho = \frac{1}{16} \frac{m' a r_1^{-2}}{a_1 (1+m)} \sum_{-\infty}^{+\infty}{}^{2}_{i,j}\; p_i q_j \times$$
$$\left\{ \begin{aligned} &\{\nu_{+j} - \nu_{+i}\} \{k_c \cos [N_1 + (i+j)g] + k_s \sin [N_1 + (i+j)g]\} \\ +&\{\nu_{+i} - \nu_{-j}\} \{k_c \cos [N_1 + (i-j)g] + k_s \sin [N_1 + (i-j)g]\} \\ +&\{\nu_{+j} - \nu_{-i}\} \{k_c \cos [N_1 - (i-j)g] + k_s \sin [N_1 - (i-j)g]\} \\ +&\{\nu_{-i} - \nu_{-j}\} \{k_c \cos [N_1 - (i+j)g] + k_s \sin [N_1 - (i+j)g]\} \end{aligned} \right\}$$

The sign Σ of finite integration here includes the separate combination of every value of i with every value of j, except those combinations which make the

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* The indices i and j, in these equations, are not to be confounded with the coefficients of λ and ω in the general terms of R and Q. We need not use the latter at present.

coefficient of the time under the sign *sin* or *cos* vanish, and so render the corresponding value of ν infinite. These cases have to be treated separately.

To find, from the expression, the coefficient of the sine or cosine of cosine $N_1 + ug$ in $r_1^2\delta\rho$, we put, in the four lines of this equation, as follows:

$$\begin{array}{lll} \text{In the first,} & i+j=u \therefore j= & u-i; \\ \text{" second,} & i-j=u \therefore j= & i-u; \\ \text{" third,} & -i+j=u \therefore j= & u+i; \\ \text{" fourth,} & -i-j=u \therefore j= & -u-i. \end{array}$$

In the above expressions i and j being independent, and including all values from $-\infty$ to $+\infty$, i and u will also be independent, and include the same range of values. Substituting for j its value in u the coefficient of

$$\frac{1}{16}\frac{m'ar_1^{-2}}{a_1(1+m)}[k_c \cos(N_1+ug)+k_s \sin(N_1+ug)]$$

becomes

$$\Sigma\left\{\begin{array}{l} p_i\, q_{(u-i)}(\nu_{(u-i)}-\nu_i \quad) \\ +p_i\, q_{(i-u)}(\nu_i \quad -\nu_{(u-i)}) \\ +p_i\, q_{(u+i)}(\nu_{(u+i)}-\nu_{-i} \quad) \\ +p_i\, q_{-(u+i)}(\nu_{-i}-\nu_{(u+i)}). \end{array}\right.$$

Since $q_j = -q_{-j}$ this expression reduces immediately to

$$2\Sigma\left\{\begin{array}{l} p_i\, q_{(i-u)}(\nu_i \quad -\nu_{(u-i)}) \\ +p_i\, q_{(i+u)}(\nu_{(u+i)}-\nu_{-i} \quad), \end{array}\right.$$

or, substituting $i-u$ for i in the second line

$$2\Sigma\,(p_i q_{(i-u)}+p_{(i-u)}q_i)(\nu_i-\nu_{u-i}).$$

Hence, writing N instead of N_1,

$$\delta\rho = \frac{1}{8}\frac{m'ar_1^{-2}}{a_1(1+m)}\Sigma^2_{i,u}(p_i q_{(i-u)}+p_{(i-u)}q_i)(\nu_i-\nu_{(u-i)})[k_c\cos(N+ug)+k_s\sin(N+ug)] \quad (19)$$

This expression fails for the particular case $N=ug$, where the value of ν_{-u} will be infinite. If we take each term of Q of the form

$$\frac{m'}{a_1}(k_c^{(u)}\cos ug+k_s^{(u)}\sin ug),$$

and substitute in the general expression (13) it will be found that the terms in $r_1^2\delta\rho$ which have the infinite values of ν as a factor are to be omitted, and replaced by

$$r_1^2\delta\rho = \frac{1}{2}\frac{m'ant}{a_1(1+m)}\{\eta\,\Sigma p_u k_c^{(u)} - \xi\,\Sigma q_u k_s^{(u)}\} \quad (20)$$

The two parts of $r_1^2\delta\rho$ thus found include all the terms of the first order with respect to the disturbing forces. But when terms of the second order are taken into account, we shall find terms in Q proceeding from secular variation in which the time appears as a factor, outside the signs sin and cos. Let us represent such of these terms as depend on any angle N by

$$Q = \frac{m'nt}{a_1}(k_c\cos N + k_s \sin N)$$

and use the symbol ν_i, as before, to represent the ratio of the mean motion of the planet to the coefficient of t in the angle $N+ig$, so that if μ' represents the coefficients of t in N we have

$$\nu = \frac{n}{\mu'},$$

$$\nu_i = \frac{n}{\mu'+in} = \frac{1}{\frac{\mu'}{n}+i},$$

we find the expression

$$\int \xi Q dt = \frac{1}{4}\frac{m'\Sigma p_i}{a_1 n}\left\{ \begin{array}{l} (\nu_i^2 k_c - \nu_i k_s nt)\cos(N+ig) + (\nu_{-i}^2 k_c - \nu_{-i} k_s nt)\cos(N-ig) \\ (\nu_i^2 k_s + \nu_i k_c nt)\sin(N+ig) + (\nu_{-i}^2 k_s + \nu_{-i} k_c nt)\sin(N-ig) \end{array}\right\}$$

$$\int \eta Q dt = \frac{1}{4}\frac{m'\Sigma q_j}{a_1 n}\left\{ \begin{array}{l} (\nu_j^2 k_c - \nu_j k_s nt)\sin(N+jg) - (\nu_{-j}^2 k_c - \nu_{-j} k_s nt)\sin(N-jg) \\ -(\nu_j^2 k_s + \nu_j k_c nt)\cos(N+jg) + (\nu_{-j}^2 k_s + \nu_{-j} k_c nt)\cos(N-jg) \end{array}\right\}.$$

If we now put for brevity

$$\nu_i^2 k_s + \nu_i k_c nt = c_i,$$

$$\nu_i^2 k_c - \nu_i k_s nt = s_i,$$

the general value of $r_0^2\delta\rho$ becomes

$$\delta\rho = \frac{1}{16}\frac{m' a r_1^{-2}}{a_1(1+m)} \times$$

$$\Sigma^2 p_i q_j \left\{ \begin{array}{l} (c_j - c_i)\cos(N+(i+j)g) + (s_i - s_j)\sin(N+(i+j)g) \\ +(c_i - c_{-j})\cos(N+(i-j)g) + (s_{-j} - s_i)\sin(N+(i-j)g) \\ +(c_j - c_{-i})\cos(N-(i-j)g) + (s_{-i} - s_j)\sin(N-(i-j)g) \\ +(c_{-i} - c_{-j})\cos(N-(i+j)g) + (s_{-j} - s_{-i})\sin(N-(i+j)g) \end{array}\right\}$$

If, as before, we transform this expression by putting

in the first line $j = u - i$;
in the second " $j = i - u$;
in the third " $j = i + u$;
in the fourth " $j = i - u$.

the value of $r_1^2\delta\rho$ reduces to

$$\frac{1}{8}\frac{m'a}{a_1(1+m)} \times$$

$$\Sigma_{i,u}^2 \left\{ \begin{array}{l} p_i q_{(u-i)}(c_{(u-i)} - c_i)\cos(N+ug) + p_i q_{(u-i)}(s_i - s_{(u-i)})\sin(N+ug) \\ p_i q_{(u+i)}(c_{(u+i)} - c_{-i})\cos(N+ug) + p_i q_{(u+i)}(s_i - s_{(u+i)})\sin(N+ug) \end{array}\right\} \tag{21}$$

or, putting $i-u$ for i in the last line,

$$\delta\rho = \frac{1}{8}\frac{m' a r_1^{-2}}{a_1(1+m)} \times$$

$$\Sigma_{u,i}^2 \{ p_i q_{(u-i)} - p_{(u-i)} q_i \} \{ (c_{(u-i)} - c_i)\cos(N+ug) + (s_i - s_{u-i})\sin(N+ug) \};$$

to which expression is to be added, in lieu of the terms which will have infinite values of ν as a factor.

$$\frac{1}{4}\frac{m'ar_1^{-2}n^2t^2}{a_1(1+m)}\{\eta\Sigma p_u k_c^{(u)} - \xi\Sigma q_u k_s^{(u)}\} \tag{22}$$

$k_c^{(u)}$ and $k_s^{(u)}$ being the factors of $\frac{m'}{a_1} nt \cos ug$ and $\frac{m'}{a_1} nt \sin ug$ in the expression for Q.

The formulæ 19, 20, 21, and 22 give the complete expressions for the perturbations of the logarithm of radius vector by successively substituting in it all the terms of Q.

Perturbations of Longitude.

We now pass to the perturbations of longitude. In the Mécanique Céleste (Première Partie, Liv. ii. Chap. vi.), Laplace gives an equation (Y) by which the perturbations of longitude, which are of the first order, may be derived from those of the radius vector without the formation of any other derivatives of R than those which enter into Q. But the formula does not seem easily adapted to the case in which the perturbations of the second order are taken into account, we shall therefore derive all the perturbations of longitude from the second of equations (1). By integration this equation gives

$$\frac{dv}{dt}=\frac{\mu}{r^2}\left\{\int\frac{\partial R}{\partial \mathrm{v}}dt+C\right\}$$

C being the arbitrary constant of the integral. Representing, as before, by subscript zeros the values of the co-ordinates corresponding to the ellipse to which the orbit is supposed to reduce itself when the disturbing forces vanish, we have

$$\frac{dv_0}{dt}=\frac{a^2n\cos\psi}{r_0^{\,2}}=\frac{\mu C}{r_0^{\,2}},$$

because the constant to which the integral must reduce itself in the elliptic motion is $\frac{a^2n\cos\psi}{\mu}$. Subtracting the last equation from the preceding, and putting $v-v_0=\delta v$, we find

$$\frac{d\delta v}{dt}=\frac{\mu}{r^2}\int\frac{\partial R}{\partial \mathrm{v}}dt+\left(\frac{1}{r^2}-\frac{1}{r_0^{\,2}}\right)a^2n\cos\psi.$$

Developing $\frac{1}{r^2}$ to terms of the second order with respect to the disturbing force

$$\frac{1}{r^2}=\frac{1}{r_0^{\,2}}\left(1-2\delta\rho+2\delta\rho^2-\text{etc.}\right),$$

which, being substituted in the last equation by putting

$$\mu=\frac{a^3n^2}{1+m},$$
$$r_0^{\,2}=a^2r_1^{\,2},$$

gives

$$r_1^{\,2}\frac{d\delta v}{dt}=\frac{an^2}{1+m}(1-2\delta\rho)\int\frac{\partial R}{\partial \mathrm{v}}dt-2n\cos\psi\,(\delta\rho-\delta\rho^2), \tag{23}$$

which is rigorous to quantities of the second order.

The most convenient mode of making the numerical computation of the second order terms by means of this equation will depend upon circumstances. If the perturbations of longitude and radius vector of both planets are already known with a sufficient degree of approximation for the computation of formula (11), it will be more convenient to form at once the complete values of all the quantities which enter into the equations (12), (13), (19) to 22), and (23), so that no steps of the process shall have to be repeated. If such perturbations are not known, they must first be computed, and it will then be necessary to begin with the perturbations of the first order, and afterward add those of the second. There is, however, one class of terms of the second order which it will be most convenient to take account of from the beginning, namely, those arising from the constant term in $\delta\rho$ and $\delta\rho'$. This is effected by correcting the mean distances for an approximate value of these constants at the beginning of the computation, and then proceeding in the usual way. This is in fact what we have supposed to be done in the preceding investigation. The values of $\delta\mathrm{v}$, $\delta\mathrm{v}'$, $\delta\rho$, $\delta\rho'$ in formula (11) will then contain only periodic terms.

In computing the terms of the first order we determine the value of $\delta\rho$ from the equations (19) and (20), using the value of Q_0 in (18). Then those of δv are obtained by integrating the equation

$$\frac{d\delta v}{dt}=\frac{an^2r_1^{-2}}{1+m}\int\frac{\partial R_0}{\partial\mathrm{v}}dt-2n\cos\psi\frac{\delta\rho}{r_1^{\,2}}. \tag{24}$$

Having found the values of δv and $\delta\rho$ for both planets, they are to be substituted in (11), to obtain δR, $\delta\frac{\partial R}{\partial\mathrm{v}}$ and $\delta\frac{\partial R}{\partial\rho}$. But, rigorously, $\delta\mathrm{v}$ and $\delta\mathrm{v}'$ are not the same with δv and $\delta v'$, owing to the movement of the orbits of the planets, and the corrections for $\delta\gamma$ are also to be added. Considering, for the present, only the perturbations of the second order, which depend on δv, $\delta v'$, $\delta\rho$, and $\delta\rho'$, we may use the following equation for δR, and similar ones for its derivatives:

$$\delta R=\frac{\partial R}{\partial\mathrm{v}}\delta v+\frac{\partial R}{\partial\mathrm{v}'}\delta v'+\frac{\partial R}{\partial\rho}\delta\rho+\frac{\partial R}{\partial\rho'}\delta\rho'. \tag{25}$$

Having thus found δR, and hence $D'_t\delta R$ by differentiation, and then $\delta\frac{\partial R}{\partial\rho}$, we form the quantity

$$\delta Q=2\int D'_t\delta R dt+\delta\frac{\partial R}{\partial\rho}-\frac{1}{\mu}\frac{d^2(r_0^{\,2}\delta\rho^2)}{dt^2}+\tfrac{1}{2}\frac{\delta\rho^2}{r_0} \tag{26}$$

which is the difference between the value of Q_0 in (18) and that of Q in (12). The terms in $\delta\rho$ arising from δQ are then to be computed by the formulæ (19), (20), (21), and (22), when we shall have $\delta\rho$ accurate to quantities of the second order. Let us represent these additional terms by $\delta^2\rho$. Subtracting (24) multiplied by $r_1^{\,2}$ from (23), recollecting that the $\delta\rho$ which appears in the second term of the former is really $\delta\rho-\delta^2\rho$, we find, neglecting quantities of the third order,

$$r_1^{\,2}\frac{d\delta^2v}{dt}=an^2\left\{\int\delta\frac{\partial R}{\partial\mathrm{v}}dt-2\delta\rho\int\frac{\partial R_0}{\delta\mathrm{v}}dt\right\}-2n\cos\psi(\delta^2\rho-\delta\rho^2)$$

from which the terms of δv of the second order are obtained by multiplying by r_1^{-2} and integrating.

Motion of the Orbital Planes.

The general theory of the motion of the planes of reference, especially of the motion of the instantaneous orbit, has been so often treated that I can scarcely hope to add anything essentially new to it. I shall, however, endeavor to present the differential equations of the motion in a simple and general form, and one in which the geometrical conceptions of the problem shall be made as clear as possible.

The orbital plane of each planet being at each moment osculatory to that part of the orbit which the planet is actually describing, its only motion is one of rotation around the radius vector of the planet as an instantaneous axis. This rotation may be resolved into two others around any pair of rectangular axes fixed in the moving plane. But the rotation produced by any one planet is most simply expressed when referred to axes, one of which coincides with the common node of the two orbits. The rotation produced by each separate planet must, therefore, be first referred to its node on the moving orbit, and then the combined rotations must be resolved into two around axes assumed at pleasure. To effect this, let us suppose positive rotation around an axis to be such that an observer looking from the origin along the positive direction of the axis sees the right hand side of the plane move downwards, and the left hand side upwards. Let us also denote the first axis in the order of longitude the principal axis, or that of X, and that 90° farther advanced the secondary axis, or that of Y. Let us now put

dq, the instantaneous rotation around the axis of X;

dp, the instantaneous rotation around the axis of Y. Let us also put, relatively to any disturbing planet,

$d\eta$, the instantaneous rotation around the ascending node of the disturbing planet on the orbit of the disturbed one.

dk, that around the corresponding secondary axis.

Then, from the known equations for the perturbations of the inclination and node of an orbit, we find, that, if any term of the perturbative function be represented, as before, by

$$\frac{m'h}{a_1}\cos(i'\lambda'+i\lambda+j'\omega'+j\omega),$$

the differential rotations η and k will be given by the equations

$$\frac{d\eta}{dt}=-\frac{m'h}{a_1}\frac{an}{\cos\psi}\left\{(i+j)\cot\gamma+(i'+j')\operatorname{cosec}\gamma\right\}\sin N$$

$$\frac{dk}{dt}=-\frac{m'an}{a_1\cos\psi}\frac{\partial h}{\partial\gamma}\cos N.$$

As R is actually developed, the mutual inclination γ does not explicitly appear, but is replaced by

$$\sigma=\sin\tfrac{1}{2}\gamma.$$

Making this substitution, and putting also

$$i'' + i + j'' + j = -\iota.$$

these equations become

$$\frac{d\eta}{dt} = \frac{m'an}{a_1 \cos\psi \cos\frac{1}{2}\gamma}\left\{\frac{\iota h}{2\sigma} + (i + j)\,\sigma h\right\}\sin N$$
$$\frac{dk}{dt} = -\frac{m'an\cos\frac{1}{2}\gamma}{2a_1\cos\psi}\,\frac{\partial h}{\partial\sigma}\cos N. \qquad (27)$$

To pass to the general rotations dp and dq, let us represent by θ_1, θ_2, etc., the longitudes of the ascending nodes of the several orbits of the disturbing planets on that of the disturbed planet. We shall then have

$$\frac{dq}{dt} = \Sigma\cos\theta_\iota\frac{d\eta_\iota}{dt} - \Sigma\sin\theta_\iota\frac{dk_\iota}{dt}$$
$$\frac{dp}{dt} = \Sigma\cos\theta_\iota\frac{dk_\iota}{dt} + \Sigma\sin\theta_\iota\frac{d\eta_\iota}{dt}. \qquad (28)$$

These equations completely define the instantaneous motion of the orbital plane. They cannot, however, be rigorously integrated in their present form because p and q as integrals have no completely defined signification. To do this it is necessary to express the differential rotations dp, dq, etc., in terms of the differentials of any elements we may select to define the position of the orbital plane, and then to integrate the equations thus formed. But, for the purpose of constructing tables of the planets we may consider p, q, etc., to represent small rotations of the planes of which the powers and products may be neglected, and the integration is then quite simple.

Perturbations of the second order depending on the motion of the orbital planes.

R being a function of the five quantities of r, r', v, v', and γ, the motion of the orbital planes introduces terms of the second order by changing the values of v, v', and γ. These terms we have hitherto neglected. To investigate them let us refer the rotations of both planes as given by (28) to the node of the disturbing on the disturbed planet as the principal axis. If we represent by $d\eta$, dk, $d\eta'$, and dk' the rotations corresponding to this axis, and designate by the subscript 1, the quantities which refer to the disturbing planet whose action we are considering, and by 2, 3, etc., the other planets, the equations (28) will be replaced by these

$$\frac{d\eta}{dt} = \frac{d\eta_1}{dt} + \Sigma\cos(\theta_\iota - \theta_1)\frac{d\eta_\iota}{dt} - \Sigma\sin(\theta_\iota - \theta_1)\frac{dk_\iota}{dt},$$
$$\frac{dk}{dt} = \frac{dk_1}{dt} + \Sigma\cos(\theta_\iota - \theta_1)\frac{dk_\iota}{dt} + \Sigma\sin(\theta_\iota - \theta_1)\frac{d\eta_\iota}{dt},$$

the summation commencing with $i = 2$.

By formulæ of the same kind we are to find the differential rotations $d\eta'$ and dk' of the orbit of the disturbing planet, produced by the action of all the planets.

These rotations will be around the same principal axis with the rotations $d\eta$ and dk, but around a secondary axis in the plane of the disturbing orbit, and therefore making an angle γ with the secondary axis of the disturbed orbit. A geometrical construction will now show quite simply that the infinitesimal rotations $\delta\eta$, δk, $\delta\eta'$, and $\delta k'$ will produce the following changes in v, v', and γ.

$$\begin{aligned} \delta \mathrm{v} &= \cot\gamma\delta k - \operatorname{cosec}\gamma\delta k' \\ \delta \mathrm{v}' &= \operatorname{cosec}\gamma\delta k - \cot\gamma\delta k' \\ \delta\gamma &= \delta\eta' - \delta\eta \end{aligned} \tag{29}$$

If we substitute these values in the general formulæ (11) the terms of the second order added to δR will be

$$\begin{aligned} \delta R = &\left(\frac{\partial R}{\partial \mathrm{v}}\cot\gamma + \frac{\partial R}{\partial \mathrm{v}'}\operatorname{cosec}\gamma\right)\delta k \\ &-\left(\frac{\partial R}{\partial \mathrm{v}}\operatorname{cosec}\gamma + \frac{\partial R}{\partial \mathrm{v}'}\cot\gamma\right)\delta k' \\ &+\frac{\partial R}{\partial\gamma}(\delta\eta' - \delta\eta). \end{aligned} \tag{30}$$

The first two terms of this expression may be put into the form

$$\left\{\tfrac{1}{2}\left(\frac{\partial R}{\partial \mathrm{v}} + \frac{\partial R}{\partial \mathrm{v}'}\right)(\operatorname{cosec}\gamma + \cot\gamma) - \tfrac{1}{2}\left(\frac{\partial R}{\partial \mathrm{v}} - \frac{\partial R}{\partial \mathrm{v}'}\right)(\operatorname{cosec}\gamma - \cot\gamma)\right\}\delta k$$

$$-\left\{\tfrac{1}{2}\left(\frac{\partial R}{\partial \mathrm{v}} + \frac{\partial R}{\partial \mathrm{v}'}\right)(\operatorname{cosec}\gamma + \cot\gamma) + \tfrac{1}{2}\left(\frac{\partial R}{\partial \mathrm{v}} - \frac{\partial R}{\partial \mathrm{v}'}\right)(\operatorname{cosec}\gamma - \cot\gamma)\right\}\delta k.$$

But,

$$\operatorname{cosec}\gamma + \cot\gamma = \cot\tfrac{1}{2}\gamma = \frac{\cos\frac{1}{2}\gamma}{\sigma}.$$

$$\operatorname{cosec}\gamma - \cot\gamma = \tan\tfrac{1}{2}\gamma = \frac{\sigma}{\cos\frac{1}{2}\gamma}.$$

and in the general term of R, by (7)

$$\frac{\partial R}{\partial \mathrm{v}} = -\frac{m'h}{a_1}(i+j)\sin N$$

$$\frac{\partial R}{\partial \mathrm{v}'} = -\frac{m'h}{a_1}(i'+j')\sin N.$$

Making these substitutions, and putting, as before,

$$i + j + i' + j' = -\iota$$

the above value of δR reduces to

$$\begin{aligned} \delta R = &\frac{m'h}{2a_1}\{\iota\cot\tfrac{1}{2}\gamma(\delta k - \delta k') + (i + j - i' - j')\tan\tfrac{1}{2}\gamma(\delta k + \delta k')\}\sin N \\ &+\frac{m'\cos\frac{1}{2}\gamma}{2a_1}\frac{\partial h}{\partial\sigma}(\delta\eta' - \delta\eta)\cos N \end{aligned} \tag{31}$$

The corresponding terms of $\delta \frac{\partial R}{\partial \nu}$ and $\delta \frac{\partial R}{\partial \rho}$, and may be obtained in the same way by substituting $\frac{\partial R}{\partial \nu}$ and $\frac{\partial R}{\partial \rho}$ for R in (30) and continuing the corresponding substitutions of the general terms of the derivatives of R as given on page 9.

The equation (31), besides being of the second order with respect to the disturbing forces, is also of the second order with respect to the mutual inclinations. For δk, $\delta k'$, $\delta \eta$, and $\delta \eta'$ are of the first order with respect to both quantities, and, whenever ι is not zero, h is a quantity of the second order, containing σ^2 as a factor. It is, therefore, only in exceptional cases that the terms of the second order depending on the motion of the orbital planes can become sensible.

Reduction of the longitude in the orbit to lo gitude on the ecliptic.

The integration of (23) gives a value of δv, which, added to the longitude in orbit corresponding to the pure elliptic motion gives the longitude in the disturbed orbit, counted from a fixed point in the moving plane of that orbit. The position of this fixed point is completely determined by the condition that the instantaneous rotation of the plane in question around the axis perpendicular to itself is always zero, so that the motion of the point of reference is always perpendicular to the direction of the plane. But, although this instantaneous rotation is zero, the integrated rotation is not rigorously zero when we consider the terms of the second order. It follows that the value of v, the longitude in orbit, and the position of the plane of the orbit do not rigorously determine the position of the planet: we must also know how the fixed point of reference has changed its position in consequence of the motions which the plane has undergone. Let us consider the relative positions of this plane at two epochs. If the fixed point were equally distant from the common node of the two planes, the integrated rotation of the plane around its own axis would be zero. But, these distances not being equal, their difference is a correction to be applied to the longitude of the planet in its orbit. Suppose, now, that at the end of any time the inclination of the actual orbit to the primitive orbit is ϕ, and the distance of its ascending node from the present position of the moving axis of x is θ. A rotation around the line of nodes will not change the quantity sought. But, if we represent the infinitesimal rotation around an axis perpendicular to it by dr we shall have

$$\cos\theta\, dp - \sin\theta\, dq = dr,$$

dq and dk being the instantaneous rotations around the respective axes of x and γ. By this rotation it is easy to see that the relative distance of any two fixed points, one on each plane, from the node, will be altered by the quantity,

$$dr\,(\operatorname{cosec}\phi - \cot\phi) = dr \tan\tfrac{1}{2}\phi,$$

the relative longitude of the fixed point on the moving plane being increased by this amount. The correction to the longitude in orbit from this cause is, therefore,

$$dl = dr \tan\tfrac{1}{2}\phi = \tan\tfrac{1}{2}\phi\,(\cos\theta\, dp - \sin\theta\, dq).$$

Counting the integrated values of p and q in a direction perpendicular to the moving plane we have

$$\sin\theta = \frac{\tan p}{\tan\phi}$$

$$\cos\theta = \frac{\tan q}{\tan\phi}$$

which, being substituted in the expression for dl, gives

$$dl = \frac{\cos\phi}{1+\cos\phi}(\tan q dp - \tan p dq).$$

The approximate value of the integrated correction is therefore

$$\delta l = \frac{1}{2}\int(q dp - p dq). \qquad (32)$$

For every pair of periodic terms in p and q, such as

$$q = s\sin\mu t,\ p = s\cos\mu t,$$

δl will contain the secular term $-\frac{1}{2}s^2\mu t$, which will be confounded with the mean motion, and, if it were not so confounded, would in few or none of the larger planets amount to a second in a thousand years. If the secular terms in p and q be

$$q = st\,;\ p = s't$$

δl will vanish. We hence conclude that these terms are entirely unimportant in the present state of astronomy, and that, if we consider the positions of the plane of the orbit at two epochs, we may consider the points of departure in them to be equally distant from their common node.

We have therefore only to consider the motion of the inclination and node due to the change of the position of the orbit and of the ecliptic. If we put

ϕ, the inclination of the orbit of the planet to the ecliptic,

θ, the longitude of its node counted on the ecliptic,

τ, the longitude of the same node counted from the same fixed point in the moving plane of the orbit from which v is counted,

Then, the longitude of the planet on the ecliptic, or L, will be given by the equation

$$\tan(L-\theta) = \cos\phi\tan(v-\tau),$$

or, when developed in powers of ϕ,

$$L = v + \theta - \tau + D, \qquad (33)$$

where D is the reduction to the ecliptic, the value of which is

$$D = -\tan^2\tfrac{1}{2}\phi\sin 2(v-\tau) + \tfrac{1}{2}\tan^4\tfrac{1}{2}\phi\sin 4(v-\tau) - \text{etc.}$$

Let us refer the instantaneous rotations of the orbit and of the ecliptic to the fixed points of reference in the two planes; q being the rotation around an axis passing through the sun and the fixed point, and p that around an axis in 90° greater longitude, and the accented quantities referring to the ecliptic. We then have

$$\frac{d\phi}{dt} = \cos\tau \frac{dq}{dt} + \sin\tau \frac{dp}{dt}$$
$$-\cos\theta \frac{dq'}{dt} - \sin\theta \frac{dp'}{dt}$$
$$\frac{d\theta}{dt} = \operatorname{cosec}\phi \left(-\sin\tau \frac{dq}{dt} + \cos\tau \frac{dp}{dt}\right) \tag{34}$$
$$+\cot\ \phi\left(\ \sin\theta \frac{dq'}{dt} - \cos\theta \frac{dp'}{dt}\right)$$
$$\frac{d\tau}{dt} = \cot\ \phi\left(-\sin\tau \frac{dq}{dt} + \cos\tau \frac{dp}{dt}\right)$$
$$+\operatorname{cosec}\phi\left(\ \sin\theta \frac{dq'}{dt} - \cos\theta \frac{dp'}{dt}\right)$$

If we differentiate (33) and substitute these values of $\frac{d\theta}{dt}$ and $\frac{d\tau}{dt}$, we shall have

$$\frac{dL}{dt} = \frac{dv}{dt} + \frac{dD}{dt} - \tan\tfrac{1}{2}\phi\left(\cos\tau \frac{dp}{dt} - \sin\tau \frac{dq}{dt} + \cos\theta \frac{dp'}{dt} - \sin\theta \frac{dq'}{dt}\right) \tag{35}$$

If we consider only quantities of the first order with respect to the disturbing forces, we may, in integrating, suppose τ and θ equal and constant, and ϕ constant. The integral will then be

$$L = v + D + \tan\tfrac{1}{2}\phi\ \{\cos\theta\,(\delta k + \delta k') - \sin\theta\,(\delta\eta + \delta\eta')\} \tag{36}$$

In the case of Uranus, $\tan\phi$ is so small that this equation will be sufficient for a long time before and after our epoch.

In the application of the method to other planets the mode of operation must depend on the circumstances of each particular case. The differential equations (34) between θ, τ, and ϕ are rigorous, and their integrals may be approximated to in various ways, out of which that best applicable to the particular case must be selected.

Expressions for the latitude.

If the position of the orbital plane and of the ecliptic were each determined by the preceding formulæ, there would be no perturbations of the latitude, the latitude itself being given rigorously by the equation

$$\sin\beta = \sin\phi\,\sin\,(v - \tau).$$
$$= \sin\phi\,\cos\tau\,\sin v - \sin\phi\,\sin\tau\,\cos v.$$

But the instantaneous values of ϕ and τ, or of $\sin\phi\,\cos\tau$ and $\sin\phi\,\sin\tau$, are troublesome to tabulate; it will therefore, in practice, be found more convenient to use only their mean values, and to consider their changes from this mean as perturbations of the latitude. Representing by the sign δ the deviations from the mean values, which are of course arbitrary, we have

$$\cos\beta\delta\beta = \cos\phi\,\sin\,(v - \tau)\,\delta\phi - \sin\phi\,\cos\,(v - \tau)\,\delta\tau.$$

Let us substitute for $\delta\phi$ and $\delta\tau$ their values given by the integration of (34) to

quantities of the first order, in which case θ and τ may be assumed equal. These values are

$$\delta\phi = \sin\tau\,\delta p + \cos\tau\,\delta q$$
$$\sin\phi\,\delta\tau = \cos\phi\,(\cos\tau\,\delta p - \sin\tau\,\delta q)$$

the terms dependent on $\delta p'$ and $\delta q'$ being omitted because, being purely secular, they may be included in the mean values of ϕ and τ. Substituting in the expression for $\delta\beta$

$$\cos\beta\delta\beta = \cos\phi\,\{\sin v\,\delta q - \cos v\,\delta p\}. \tag{37}$$

In the case of all the larger planets both $\cos\beta$ and $\cos\phi$ may here be put equal to unity, when the expression for $\delta\beta$ will become

$$\delta\beta = \sin v\,\delta q - \cos v\,\delta p. \tag{38}$$

To develop this expression in purely periodic terms we must substitute for v its value in terms of the mean longitude or mean anomaly, namely,

$$v = l + 2e\sin g + \frac{5}{4}e^2\sin 2g + \text{etc.};$$

suppose the terms of δp and δq depending on any argument, N to be

$$\left.\begin{aligned}\delta p &= -a_s\sin N - a_c\cos N\\ \delta q &= a'_s\sin N + a'_c\cos N\end{aligned}\right\} \tag{39}$$

and put π for the longitude of the perihelion, so that

$$l = \pi + g$$

then, to terms of the first order with respect to the eccentricities, we have

$$\begin{aligned}\delta\beta = &-e\,(a_s\cos\pi + a'_s\sin\pi)\sin N - e\,(a_c\cos\pi + a'_c\sin\pi)\cos N\\
&+\tfrac{1}{2}\ \{(a_s + a'_c)\cos\pi + (a'_s - a_c)\sin\pi\}\sin(N + g)\\
&+\tfrac{1}{2}\ \{(a_c - a'_s)\cos\pi + (a'_c + a_s)\sin\pi\}\cos(N + g)\\
&+\tfrac{1}{2}\ \{(a_s - a'_c)\cos\pi + (a'_s + a_c)\sin\pi\}\sin(N - g)\\
&+\tfrac{1}{2}\ \{(a_c + a'_s)\cos\pi + (a'_c - a_s)\sin\pi\}\cos(N - g)\\
&+\tfrac{1}{2}e\{(a_s + a'_c)\cos\pi + (a'_s - a_c)\sin\pi\}\sin(N + 2g)\\
&+\tfrac{1}{2}e\{(a_c - a'_s)\cos\pi + (a'_c + a_s)\sin\pi\}\cos(N + 2g)\\
&+\tfrac{1}{2}e\{(a_s - a'_c)\cos\pi + (a'_s + a_c)\sin\pi\}\sin(N - 2g)\\
&+\tfrac{1}{2}e\{(a_c + a'_s)\cos\pi + (a'_c - a_s)\sin\pi\}\cos(N - 2g)\end{aligned} \tag{40}$$

The point of the orbit from which π and v are counted is entirely arbitrary, and, in considering the action of but a single planet, it will be most convenient to count them from the common node, in which case π must be replaced by ω, and δp and δq by δk and $\delta\eta$. Thus, deducing the perturbations of the latitude immediately from the formulæ (27), we shall have

$$\delta\beta = \sin v\,\delta\eta - \cos v\,\delta k.$$

CHAPTER II.

APPLICATION OF THE PRECEDING METHOD TO THE COMPUTATION OF THE PERTURBATIONS OF URANUS BY SATURN.

Data of Computation.

THE elements of Uranus, adopted in this computation, were deduced from the comparison of nine normal heliocentric longitudes at intervals of 3697 days extending from 1781, December 26, to 1862, December 18, with corresponding provisional places derived from the elements given in the "Investigation of the Orbit of Neptune," with perturbations produced by Jupiter, Saturn, and Neptune. As the perturbations are to be entirely re-computed, and the elements to be re-corrected from more extended series of observations, all the details of this first approximation will be omitted. The resulting elements of Uranus are given in the following table, together with the adopted elements of Saturn, which are nearly the same as those employed in the theory of Neptune, except that the inclination and longitude of the node have been corrected to agree with observations:—

	Elements II. of Uranus.	Elements I. of Saturn.
π	168° 16′ 31″	90° 4′ 0″
ε	28 25 36.0	14 48 45.0
θ	73 11 58	112 20 0
ϕ	0 46 20	2 29 39.2
e	.0469276	.0560050
e in seconds,	9679.5	11551.9
n	15426.10	43996.13
m	$\frac{1}{21000}$	$\frac{1}{3501.6}$

In computing the perturbations of the radius vector, one of the largest terms will be a constant. To avoid the necessity of computing separately the perturbations of the second order, which depend on this constant, we shall include an approximate value of it in the mean distance. This approximate value is, in the action of an outer or an inner planet, $\delta \log a = -\frac{1}{6} m'M\alpha^2 D_\alpha\, b_{\frac{1}{2}}^{(0)}$. In the action of an inner or an outer planet, $\delta \log a' = +\frac{1}{6}\, mM (b_{\frac{1}{2}}^{(0)} + \alpha\, D_\alpha b_{\frac{1}{2}}^{(0)})$. M being the modulus of the system of logarithms.

Using the values of $b_{\frac{1}{2}}^{(0)}$ and $\alpha\, D_\alpha\, b_{\frac{1}{2}}^{(0)}$, which are found in different works relating to Celestial Mechanics, we find that the different planets produce the following changes in 6 log a, the units being those of the seventh place of decimals:—

		On Saturn.	On Uranus.
Action of	Venus,	+ 22	+ 22
"	Earth,	+ 24	+ 25
"	Mars,	+ 3	+ 3
"	Jupiter,	+10865	+8780
"	Saturn,		+3081
"	Uranus,	— 35	......
"	Neptune,	— 9	— 119
"	Sum,	+.0010870	+.0011792
$\delta \log a$		+.0001812	+.0001965

The uncorrected mean distance is deduced from the mean motion by the relation

$$a^3 = \frac{\mu(1+m)}{n^2}.$$

We thus have

	Saturn.	Uranus.
Uncorrected mean dist. (log)	0.979496	1.282901
Action of the planets	+ 181	+ 197
Corrected log a	0.979677	1.283098

The following functions of the elements are derived from the preceding elements by well known formulæ:—

γ (mutual inclination)	1° 57′ 24.4″
$\log \sin \frac{1}{2}\gamma = \sigma$	8.232373
$\log \cos \frac{1}{2}\gamma$	9.9999367
τ (long. of ascending node of Saturn on Uranus)	126° 44′ 51″
ω	41 31 40
ω'	323 18 21
$\omega' - \omega = (\omega)$	281 46 41
$\log \sin (\omega)$	—9.990759
$\log \cos (\omega)$	+9.309888
$\sin 2(\omega)$	—0.39966
$\cos 2(\omega)$	—0.91667
α	0.497249
$\frac{1}{\alpha^2}$	4.04438

The following functions of α, necessary in computing the coefficients h, are derived from Runkle's Tables, published by the Smithsonian Institution:—

Values of $\alpha^n D_\alpha^n b_{\frac{1}{2}}^{(i)}$.

i	$b_{\frac{1}{2}}^{(i)}$	$\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{i}$	$\alpha^5 D_\alpha^5 b_{\frac{1}{2}}^{(i)}$
0	2.14447	0.33969	0.5878	1.081	3.44	13.6
1	0.55207	.68314	.4990	1.177	3.40	13.8
2	0.20836	.47198	.7396	1.152	3.59	13.9
3	0.08687	.28491	.7123	1.463	3.68	14.5
4	0.03793	.16270	.5632	1.596	4.30	15.1
5	0.01702	.09010	.3998	1.485	4.87	16.9
6	0.00777	.04896	.2653	1.231	4.98	19.1
7	0.00359	.02624	.1682	0.940	4.60	20.5
8	0.00168	.01392	.1022	0.675	3.91	20.4
9	0.00079	0.00733	0.0615	0.463	3.11	18.8

Derivatives with respect to $(\log \alpha = v)$ *of* $\alpha^n D_\alpha^n b_{\frac{1}{2}}^{(i)}$.

i	$n=$ 0	1	2	3	4
	$D_v b_{\frac{1}{2}}^{(i)}$	$D_v(\alpha D_\alpha b_{\frac{1}{2}}^{(i)})$	$D_v(\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)})$	$D_v(\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)})$	$D_v(\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)})$
0	0.33969	0.9275	2.257	6.68	27.4
1	.68314	1.1821	2.175	6.93	27.4
2	.47198	1.2116	2.631	7.05	28.3
3	.28491	0.9972	2.888	8.07	29.2
4	.16270	0.7259	2.722	9.09	32.3
5	.09010	0.4899	2.285	9.33	36.4
6	.04896	0.3143	1.762	8.67	39.0
7	.02624	0.1944	1.276	7.41	38.9
8	.01392	0.1161	0.879	5.93	36.0
9	0.00733	0.0688	0.586	4.50	31.2

Second derivatives.

i	$D_v^2 b_{\frac{1}{2}}^{(i)}$	$D_v^2(\alpha D_\alpha b_{\frac{1}{2}}^{(i)})$	$D_v^2(\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)})$	$D_v^2(\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)})$
0	0.9275	3.184	11.19	47.4
1	1.1821	3.357	11.28	48.2
2	1.2116	3.843	12.31	49.4
3	0.9972	3.885	13.85	53.4
4	0.7259	3.448	14.53	59.6
5	0.4899	2.775	13.90	64.4
6	0.3143	2.076	12.19	65.0

Values of $\alpha^{n+1} D_\alpha^n b_{\frac{3}{2}}^{(i)}$

i	$\alpha b_{\frac{3}{2}}^{(i)}$	$\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\alpha^4 D_\alpha^3 b_{\frac{3}{2}}^{(i)}$
0	1.865	2.674	8.104	30.8
1	1.267	2.844	7.77	30.8
2	0.761	2.412	7.63	29.9
3	0.433	1.790	6.92	28.7
4	0.240	1.224	5.73	26.8
5	0.130	0.792	4.41	23.5
6	0.070	0.493	3.20	19.6

Derivatives with respect to (log $a = v$.)

i	$D_v(ab_{\frac{1}{2}}^{(i)})$ $=a_1 B_1^{(i)}$	$D_v(a^2 D a b_{\frac{3}{2}}^{(i)})$ $=2a_1 B_2^{(i)}$	$D_v(a^3 D a b_{\frac{3}{2}}^{(i)})$ $=6a_1 B_3^{(i)}$
0	4.539	13.452	55.1
1	4.111	13.46	54.1
2	3.173	12.45	52.8
3	2.223	10.50	49.5
4	1.464	8.18	44.0
5	0.922	5.99	36.7
6	0.563	4.19	29.2

Second derivatives.

i	$a_1 D_v B^{(i)}$	$a_1 D_v B_1^{(i)}$	$2a_1 D_v B_2^{(i)}$
0	4.539	17.99	82.0
1	4.111	17.57	81.0
2	3.173	15.62	77.7
3	2.223	12.72	70.5
4	1.464	9.64	60.4
5	0.922	6.91	48.7
6	0.563	4.75	37.6

	$a_1 E^{(i)}$	$a_1 E_1^{(i)}$	$2a_1 E_2^{(i)}$	$6a_1 E_3^{(i)}$	$a_1 D_v E^{(i)}$	$a_1 D_v E_1^{(i)}$	$2a_1 D_v E_2^{(i)}$
0	1.267	4.111	13.46	54.1	4.111	17.57	81.0
1	1.313	3.856	12.95	54.0	3.856	16.80	79.8
2	0.850	3.167	11.98	51.8	3.167	15.15	75.8
3	0.500	2.318	10.31	48.4	2.318	12.63	69.0
4	0.281	1.573	8.24	43.1	1.573	9.82	59.6
5	0.155	1.014	6.18	36.6	1.014	7.20	49.0

The notation $B_n^{(i)}$ and $E_n^{(i)}$ is that of Le Verrier in his development in the first volume of "*Annales de l'Observatoire Imperial de Paris.*"

Numerical expression of R and its derivatives.

We next proceed to the computation of the coefficients h and their derivatives. As an example of the most convenient form of computation we present in full that of the coefficient of $\frac{m'}{a_1}$ cos $(i\lambda'-(i-1)\lambda-\omega)$ in the expression of R for the action of Saturn on Uranus. In this computation I use the tables given by Le Verrier in his "*Annales de l'Observatoire,*" tome i, pages 358–383, comparing the development with that of Professor Peirce in the Astronomical Journal, vol. i, as a control.

$j = 0;\ j = 1.$

i	−3	−2	−1	0	+1	2	3	4	5	6
i'	+4	3	2	1	0	−1	−2	−3	−4	5
$(0) \times b_{\frac{1}{2}}^{(i)}$	+0.5212	+0.8334	+ 1.1041	0	−1.10414	−0.83344	−0.5212	−0.3034	−0.1702	−0.0929
$(1) \times \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	−0.2849	−0.4720	− 0.6831	−0.3397	−0.68314	−0.47198	−0.2849	−0.1627	−0.0901	−0.0490
$\Delta a_1\,(50)^{(i)}$	......	......	−16.1775	......	......	......	......	......	......	......
$a_1\,(50)^{(i)}$	+0.2363	+0.3614	−15.7565	−0.3397	−0.78728	−1.30542	−0.8061	−0.4661	−0.2603	−0.1419
$(0) b_{\frac{1}{2}}^{(i)}$	−13.55	−11.25	− 5.52	0.00	0.00	+ 2.92	+ 5.73	+ 6.83	+ 6.46	
$(1) \times \alpha D_\alpha$	+ 8.54	+ 7.78	+ 4.78	+0.51	0.00	+ 1.18	+ 2.56	+ 3.25	+ 3.06	
$(2) \times \alpha^2 D^2_\alpha$	+ 1.43	+ 0.74	− 0.00	−0.59	− 1.00	− 2.22	− 2.85	− 2.82	− 2.40	
$(3) \times \alpha^3 D^3_\alpha$	− 0.73	− 0.58	− 0.59	−0.54	− 0.59	− 0.58	− 0.73	− 0.80	− 0.74	
$\Delta a_1(51)$	......	......	+ 48.53	......	......	......	......	......	......	
$a_1(51)$	− 4.31	− 3.31	+ 47.20	−0.62	− 1.59	+ 1.30	+ 4.71	+ 6.46	+ 6.38	
$(0) \times b_{\frac{1}{2}}^{(i)}$	−18.76	−13.33	− 4.42	0.00	+ 4.42	+13.33	+18.76	+19.41	+17.01	
$(1) \times \alpha D_\alpha$	+13.10	+10.38	+ 4.10	−0.68	− 1.37	+ 2.83	+ 6.27	+ 7.48	+ 7.02	
$(2) \times \alpha^2 D^2_\alpha$	+ 1.43	0.00	− 1.00	−2.35	− 3.00	− 5.92	− 7.13	− 6.76	− 5.60	
$(3) \times \alpha^3 D^3_\alpha$	− 1.46	− 1.15	− 1.18	−1.08	− 1.18	− 1.15	− 1.46	− 1.60	− 1.48	
$\Delta a_1(52)$	......	......	+ 32.36	......	......	......	......	......	......	
$a_1(52)$	− 5.69	− 4.10	+ 29.86	−4.11	− 1.13	+ 9.09	+16.44	+18.53	+16.95	
$(0) \times \alpha E_1^{(i)}$	−3.00	− 3.40	− 2.63	0.00	+ 2.63	+ 3 40	+ 3.00	+ 2.25	+ 1.55	
$(1) \times a_1 E_1^{(i)}$	+2.32	+ 3.17	+ 3.86	+4.11	+ 3.86	+ 3.17	+ 2.32	+ 1.57	+ 1.01	
$\Delta a_1(60)$	......	......	+ 16.18	......	......	......	......	......	......	
$a_1(60)$	−0.68	− 0.23	+ 17.41	+4.11	+ 6.49	+ 6.57	+ 5.32	+ 3.82	+ 2.56	
$\frac{1}{2} e \times (50)$	+6.61[1]	+10.12	−441.22	−9.51	−50.049	−36.556	−22.573	−13.05	− 7.28	−3.97
$\frac{1}{2} e^3 \times (51)$	−0.09	− 0.07	+ 1.04	−0.01	− 0.035	+ 0.028	+ 0.103	+ 0.14	+ 0.14	
$\frac{1}{2} e e'^2 \times (52)$	−0.08	− 0.06	+ 0.45	−0.06	− 0.017	+ 0.138	+ 0.248	+ 0.28	+ 0.26	
$\frac{1}{2} e e^2 \times (60)$	−0.05	− 0.02	+ 0.14	+0.03	+ 0.051	+ 0.052	+ 0.043	+ 0.03	+ 0.02	
h	+6.39	+ 9.97	−439.59	−9 55	−50.050	−36.338	−22 179	−12.60	− 6.86	−3.60

The derivatives of h with respect to (log. $a = \nu$) are computed in precisely the same way by simply substituting for $b_{\frac{1}{2}}^{(i)}$, $\alpha D\alpha b_{\frac{1}{2}}^{(i)}$, etc., their derivatives with respect to ν as given in the above table of constants.

The quantities $\Delta a_1\,(50)^{(i)}$, etc., which appear in the third series of terms above express that part of the perturbations of Uranus caused by the action of Saturn

[1] In units of the third place of decimals.

on the sun. They are each of the form $N \times \alpha^{-2}$, N being a numerical coefficient given by Le Verrier under the coefficient for each term. The derivative of this expression with respect to ν is $-2N \times \alpha^{-2}$, so that for the corresponding terms in $D_\nu h$ and $D_\nu^2 h$ we have

$$\Delta D_\nu h = -2\Delta h$$
$$\Delta D_\nu^2 h = +4\Delta h$$

The values of h and its derivatives, corresponding to any one argument i' and i, are to be combined into two terms depending the one on the cosine, the other on the sine of the argument. Let us represent by g the mean anomaly of Uranus, and let us put l' for the mean longitude of Saturn counted from the perihelion of Uranus, or, more exactly, for the arc $\lambda' - \omega$. Put also

$$\begin{aligned} N &= ig + i'l', \\ i + i' + j &= j'', \\ P &= j\omega + j'\omega', \\ P' &= j''\omega + j'\omega'. \end{aligned}$$

Then, for each value of N there will be several values of P corresponding to different powers and products of the eccentricities and inclinations in h. Distinguishing these values and the corresponding values of h by subscript numerals, we shall have a series of terms of R of the following form —

$$R = \frac{m}{a_1} \left\{ \begin{array}{l} h_1 \cos(N + P'_1) \\ + h_2 \cos(N + P'_2) \\ + h_3 \cos(N + P'_3) \\ + \text{etc.} \qquad \text{etc.} \end{array} \right\}$$

and by putting

$$\left.\begin{aligned} h_c &= h_1 \cos P'_1 + h_2 \cos P'_2 + h_3 \cos P'_3 + \text{etc.} \\ h_s &= -h_1 \sin P'_1 - h_2 \sin P'_2 - h_3 \sin P'_3 - \text{etc.} \end{aligned}\right\} \quad (41)$$

The above terms may be condensed into

$$R = \frac{m}{a_1} h_c \cos N + \frac{m}{a_1} h_s \sin N,$$

which are of the form supposed in the preceding theory.

In order that the derivative of R, with respect to the true longitude of Uranus, may be expressed in the form

$$\frac{\partial R}{\partial v} = \frac{m}{a_1} v_s \sin N + \frac{m}{a_1} v_c \cos N$$

we must, by (7), put

$$\left.\begin{aligned} v_s &= -(i + j_1) h_1 \cos P'_1 - (i + j_2) h_2 \cos P'_2 - \text{etc.} \\ v_c &= -(i + j_1) h_1 \sin P'_1 - (i + j_2) h_2 \sin P'_2 - \text{etc.} \end{aligned}\right\} \quad (42)$$

j_1, j_2, representing the several values of j in the different terms which correspond to one and the same set of values of i and i'.

To obtain the derivative with respect to γ we notice that all the appreciable terms in the different values of h, which depend upon the mutual inclination, are of the form

$$h = \sigma^2 A,$$

where $\sigma = \sin \frac{1}{2}\gamma$. These equations give

$$\frac{\partial h}{\partial \gamma} = \frac{\partial h}{\partial \sigma}\frac{\partial \sigma}{\partial \gamma} = A\sigma \cos \tfrac{1}{2}\gamma = \tfrac{1}{2} A \sin \gamma.$$

Consequently

$$\frac{\partial R}{\partial \gamma} = \tfrac{1}{2} A \sin \gamma \cos (N + P').$$

and the various terms depending on the same argument (i', i) may be condensed into two, exactly as in the case of R itself.

The different co-efficients h and $D_\gamma h$, computed in the way already described, are given *in extenso* in the following table. At the top of each individual column is given the value of P, or of $j\omega + j'\omega'$, corresponding to the values of h below, and immediately under P is given its modified value, or P', to be used in condensing the terms, putting for brevity

$$(\omega) = \omega - \omega'.$$

P and P' are therefore regarded as constant angles the numerical values of the sines and cosines of which may be obtained from the values of ω and ω' already given.

The condensed h_c and h_s are given in the two right hand columns.

All the numbers are given in units of the third place of decimals.

Values of h.

P / P'	0 / $= 0$	$\omega' - \omega$ / (ω)		
i i'	h	h	h_c	h_s
—4, +4		+ 0.62		
—3, 3		+ 0.82		
—2, 2		+ 0.88		
—1, 1		+ 0.53		
0, 0	+1072.90	— 0.4975	+ 1072.80	
+1, —1	—3481.36	— 0.83	— 3481.42	— 1.33
2, —2	+ 205.92	— 9.63	+ 204.14	— 10.29
3, —3	+ 84.27	+ 1.62	+ 84.77	+ 0.79
4, —4	+ 35.80	+ 1.54	+ 36.24	+ 0.90
5, —5	+ 15.48	+ 1.2	+ 15.81	+ 0.81

P / P'	$-\omega$ / 0	$-\omega'$ / $-(\omega)$	$\omega - 2\omega'$ / $-2(\omega)$	$\omega' - 2\omega$ / $+(\omega)$	ω' / $\omega + \omega'$	ω / $+2\omega$		
i i'	h	h	h	h	h	h	h_c	h_s
—3, +4	— 2.31	+ 6.39	0	0	0	0	— 1.01	— 6.26
—2, 3	— 3.42	+ 9.97	—0.95	0	0	+.02	— 0.51	— 9.40
—1, 2	— 3.54	—439.59	0	0	—.03	+.02	— 93.30	+430.35
0, 1	+287.30	— 9.55	+0.13	— .02	—.04	+.09	+285.20	+ 9.19
+1, 0	+ 58.38	— 50.05	—0.02	— .05	—.03	0	+ 48.15	+ 48.96
2, —1	— 39.760	— 36.338	—0.074	— .071	—.016	0	— 47.138	+ 35.531
3, —2	+ 35.13	— 22.179	—0.105	— .313	—.007	0	+ 30.63	+ 21.44
4, —3	+ 20.39	— 12.60	—0.11	+ .17	0	0	+ 17.96	+ 12.55
5, —4	+ 11.25	— 6.86	—0.10	+ .23	0	0	+ 9.98	+ 7.00
6, —5	+ 6.00	— 3.6	0	+ .22	0	0	+ 5.38	+ 3.78

VALUES OF h.

P / P'	-2ω / 0	$-\omega'-\omega$ / $-(\omega)$	$-2\omega'$ / $-2(\omega)$	0 / $+2\omega$		
i i'	h	h	h	h	h_c	h_s
−1, 3	+0.04	− 0.27	−42.24	+0.11	+38.72	+17.03
0, 2	+0.06	+31.65	− 0.04	+0.18	+ 6.58	−31.15
+1, 1	−0.74	− 0.83	− 1.08	−0.91	− 0.03	+ 2.15
2, 0	+3.09	− 5.19	+ 1.89	+0.18	+ 0.32	+ 4.14
3,−1	+1.268	− 6.290	+ 2.082	+0.111	− 1.912	+ 5.216
4,−2	+4.16	− 5.458	+ 1.767	+0.061	+ 1.44	+ 4.576
5,−3	+3.13	− 4.03	+ 1.29	+0.04	+ 1.13	+ 3.39
6,−4	+2.11	− 2.74	+ 0.87	+0.02	+ 0.75	+ 2.31
7,−5	+1.35	− 1.76	+ 0.55	+0.01	+ 0.49	+ 1.49

P / P'	-3ω / 0	$-\omega'-2\omega$ / $-(\omega)$	$-2\omega'-\omega$ / $-2(\omega)$	$-3\omega'$ / $-3(\omega)$	$-\omega'$ / $3\omega-\omega'$	$-\omega$ / 2ω		
i i'	h	h	h	h	h	h	h_c	h_s
+1,+2	0	−.12	0	−.11	−.14	+.02	+.17	+.06
2, 1	+.01	−.07	+.03	−.02	−.02	+.01	0	+.04
3, 0	+.18	−.46	+.34	−.08	−.02	+.04	−.15	+.213
4,−1	+.222	−.760	+.520	−.117	−.016	+.029	−.325	+.416
5,−2	+.424	−.847	+.558	−.122	−.012	+.021	−.176	+.490
6,−3	+.398	−.773	+.498	−.107	−.008	+.014	−.145	+.460
7,−4	+.327	−.627	+.395	−.086	−.005	+.009	−.107	+.379
8,−5	+.24	−.47	+.29	−.06	0	0	−.083	+.289

A / P'	-4ω / 0	$-\omega'-3\omega$ / $-(\omega)$	$-2\omega'-2\omega$ / $-2(\omega)$	$-3\omega'-\omega$ / $-3(\omega)$	$-4\omega'$ / $-4(\omega)$		
i i'	h	h	h	h	h	h_s	h_c
4, 0	0	−.04	+.04	−.02	0	−.04	0
5,−1	+.02	−.08	+.08	−.04	+.01	−.03	+.003
6,−2	+.039	−.105	+.106	−.047	+.008	−.045	+.028
7,−3	+.043	−.114	+.112	−.048	+.008	−.049	+.034
8,−4	+.042	−.106	+.103	−.044	+.007	−.044	+.032
9,−5	+.035	−.090	+.086	−.036	+.006	−.037	+.029

VALUES OF $D_v h$.

P / P'	0 / 0	$\omega'-\omega$ / (ω)		
i i'	$D_v h$	$D_v h$	$D_v h_c$	$D_v h_s$
−4,+4		+ 3.18		
−3, 3		+ 3.24		
−2, 2		+ 2.39		
−1, 1		+ 0.40		
0, 0	+ 171.93	− 2.075	+ 171.51	
+1,−1	+8749.59	− 2.69	+ 8749.12	− 3.02
2,−2	+ 467.72	+ 20.83	+ 472.46	+ 18.05
3,−3	+ 277.00	+ 2.87	+ 278.25	− 0.36
4,−4	+ 153.87	+ 4.62	+ 155.46	+ 1.41
5,−5	+ 82.12	+ 4.90	+ 83.54	+ 1.80

VALUES OF D_0h.

P	$-\omega$	$-\omega'$	$\omega-2\omega'$	$\omega'-2\omega$	$+\omega'$	ω		
P'	0	$-(\omega)$	$-2(\omega)$	$+(\omega)$	$\omega+\omega'$	2ω		
i i'	D_0h	D_0h	D_0h	D_0h	D_0h	D_0h	D_0h_e	$D_0h_{,}$
—3,+4	— 9.28	+ 19.4	0	0	0	0	— 5.32	— 19.00
—2, 3	— 9.74	+ 18.5	+2.05	— .06	—.12	+.08	— 7.97	— 17.43
—1, 2	— 4.63	+907.62	+ .05	— .08	—.13	+.07	+180.44	—888.65
—0, 1	—556.40	— 26.17	— .10	— .09	—.14	—.12	—561.82	+ 25.60
1, 0	+ 30.16	— 71.56	+ .04	— .19	—.11	+.02	+ 15.37	+ 69.86
2,—1	+265.17	— 86.47	— .19	— .26	—.07	0	+247.58	+ 84.30
3,—2	+ 83.10	— 74.60	— .42	+ .63	—.04	0	+ 68.34	+ 73.49
4,—3	+ 68.34	— 54.9	— .55	+ .29	—.02	0	+ 57.70	+ 53.9
5,—4	+ 49.04	— 36.9	— .58	+ .68	—.01	0	+ 42.19	+ 36.7
6,—5	+ 32.1	— 23.2	— .53	+ .90	0	0	+ 28.1	+ 23.4

P	-2ω	$-\omega-\omega'$	$-2\omega'$	0		
P'	0	$-(\omega)$	$-2(\omega)$	$+2\omega$		
i i'	D_0h	D_0h	D_0h	D_0h	D_0h_e	$D_0h_{,}$
—1,+3	+ 0.16	— 0.66	+86.02	+0.47	—78.76	—34.19
0, 2	+ 0.32	—64.55	+ 0.15	+0.60	—12.92	+62.54
+1, 1	+ 3.27	— 2.70	+ 4.69	+3.02	— 1.22	— 2.26
2, 0	+ 2.56	— 8.78	+ 5.00	+0.60	— 3.74	+ 6.00
3, 1	+13.745	—15.870	+ 7.232	+0.463	+ 3.933	+12.153
4,—2	+10.25	—18.93	+ 7.85	+0.32	— 0.77	+15.07
5,—3	+10.76	—18.07	+ 6.99	+0.21	+ 0.69	+14.68
6,—4	+ 9.39	—15.0	+ 5.59	+0.13	+ 1.22	+12.4
7,—5	+ 7.3	—11.5	+ 4.21	+0.09	+ 1.2	+ 9.6

P	-3ω	$-2\omega-\omega'$	$-\omega-2\omega'$	$-3\omega'$	$-\omega'$	$-\omega$		
P'	0	$-(\omega)$	$-2(\omega)$	$-3(\omega)$	$3\omega-\omega'$	$+2\omega$		
i i'	D_0h	D_0h	D_0h	D_0h	D_0h	D_0h	D_0h_e	$D_0h_{,}$
+1,+2	+0.03	+0.18	+0.05	+0.22	+.22	+.08	—0.31	—0.17
2, 1	+0.12	—0.26	+0.33	—0.09	—.08	+.17	—0.08	—0.09
3, 0	+0.21	—0.89	+0.98	—0.31	—.09	+.13	—0.59	+0.13
4,—1	+0.89	—2.02	+1.89	—0.54	—.09	+.13	—0.84	+0.69
5,—2	+1.08	—3.00	+2.53	—0.67	—.08	+.11	—1.38	+1.30
6,—3	+1.40	—3.47	+2.73	—0.69	—.07	+.09	—1.33	+1.68
7,—4	+1.46	—3.44	+2.60	—0.62	—.05	+.06	—1.20	+1.78
8,—5	+1.32	—3.00	+2.20	—0.51	0	+.04	—0.99	+1.62

P	-4ω	$-3\omega-\omega'$	$-2\omega-2\omega'$	$-\omega-3\omega'$	$-4\omega'$		
P'	0	$-(\omega)$	$-2(\omega)$	$-3(\omega)$	$-4(\omega)$		
i i'	D_0h	D_0h	D_0h	D_0h	D_0h	D_0h_e	$D_0h_{,}$
4, 0	+.02	—.09	+.13	—.08	+.02	—.06	—.01
5,—1	+.05	—.22	+.30	—.18	+.04	—.14	—.02
6,—2	+.10	—.38	+.48	—.26	+.05	—.24	+.01
7,—3	+.15	—.52	+.62	—.32	+.06	—.30	+.04
8,—4	+.18	—.59	+.66	—.33	+.06	—.32	+.09
9,—5	+.19	—.59	+.65	—.31	+.05	—.31	+.11

The values of D_0^2h, needed in computing the perturbations of the second order with respect to the masses being obtained in the same way, by the simple substitution of the second derivatives of the functions b,$aDab$, etc., for those functions themselves in the expressions for h, it is not necessary to present the details of the computation.

After obtaining h and its derivatives, it will be found convenient to change the arrangement of the terms. Hitherto we have kept in one series those in which the sum of the indices are a constant. Now, we shall put together all those in which

the index of the disturbing planet has the same value, arranging the individual terms of each series according to the index of the disturbed planet. Thus, the index of the product of any term, as $h \cos N$, by any multiple of the mean anomaly of the disturbed planet, as jg, will be found in the same series with that of N itself, and j lines above and below.

The next process will be the formations of the required functions of the mean anomaly of Uranus, $\frac{dv}{dt}$, $\frac{d\rho}{dt}$, $\frac{a^2}{r^2}$, $\log r$. Their values are as follows:—

$r_1^{-2} =$	$\rho_0 = n$	$\frac{d\rho_0}{ndt} =$	$\frac{dv_0}{ndt} = 1$
1.001103	+.0005507		
+.093933 cos g	—.0468889 cos g	+.0468889 sin g	+.0938294 cos g
+.005507 cos $2g$	—.0016494 cos $2g$	+.0032988 sin $2g$	+.0055012 cos $2g$
+.000336 cos $3g$	—.0000732 cos $3g$	+.0002196 sin $3g$	+.0003357 cos $3g$
+.000020 cos $4g$	—.0000035 cos $4g$	+.0000142 sin $4g$	+.0000206 cos $4g$

Considering only those terms which are of the first order, the value of $D'_t R$ may be found in two ways, the agreement of which will afford a check upon the entire development of the perturbative function, and upon the computations of R and $\frac{\partial R}{\partial v}$. These are (1) by direct differentiation, with respect to the time as contained in the mean anomaly of a single planet, whereby each term in R of the form

$$R = \frac{m}{a_1} h \cos N$$

will produce in $D'_t R$ the term

$$D'_t R = -\frac{m}{a_1} inh \sin N.$$

and (2) by forming the expression

$$D'_t R = \frac{\partial R}{\partial v} \frac{dv_0}{dt} + \frac{\partial R}{\partial \rho} \frac{d\rho_0}{dt}.$$

As several "mechanical multiplications," like those indicated in this last expression, are to be performed, the following example of the form of computation is presented. It exhibits the formation of the product of those terms of $\frac{\partial R}{\partial v}$ in which $i' = -1$ by $\frac{dv_0}{dt}$.

i'	—2	—1	0	+1	2	3	4	5
$\frac{a_1}{m} \frac{\partial R}{\partial v}$ (sin)	+.02	+ 1.63	—287.42	+3481.14	+ 54.42	+ 6.988	+1.18	+0.10
× .046915	0	0	+ 08	— 13.48	+163.32	+ 2.553	+0.33	+0.06
	+.08	—13.48	+163.32	+ 2.55	+ 0.33	+ 0.056	0	0
× .002750	0	0	0	0.00	— 0.79	+ 9.576	+0.15	+ .02
	—.79	+ 9.58	+ 0.15	+ .02	0	0	0	0
× .000168	+.58	+ 0.01	0	0	0	— .048	+ .58	0
$\frac{a_1}{m'n} \frac{\partial R}{\partial v} \frac{dv}{dt}$ (sin)	—.11	— 2.26	—123.87	+3470.23	+217.28	+19.125	+2.24	+0.18

The multipliers on the left are each one-half the coefficient cos jg in the expression for $\frac{dv_0}{dt}$, and each product is placed in the two columns corresponding respectively to $N+jg$ and $N-jg$.

All the derivations of R_0 necessary in the computation of the perturbations of the first order are given in the following tables. First we have the values of D'_tR obtained by direct differentiation, as indicated in the preceding formulæ. Next we have $\frac{\partial R}{\partial v}$ and $\frac{\partial R}{\partial \rho}$, obtained by the formulæ (7) and (42). The products $\frac{\partial R}{\partial v}$ by $\frac{dv_0}{dt}$ and of $\frac{\partial R}{\partial \rho}$ by $\frac{\partial \rho}{dt}$, being formed in the simple way just pointed out, and with the values of the component factors just given, their sum is next shown. This sum should agree accurately with D'_tR. The discrepancies are shown in the next two columns. The only apparently large discrepancy is found in the argument $5g'-5l$. It probably arises from the incompleteness of the computation of R and $\frac{\partial R}{\partial v}$, so far as they depend on this argument. As the entire term does not amount to 0".01, I have not sought to correct it.

The great value of this check arises from the fact that it gives a complete control of the correctness of the development of the perturbative function, *ab initio*, since the two valves of D'_tR are derived from different terms of that development. It also controls all the computations except that of $\frac{\partial R}{\partial \rho}$. This quantity being multiplied by quantities of the order of the eccentricities in the second value of D'_tR, an error in its value will produce a discrepancy of only $\frac{1}{20}$ its own amount in D'_tR, and may therefore be overlooked. The derivative in question must therefore be checked by a complete duplicate computation.

In the column next following are given the integrating factors ν, for which the expression is

$$\nu = \frac{n}{i'n' + in} = \frac{1}{i + i'\frac{n'}{n}}.$$

For each value of i' the values of ν are therefore the reciprocals of a series of numbers in arithmetical progression, the common difference being unity.

	$D_t'R = \frac{m'}{a_1} n \times$		$\frac{\partial R}{\partial \nu} = \frac{m'}{a_1} \times$		$\frac{\partial R}{\partial \rho} = \frac{m'}{a_1} \times$	
g l'	sin	cos	sin	cos	cos	sin
0, 0	0	0	....	+ 0.487	—1244.31	....
1,	— 48.15	+ 48.96	+ 10.20	+ 49.07	— 63.52	—118.82
2,	— 0.64	+ 8.28	+ 4.48	+ 3.20	+ 3.42	— 10.14
3,	+ 0.45	+ 0.64	+ 0.50	— 0.08	+ 0.74	+ 0.34
4,	+ 0.16	0	+ 0.06	— 0.06	+ 0.10	+ 0.01
— 2,—1	0	+ 0.08	+ 0.02	— 0.04	+ 0.08	— 0.05
— 1,	— 0.03	+ 2.15	+ 1.63	+ 1.34	+ 1.25	— 0.11
0,	0	0	— 287.42	— 0.10	+ 276.62	+ 34.79
1,	+3481.42	— 1.33	+3481.14	— 1.04	—5267.70	+ 4.35
2,	+ 94.28	+ 71.06	+ 54.42	+ 71.23	— 200.44	—119.83
3,	+ 5.74	+ 15.65	+ 6.988	+ 9.490	— 2.02	— 17.37
4,	+ 1.30	+ 1.66	+ 1.18	+ 0.42	+ 1.16	— 1.11
5,	+ 0.15	+ 0.15	+ 0.10	0	+ 0.17	— 0.01
—1,—2	+ 0.17	+ 0.06	+ 0.21	— 0.16	+ 0.14	— 0.11
0,	0	0	— 6.58	+ 30.99	+ 6.34	+ 31.39
1,	+ 93.30	—430.35	+ 96.81	—430.37	— 87.14	—458.30
2,	— 408.28	— 20.58	— 410.42	— 12.01	— 676.60	— 7.76
3,	— 91.89	+ 64.32	— 57.01	— 64.98	— 98.97	— 94.93
4,	— 5.76	+ 18.30	+ 1.45	+ 12.96	— 0.67	— 19.65
5,	+ 0.88	+ 2.45	+ 1.30	+ 1.04	+ 1.56	— 1.79
6,	+ 0.27	+ 0.17	+ 0.20	— 0.02	+ 0.28	— 0.04
1,—3	— 38.72	— 17.03	— 38.75	— 17.29	+ 40.04	— 17.16
2,	+ 1.02	+ 18.80	+ 5.31	+ 19.14	+ 8.48	— 26.83
3,	— 254.31	+ 2.37	— 254.15	— 0.02	— 363.02	— 0.43
4,	— 71.84	+ 50.20	— 51.49	+ 49.90	— 75.66	— 66.45
5,	— 5.65	+ 16.95	— 0.23	+ 13.0	— 1.82	— 18.07
6,	+ 0.87	+ 2.76	+ 1.29	+ 1.5	+ 1.48	— 2.14
7,	+ 0.34	+ 0.24	+ 0.27	0	+ 0.35	— 0.07
3,—4	+ 3.03	+ 18.78	+ 1.34	+ 18.78	+ 6.33	— 25.26
4,	— 144.96	+ 3.60	— 144.78	+ 1.5	— 191.70	— 2.31
5,	— 49.90	+ 35.00	— 38.64	+ 34.58	— 52.17	— 43.7
6,	— 4.50	+ 13.86	— 0.9	+ 11.2	— 1.97	— 14.7
7,	+ 0.75	+ 2.65	+ 1.1	+ 1.6	+ 1.31	— 2.2
8,	+ 0.35	+ 0.26	+ 0.3	+ 0.1	+ 0.36	— 0.1
5,—5	— 79.05	+ 4.05	— 78.9	+ 2.4	— 99.3	— 2.1
6,	— 32.3	+ 22.7	— 26.3	+ 22.3	— 33.5	— 27.2
7,	— 3.4	+ 10.4	— 1.1	+ 8.7	— 1.7	— 11.1
8,	+ 0.7	+ 2.3	+ 0.9	+ 1.5	+ 1.1	— 1.0
9,	+ 0.3	+ 0.3	+ 0.3	+ 0.1	+ 0.3	+ 0.1

	$\frac{\partial R}{\partial v}\frac{dv_0}{dt}+\frac{\partial R}{\partial \rho}\frac{d\rho_0}{dt} = \frac{m'}{a_1}n \times$		Discrepancy.		ν	k_c	k_s
g l'	sin	cos	sin	cos			
0, 0	0	— 0.01	0	—.01	....	—1244.31	
1,	— 48.05	+ 48.92	+.10	—.04	+1.0	+ 32.78	— 20.90
2,	— 0.63	+ 8.27	+.01	—.01	+0.5	+ 4.06	— 1.86
3,	+ 0.45	+ 0.65	0	+.01	$+\frac{1}{3}$	+ 0.44	+ 0.08
4,	+ 0.11	— 0.03	—.05	—.03	+0.25		
—2,—1	— 0.03	+ 0.02	—.03	—.06	—0.206098	+ 0.08	— 0.08
—1,	— 0.05	+ 2.16	—.02	+.01	—0.259601	+ 1.23	— 1.23
0,	— 0.06	+ 0.09	—.06	+.09	—0.350623	+ 276.62	+ 34.79
+1	+3481.41	— 1.33	—.01	0	—0.539942	—1508.18	+ 5.78
2,	+ 94.20	+ 71.06	—.08	0	—1.173630	+ 20.85	—286.59
3,	+ 5.78	+ 15.63	+.04	—.02	+6.75940	— 79.56	+194.17
4,	+ 1.27	+ 1.66	—.03	0	+0.871126	— 1.10	+ 1.79
5,	+ 0.14	+ 0.11	—.01	—.04	+0.4656	+ 0.03	+ 0.13
—1,—2	+ 0.16	+ 0.10	—.01	+.04	—0.149162	+ 0.19	— 0.13
0,	+ 0.01	+ 0.07	+.01	+.07	—0.175312	+ 6.34	+ 31.39
+1,	+ 93.24	—430.37	—.06	—.02	—0.212580	— 47.47	—275.33
2,	— 408.28	— 20.60	0	—.02	—0.269971	— 897.05	+ 3.35
3,	— 91.90	+ 64.33	—.01	+.01	—0.369806	— 166.93	—142.50
4,	— 5.77	+ 18.17	—.01	—.13	—0.586813	— 7.44	— 41.13
5,	+ 0.91	+ 2.45	+.03	0	—1.42022	+ 4.05	— 8.75
6,	+ 0.29	+ 0.15	+.02	—.02	+3.3797	— 1.53	+ 1.09
1,—3	— 38.78	— 17.03	—.06	0	—0.132342	+ 29.78	— 12.65
2,	+ 1.02	+ 18.68	0	—12	—0.152528	+ 8.79	— 32.56
3,	— 254.29	+ 2.37	+.02	0	—0.179981	— 454.55	— 1.28
4,	— 71.88	+ 50.13	—.04	—.07	—0.219482	— 107.20	— 88.50
5,	— 5.69	+ 17.01	—.04	+.06	—0.281202	— 5.00	— 27.61
6,	+ 0.91	+ 2.73	+.04	—.03	—0.391210	+ 2.15	— 4.30
7,	+ 0.35	+ 0.19	+.01	—.05	—0.6426	+ 0.79	— 0.38
3,—4	+ 3.10	+ 18.78	+.07	0	+0.11893	+ 7.05	— 29.73
4,	— 145.0	+ 3.51	—.04	—.09	—0.13500	— 230.83	— 3.28
5,	— 49.9	+ 34.85	0	—.15	—0.15605	— 67.74	— 54.60
6,	— 4.7	+ 13.95	—.20	+.09	—0.18490	— 3.63	— 19.80
7,	+ 0.8	+ 2.62	+.05	—.03	—0.22685	+ 1.65	— 3.36
8,	+ 0.3	+ 0.2	—.05	—.06	—0.2934	+ 0.57	— 0.27
5,—5	— 79.3	+ 2.7	—.25	—1.35	—0.1080	— 116.4	— 3.5
6,	— 32.4	+ 22.7	—.10	0	—0.1211	— 41.3	— 32.7
7,	— 3.5	+ 10.5	—.10	+.10	—0.1377	— 2.6	— 13.9
8,	+ 0.8	+ 2.3	+.10	0	—0.1597	+ 1.3	— 2.6
9,	+ 0.3	+ 0.2	0	—.10	—0.1901	+ 0.5	— 0.2

The values of $\frac{a_1}{m}\int D'_i R_0 dt$ are formed from $\frac{a_1}{m'n} D'_i R$ by simple multiplication by ν, and proper changes of sign. The values of k_c and k_s are then formed by adding the terms of $2\frac{a_1}{m'}\int D'_i R_0 dt$ to the corresponding terms of $\frac{a_1}{m'}\frac{\partial R}{\partial \rho}$.

Perturbations of radius vector.

Let us now resume equation (19), and put for brevity

$$M=\frac{m'a}{a_1(1+m)}. \tag{44}$$

If we give to u the successive values 0, +1, —1, +2, —2, +3, —3, we have

$$r_1^2\delta\rho=M\times\left\{\begin{array}{l}\quad\frac{1}{4}\Sigma_i\, p_iq_i\,(\nu_i-\nu_{-i})\,\{k_c\cos N+k_s\sin N\}\\ +\frac{1}{8}\Sigma_i(p_iq_{(i-1)}+p_{(i-1)}q_i)(\nu_i-\nu_{(1-i)})\,\{k_c\cos(N+g)+k_s\sin(N+g)\}\\ +\frac{1}{8}\Sigma_i(p_iq_{(i+1)}+p_{(i+1)}q_i)(\nu_i-\nu_{-(i+1)})\,\{k_c\cos(N-g)+k_s\sin(N-g)\}\\ +\frac{1}{8}\Sigma_i(p_iq_{(i-2)}-p_{(i-2)}q_i)(\nu_i-\nu_{(2-i)})\,\{k_c\cos(N+2g)+k_s\sin(N+2g)\}\\ +\quad\text{etc.}\qquad\text{etc.}\qquad\text{etc.}\qquad\text{etc.}\end{array}\right\}$$

the finite integral being taken with respect to all values of i from $-\infty$ to $+\infty$, and the terms in which the angles $N\pm ug$ vanish being omitted. Proceeding farther to expand with respect to i, if we collect similar terms we shall find the individual terms in $r^2\delta\rho$ to be as follows:

$$\begin{aligned}
r_1^2\delta\rho=\tfrac{1}{2}M&\left\{\begin{array}{l}\quad p_1q_1\,(\nu_1-\nu_{-1})\\ +\ p_2q_2\,(\nu_2-\nu_{-2})\end{array}\right\}\{k_c\cos N+k_s\sin N\}\\
+\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_1\ (\nu_1-\nu_0)\\ +(p_1q_2+p_2q_1)\ (\nu_2-\nu_{-1})\\ +(p_2q_3+p_3q_2)\ (\nu_3-\nu_{-2})\\ +\quad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N+g)+k_s\sin(N+g)\}\\
+\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_1\ (\nu_0-\nu_{-1})\\ +(p_1q_2+p_2q_1)\ (\nu_1-\nu_{-2})\\ +(p_2q_3+p_3q_2)\ (\nu_2-\nu_{-3})\\ +\quad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N-g)+k_s\sin(N-g)\}\\
+\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_2\ (\nu_2-\nu_0)\\ +(p_1q_3+p_3q_1)\ (\nu_3-\nu_{-1})\\ +\quad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N+2g)+k_s\sin(N+2g)\}\\
+\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_2\ (\nu_0-\nu_{-2})\\ +(p_1q_3+p_3q_1)\ (\nu_1-\nu_{-3})\\ +\quad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N-2g)+k_s\sin(N-2g)\}\\
+\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_3\ (\nu_3-\nu_0)\\ +(p_1q_4+p_4q_1)\ (\nu_4-\nu_{-1})\\ +(p_1q_2-p_2q_1)\ (\nu_2-\nu_1)\\ +\quad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N+3g)+k_s\sin(N+3g)\}
\end{aligned}$$

$$+\frac{1}{4}M\left\{\begin{array}{l} p_0q_3\ (\nu_0-\nu_{-3}) \\ +(p_1q_4+p_4q_1)\ (\nu_2-\nu_{-4}) \\ +(p_1q_2-p_2q_1)\ (\nu_{-1}-\nu_{-2}) \\ +\quad \text{etc.} \qquad \text{etc.} \end{array}\right\}\{k_c\cos(N-3g)+k_s\sin(N-3g)\}$$

A law of the factors of $k_c\cos(N+ug)+k_s\sin(N+ug)$ which will be noticed in the above expression, is this: Representing this factor by K_u, we have

$$K_u^i = K_{-u}^{(i+u)},$$

the index i representing the coefficient of g in N, so that only half the values of K need be separately computed.

As the computation of $r_1^2\delta\rho$ from these formulæ can be arranged in such a way as to be very simple, the computation of the terms in which the index i' is -1 is here presented quite fully. The logarithms only are omitted, being used only in the cases in which they are more convenient than a table of products. In practice I find it convenient to write them in red ink immediately under the numbers which they represent.

First, to find M, it will be noticed that in the expression $\frac{m'a}{a_1(1+m)}$, the a in the numerator represents the mean distance of the *disturbed* planet, as deduced from the observed mean motion by the equation $a^3n^2=\mu\,(1+m)$ while a_1 represents the mean motion of the *outer* planet. When the outer planet is the disturbed one, the ratio $\frac{a}{a_1}$ would be unity, but that, to avoid a large class of second order terms, a_1 has been corrected for perturbations in the beginning (p. 32). In the case of Uranus disturbed by Saturn, we have in consequence

$$\log\frac{a}{a_1}=9.999803.$$

Whence

$$M=285.44$$

in units of the sixth place of decimals.

Computing the values of p_i and q_i from (16)' we find, for Uranus,

$$\left.\begin{array}{lll} \frac{1}{2}M\ p_1q_1 & =+ & 142.56 \\ \frac{1}{2}M\ p_2q_2 & =+ & 0.0784 \\ \frac{1}{4}M\ p_0q_1 & =- & 10.044 \\ \frac{1}{4}M(p_1q_2+p_2q_1) & =+ & 3.3433 \\ \frac{1}{4}M(p_2q_3+p_3q_2) & =+ & 0.0028 \\ \frac{1}{4}M\ p_0q_2 & =- & 0.2358 \\ \frac{1}{4}M(p_1q_3+p_3q_1) & =+ & 0.118 \\ \frac{1}{4}M\ p_0q_2 & =- & 0.008 \\ \frac{1}{4}M(p_1q_4+p_4q_1) & =+ & 0.005 \\ \frac{1}{4}M(p_1q_2-p_2q_1) & -+ & 0.003 \end{array}\right\}\text{In units of the sixth place of decimals.}$$

In the computation the first three lines are copied from previous pages.

$i' = -1$										
i	—4	—3	—2	—1	0	+1	2	3	4	5
ν	—0.1459	—0.1709	—0.20610	—0.25960	—0.35062	—0.53994	—1.17363	+6.75940	+0.87113	+0.4656
		k_c	+0.08	+1.23	+276.62	—1508.18	+ 20.85	— 79.56	—1.10	+0.03
		k_s	—0.08	—1.23	+ 34.79	+5.79	—286.59	+194.17	+1.79	+0.13
	+142.56	$\nu_1-\nu_{-1}$	— .0887	— .14452	— .28034	— .82301	+7.29934	+2.04476	—6.2938	—0.5535
	+ 0.0784	$\nu_2-\nu_{-2}$	— .2047	— .3690	— .9675	+7.0190	+1.2218	+1.0055	+1.4913	—6.5186
	— 10.044	$\nu_1-\nu_0$	— .0535	— .0910	— .1893	— .63368	+7.9330	—5.8883	—0.4056	
	+ 3.3433	$\nu_2-\nu_{-1}$	— .1797	— .3338	— .9140	+7.1100	+1.4110	+1.6392	—6.4417	
	+ 0.0028	$\nu_3-\nu_{-2}$	— .39	—1.00	+6.96	+1.13	+0.82	+0.86	+1.41	
	— 0.2358	$\nu_2-\nu_0$	— .144	— .280	— .823	+7.299	+2.045	—6.294		
	+ 0.118	$\nu_3-\nu_{-1}$	— .369	— .968	+7.019	+1.222	+1.005	+1.490		
	— 0.008	$\nu_3-\nu_0$	— .33	— .91	………	+1.41	+1.64			
	+ 0.005	$\nu_4-\nu_{-1}$	—1.00	+6.97	………	+0.82	+0.86			
	+ 0.0003	$\nu_2-\nu_{-1}$	— .09	— .19	………	+7.93	—5.89			
	142.56	$\times(\nu_2-\nu_{-1})$	—12.35	—20.60	—39.965	—117.327	+1040.59	+291.49	—897.2	—78.8
	0.0784	$\times(\nu_3-\nu_{-2})$	— 0.02	— 0.03	— 0.075	+0.550	+0.10	+0.08	+0.1	— 0.5
		K_0	**—12.37**	**—20.63**	**—40.040**	**—116.777**	**+1040.69**	**+291.57**	**—897.1**	**—79.3**
	— 10.044	$\times(\nu_1-\nu_0)$	+0.532	+0.914	+1.901	+ 6.364	—79.68	+59.14	+ 4.1	
	+ 3.3433	$\times(\nu_2-\nu_{-1})$	—0.601	—1.126	—3.056	+23.770	+ 4.72	+ 5.48	—21.5	
	+ 0.0028	$\times(\nu_3-\nu_{-2})$	—0.001	—0.003	+0.020	+ 0.003	0	0	0	
		K_1	………	**—0.20**	**—1.135**	**+30.137**	**—74.96**	**+64.62**	**—17.4**	
		K_{-1}	………	**—0.07**	**—0.205**	**— 1.135**	**+30.14**	**—74.96**	**+64.6**	
	— .2358	$\times(\nu_2-\nu_0)$	+0.034	+0.066	+0.194	—1.721	—0.48	+ 1.48		
	+ .118	$\times(\nu_3-\nu_{-1})$	—0.043	—0.114	+0.828	+0.144	+0.12	+ 0.18		
		K_2	………	………	**+1.022**	**—1.577**	**—0.36**	**+ 1.66**		
		K_{-2}	………	………	**—0.009**	**—0.048**	**+1.02**	**+ 1.58**		
	— .008	$\times(\nu_1-\nu_0)$	+ .003	+ .007	………	—.011	—.013			
	+ .005	$\times(\nu_4-\nu_1)$	— .005	+ .035	………	+.004	+.004			
		K_3	………	………	………	**—.007**	**—.009**			
		K_{-3}	………	………	………	**—.002**	**+.042**			
		$K_0\ k_c$	— 1	— 26	—11076	+ 176117	+21698	—23197	+991	—2
		$K_1\ k_c$	0	0	0	—314	—45452	—1563	—5142	+19
		$K_{-1}\ k_c$	0	— 56	+1712	+628	+5964	—71	0	0
		$K_2\ k_c$	0	0	0	0	+ 282	+2378	—8	—132
		$K_{-2}\ k_c$	— 3	+ 72	+21	+126	0	0	0	0
		$K_3\ k_c$	0	0	0	0	0	0	+8	0
		$K_{-3}\ k_c$	+ 3	+ 1	0	0	0	0	0	0
		$r_1^2\,\delta\rho$ (cos)	**— 1**	**— 9**	**—9343**	**+176557**	**—17508**	**—22453**	**—4151**	**—115**
		$\times .046915$	0	0	0	—438	+8283	—821	—1053	—195
			0	—438	+8283	—821	—1053	—195	— 5	
		$\times .002751$	0	0	0	0	—26	+485	—48	—62
			—26	+485	—48	—62	—11	0	0	0
		$\times .000168$	+30	— 3	— 4	— 1	0	—2	+30	—3
		$\cos\psi\,\delta\rho$ (cos)	**+ 3**	**+ 34**	**—1112**	**+175235**	**—10315**	**—22986**	**—5227**	**—375**
		$K_0\ k_s$	+ 1	+ 26	—1393	—676	—298254	+56614	—1606	—10
		$K_1\ k_s$	0	0	0	— 39	+175	+21483	+12548	—30
		$K_{-1}\ k_s$	0	— 7	—6	—8637	—14556	+116	—2	0
		$K_2\ k_s$	0	0	0	0	+36	—9	+104	+322
		$K_{-2}\ k_s$	0	0	—293	—306	— 1	0	0	0
		$K+_3\ k_s$	0	— 11	0	0	0	0	0	—1
		$r_1^2\,\delta\rho$ (sin)	**+ 1**	**+ 8**	**—1692**	**—9658**	**—312600**	**+78204**	**+11044**	**+281**
		$\times .046915$	0	0	0	— 79	—453	—14666	+3669	+518
			0	— 79	—453	—14666	+3669	+518	+13	0
		$\times .002751$	0	0	0	0	— 5	—27	—860	+215
			— 5	— 27	—860	+215	+30	0	0	0
		$\times .000168$	— 5	— 53	+ 13	+2	0	0	—2	—3
		$\cos\varphi\,\delta\rho$ (sin)	**— 9**	**— 151**	**—2992**	**—24186**	**—309359**	**+64029**	**+13864**	**+961**

In forming the next ten lines, it will be noticed that the value of ν_u corresponding to any vertical column is found u columns to the right. It is therefore necessary to extend the line ν two columns at each end. The extension on the right is, however, omitted for want of space. In performing the subtractions it will be convenient to copy the ν's again on the lower edge of a horizontal strip of paper, and, in forming the differences $\nu_u - \nu_u'$ to lay the strip above the line of ν's, and $u-u'$ columns to the right.

On the left of each line of differences is written the factor by which that line is to be multiplied.

The mode of formation of the K's is evident from the formula.

It will be seen that the same computation which gives K_u gives also K_{-u}, only the latter belongs u columns to the right.

Each k_c and k_s is multiplied in succession by all the K's which lie below it in the same column, but the product by K_u is to be written u columns to the right, and that by K_{-u} u columns to the left. The sum of the products in any one column gives the coefficient of $\frac{\cos}{\sin}(ig+i'l')$ in the development of $r_1^2\delta\rho$.

This quantity being multiplied by $\frac{dv_0}{ndt} = \frac{a^2 \cos\psi}{r_0^2}$ we have $\cos\psi\delta\rho$, which only needs to be multiplied by $\sec\psi = 1.001103$ to give $\delta\rho$. The units of $r_1^2\delta\rho$ and $\delta\rho$ correspond to the ninth place of decimals.

All the periodic terms are to be treated in this manner, all the series of values of k_c and k_s, including the constant term, being subjected to the same process. But, when i' and i are both zero, ν will be infinite. Here we simply omit the ν, treating it as if it were zero. We thus obtain the complete value of the terms with constant coefficients in $r_1^2\delta\rho$ and $\delta\rho$ which are given in the following table. The terms multiplied by the time are still to be computed. They are derived from (20), which may be put in the form

$$r_1^2\delta\rho = \tfrac{1}{2} M\{\eta\Sigma p_u k_c^{(u)} - \xi\Sigma q_u k_s^{(u)}\} nt.$$

This expression is computed thus:

u	p_u	$k_c^{(u)}$	q_u	$k_s^{(u)}$	$p_u k_c^{(u)}$	$-q_u k_s^{(u)}$
0	$-.140783$	-1244.31	0	0	$+175.18$	0
1	$+.99917$	$+\ 32.78$	$+.99972$	-20.90	$+\ 32.75$	$+20.89$
2	$+.0234$	$+\ 4.06$	$+.0234$	$-\ 1.86$	$+\ 0.09$	$+\ 0.04$

We have now

$$\Sigma p_u k_c^{(u)} = +208.02; \qquad \tfrac{1}{2} M\Sigma p_u k_c^{(u)} = +29689$$
$$-\Sigma q_u k_s^{(u)} = +\ 20.93; \qquad \tfrac{1}{2} M\Sigma q_u k_s^{(u)} = +\ 2988$$

$$\begin{aligned} r_1^2\delta\rho = &-210\,nt \\ &+2986\,nt\cos g \quad +29681\,nt\sin g \\ &+\ 70\,nt\cos 2g \quad +\ 697\,nt\sin 2g \\ &+\ 2\,nt\cos 3g \quad +\ 24\,nt\sin 3g, \end{aligned}$$

in units of the ninth place of decimals. The value of $\cos\psi\delta\rho$ is obtained from them by multiplying by r_1^{-2}, exactly as in the case of the constant terms.

	$r_1^2\delta\rho$		cos $\psi\delta\rho$		log M_ν	ν_0	ν_1
g l'	cos	sin	cos	sin			
0, 0	—210 nt		—70 nt			+139$n't$	
1,	+2986 nt	+29681 nt	+2978 nt	+29633 nt			
2,	+70 nt	+697 nt	+209 nt	+2084 nt			
3,	+ 2 nt	+24 nt	+14 nt	+139 nt			
0, 0	—355045		—354347		∞	0	
1,	+14875	—1503	—18411	—1496	2.45551	—2912	+14206
2,	—283	+49	—1536	—21	2.25448	—639	+457
3.	—20	— 1	—112	— 2	1.97839	—46	—8
—2,—1	— 1	+ 1	+ 3	— 9	1.7696	+ 1	+2
—1,	—10	+ 8	+34	—151	1.8698	+12	—99
0,	—9343	—1692	—1112	—2992	2.00035	—2876	+10
+1,	+176557	—9658	+175235	—24186	2.18786	+536512	+160
2,	—17508	—312600	—10315	—309359	2.52504	+1823	—23860
3,	—22453	+78204	—22986	+64029	3.28542	—13482	+18310
4,	—4151	+11044	—5227	+13864	2.39559	—293	+104
5,	—115	+281	—373	+961	2.1235	—13	0
—1,—2	— 1	+ 1	+ 1	— 2	1.6292	+ 9	+7
0,	—56	—276	+32	—100	1.6993	—329	—1551
+1,	+710	+3723	+1675	+3728	1.78303	+5874	+26114
2,	+20180	—13	+20583	+484	1.88683	—31660	+926
3,	+7797	+6512	+8824	+6796	2.02348	—6019	—6860
4,	+1534	+6202	+2054	+6230	2.22401	+243	—2172
5,	+2071	—5954	+2134	—5610	2.60786	+527	—421
6,	—653	+755	—550	+494	2.98438	—190	—20
1,—3	—151	+64	—137	+75	1.5772	—1464	+653
2,	—52	+221	+149	+229	1.63886	+231	—833
3,	+4348	+15	+4420	+88	1.71074	—13050	+1
4,	+1564	+1280	+1774	+1314	1.79691	—3225	—3126
5,	+133	+685	+214	+756	1.9045	—18	—1043
6,	—111	+232	—109	+272	2.0479	+144	—163
7,	—163	+87	—168	+100	2.2634	+50	—7
3,—4	—28	+121	+31	+122	1.5408	+181	—638
4,	+1223	+17	+1245	+41	1.5858	—5578	—58
5,	+485	+389	+545	+400	1.6488	—1721	—1540
6,	+38	+201	+63	+221	1.7225	—50	—590
7,	—25	+53	—23	+63	1.8012	+70	—100
8,	—15	+ 7	—16	+ 9	1.9229		
5,—5	+392	+12	+400	+18	1.4889	—2432	—74
6,	+175	+138	+194	+142	1.5385	—908	—760
7,	+14	+77	+22	+84	1.5946	—40	—340
8,	—10	+20	— 9	+24	1.6589	+40	—70
9,	— 5	+ 3	— 5	+ 4	1.7345		

Perturbations of Longitude.

The perturbations of the longitude are now to be computed by formulæ (24). To do this in the most simple way we remark that the numbers given on page 42, under the heading $\frac{\partial R}{\partial \mathrm{v}}$, are those represented in formula (42) by v_s and v_c. If we put

$$nt = t'$$

equation (24) may be put into the form

$$\frac{d\delta v}{dt'}, = r_1^{-2}\left\{\frac{a}{1+m}\int\frac{\partial R}{\partial \mathrm{v}}\,dt' - 2\cos\psi\delta\rho\right\};$$

but we have from (42)

$$\int\frac{\partial R}{\partial \mathrm{v}}\,dt = \Sigma\left(\frac{m'\nu}{a_1}v_c\sin N - \frac{m'\nu}{a_1}v_s\cos N\right).$$

If now we represent the numerical values of $\cos\psi\delta\rho$, already found, by

$$\Sigma\,(\rho_s\sin N + \rho_c\cos N),$$

and if we substitute these expressions in the above value of $\frac{d\delta v}{dt'}$, the latter will become

$$\frac{d\delta v}{dt'} = r_1^{-2}\Sigma\,\{(\mathrm{v}_s - 2\rho_s)\sin N + (\mathrm{v}_c - 2\rho_c)\cos N\},$$

where we put for brevity

$$\mathrm{v}_s = \quad M\nu v_c,$$
$$\mathrm{v}_c = -M\nu v_s.$$

The numerical expression for r_1^{-2} is given on page 40, and by multiplying the quantities within brackets by this expression, after the manner explained on pages 40 and 41, we form the terms of $\frac{d\delta v}{dt'}$. Multiplying each of these terms by its corresponding value of ν, changing *cos* to *sin* and *sin* to *cos*, we have the coefficients in the expressions for δv given on page 50.

As previously mentioned, before commencing the above computation, I had computed all the perturbations of Uranus by the method of "perturbations of the elements," using the formulæ developed in my Investigation of the Orbit of Neptune. The two results are here placed side by side, for the purpose of comparison. The discrepancies in the various coefficients, expressed in thousandths of a second, are shown in the sixth and seventh columns.

It will be seen that the largest discrepancies, and indeed the only ones (with a single exception) exceeding one-tenth of a second, occur in the coefficients of the terms $2y-l$ aug $3g-l$. Here the errors are almost certainly in the computation from perturbations of the elements. Owing to the long period of the term $3g-l$ they would not become sensible in the course of any one century.

Perturbations of the Longitude of Uranus produced by the action of Saturn, and depending on the first power of the disturbing forces.

	From comp. preceding δv		From pert. of elements δv		Discrepancy.		0.434294 $\delta\rho$	
g, l'	sin	cos	sin	cos	sin	cos	cos	sin
	"	"	"	"	"	"		
0, 0	......	+ 10.9690t	......	+10.9645t	......	.0045t		
1,	— 1.230 nt	+ 12.231 nt	—1.228 nt	+12.271 nt	.002 nt	.040 nt	+13 nt	+128nt
2,	— 0.072 nt	+ 0.717 nt	—0.072 nt	+ 0.720 nt	0	.003 nt	+1 nt	+9nt
3,	— 0.004 nt	+ 0·043 nt	—0.004 nt	+ 0.043 nt	0	0	0	0
0, 0	......	......	......	......	....	....	—1541	
1,	+ 8.545	— 4.735	+ 2.844	+ 1.013	....	....	— 80	— 6
2,	+ 0.461	— 0.169	+ 0.133	+ 0.166	....	....	— 7	0
3,	+ 0.028	— 0.005	+ 0.013	+ 0.014	....	....	0	0
—1,—1	+ 0.036	+ 0.039	+ 0.032	+ 0.005	4	34	0	— 1
0,	+ 1.282	+ 0.718	+ 1.280	+ 0.719	2	1	— 5	— 13
1,	—20.817	+ 8.522	— 20.873	+ 8.595	56	73	+761	— 106
2,	—11.890	+143.463	— 11.093	+143.465	797	2	— 45	—1351
3,	+ 49.30	+115.86	+ 49.62	+116.08	320	220	—103	+ 280
4,	+ 2.133	+ 5.616	+ 2.195	+ 5.621	62	5	— 24	+ 61
5,	+ 0.126	+ 0.329	+ 0.109	+ 0.331	17	2	— 2	+ 4
0,—2	+ 0.017	— 0.017	+ 0.025	— 0.033	8	16	0	0
1,	+ 0.042	+ 0.814	+ 0.034	+ 0.818	8	4	+ 7	+ 16
2,	+ 4.110	— 0.009	+ 4.103	— 0.012	7	3	+ 89	+ 2
3,	+ 2.079	— 1.607	+ 2.106	— 1 676	27	69	+ 38	+ 30
4,	+ 0.648	— 1.830	+ 0.643	— 1.902	5	72	+ 9	+ 27
5,	+ 1.163	+ 2.956	+ 1.274	+ 2.991	111	35	+ 9	— 24
6,	+ 0.503	+ 0.378	+ 0.556	+ 0.445	53	67	— 2	+ 2
1,—3	+ 0.034	+ 0.012	+ 0.036	+ 0.015	2	3	0	0
2,	+ 0.037	— 0.041	+ 0.014	— 0.050	23	9	+ 1	+ 1
3,	+ 0.824	— 0.019	+ 0.812	— 0.017	12	2	+ 19	0
4,	+ 0.355	— 0.267	+ 0.351	— 0.263	4	4	+ 8	+ 6
5,	+ 0.047	— 0.165	+ 0.039	— 0.191	8	26	+ 1	+ 3
6,	— 0.026	— 0.063	— 0.028	— 0.066	2	3	0	+ 1
7,	— 0.053	— 0.032	— 0.053	— 0.018	0	14	— 1	0
3,—4	+ 0.006	— 0.022	— 0.005	— 0.023	11	1	0	+ 1
4,	+ 0.228	— 0.008	+ 0.221	+ 0.002	7	10	+ 5	0
5,	+ 0.103	— 0.077	+ 0.084	— 0.075	19	2	+ 2	+ 2
6,	+ 0.013	— 0.044	+ 0.013	— 0.057	0	13	0	+ 1
7,	— 0.005	— 0.013	— 0.001	— 0.015	4	2	0	0
8,	— 0.003	— 0.002	0	+ 0.002	3	4	0	0
5,—5	+ 0.074	— 0.003	+ 0.071	0	3	3	+ 2	0
6,	+ 0.038	— 0.027	+ 0.023	— 0.026	15	1	+ 1	+ 1
7,	+ 0.005	— 0.016	+ 0.005	— 0.025	0	9	0	0
8,	— 0.002	— 0.004	0	— 0.003	2	1	0	0

Perturbations of Latitude.

These are computed from the formulæ (27) and (40), no reductions being made from δk and $\delta\eta$ to δp and δq, but the perturbations of the latitude being computed directly from the former by (40). We have only to represent the expressions for δk and $\delta\eta$ by

$$\begin{aligned} \delta k &= -\Sigma a_c \cos N - \Sigma a_s \sin N \\ \delta\eta &= \quad \Sigma a'_c \cos N + \Sigma a'_s \sin N \end{aligned}$$

and substitute ω for π in the equations (40) from which $\delta\beta$ is computed.

The principal steps of the computation are shown quite fully in the following table. The values of

$$\frac{\partial h}{\partial \gamma} = \frac{1}{2} \cos \tfrac{1}{2}\gamma \frac{\partial h}{\partial \sigma}$$

are first formed from those terms of h, on pages 37 and 38, which contain σ as a coefficient. Then, having for each original term of R

$$\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \frac{\partial h}{\partial \gamma} \cos N$$

all the terms which have the same coefficients of λ and λ' in N are combined into two depending on g and l' as shown in the case of R on page 36. The coefficients of these terms, in units of the third place of decimals, are given in the columns headed $\frac{\partial R}{\partial \gamma}$.

The value of $\frac{ih}{2\sigma}$ sin N being formed for each term of R, all the terms depending on the same multiples of λ and λ' are combined into two, of which the coefficients are given under the proper heading. The terms of $(i+j)\,\sigma h$ sin N being formed in like manner, we have, by adding the last two expressions, all the quantities which enter into the formulæ (27). To integrate these equations thus forming the numerical values of δk and $\delta\eta$ we have only to multiply each term in the second, third, eighth, and ninth columns of the table by the corresponding values of $\frac{m'a\nu}{a_1 \cos \psi}$, for which we may use the value of $\frac{M\nu}{\sin 1''}$ already given.

The quantities given in the four columns under δk and $-\delta\eta$ show the values of $-a_s$, $-a_c$, $-a'_c$, $-a'_s$, corresponding to each argument. From these the terms of $\delta\beta$ are formed by equation (40) with the modification mentioned above.

Perturbations of the Latitude produced by Saturn.

g l'	$\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \times$		$\Sigma \frac{ih}{2\sigma} \sin N$		$\Sigma (i+j)\tau h \sin N$		$\frac{dn}{dt} = Mn \times$		δk		$-\delta\eta$		$\delta\beta$	
i i'	cos	sin	sin	cos	sin	cos	sin	cos	sin ″	cos ″	cos ″	sin ″	sin ″	cos ″
0, 0	−10.82	0	0	0	0	+0.008	0.00	+ .008						+0.192
1,	− 3.45	− 2.92	−1.94	− 0.11	+ 0.17	+0.84	− 1.77	+ 0.73	+0.203	−0.172	−0.104	−0.043	+0.246	+0.209
2,	+ 1.42	−11.10	+1.30	+10.73	+ 0.07	+0.05	+ 1.37	+10.78	−0.042	−0.326	+0.042	−0.317	−0.049	+0.015
3,	+ 1.31	− 1.71	+1.31	+ 1.71	+ 0.01	0.00	+ 1.32	+ 1.71	−0.026	−0.033	+0.026	−0.033	−0.008	+0.003
−2, −1	+ 1.0	+ 0.1	−1.0	− 0.1	0.00	0.00	− 1.0	− 0.1	+0.012	−0.001	+0.012	−0.001	0.000	0.000
−1,	− 6.43	−52.6	+6.4	−52.8	+ 0.03	+0.02	+ 6.4	−52.8	−0.098	+0.804	−0.098	−0.804	−0.004	+0.018
0,	− 6.97	+ 7.06	+1.53	+ 5.14	− 4.91	0.00	− 3.38	+ 5.14	−0.144	−0.145	+0.070	+0.106	−0.661	+0.576
+1,	+46.67	0	0.00	0.00	+59.45	−0.02	+59.45	− 0.02	+1.482	0	−1.887	−0.001	+0.084	−0.026
2,	− 1.84	− 2.86	−0.99	− 0.20	+ 0.93	+1.22	− 0.06	+ 1.02	−0.127	+0.197	+0.004	+0.070	+2.218	+1.920
3,	+ 0.90	− 6.98	+0.78	+ 6.44	+ 0.12	+0.16	+ 0.90	+ 6.60	−0.358	−2.781	+0.358	−2.630	+0.091	−0.071
4,	+ 1.09	− 1.38	+1.09	+ 1.38	+ 0.02	+0.01	+ 1.11	+ 1.39	−0.056	−0.071	+0.057	−0.071	−0.048	+0.056
−1, −2	+ 7.6	− 1.5	−7.6	+ 1.5	0.00	0.00	− 7.6	+ 1.5	+0.067	+0.013	+0.067	+0.013	+0.003	+0.004
0,	+ 1.3	+10.9	−1.3	+10.7	− 0.11	+0.53	− 1.4	+11.2	+0.013	−0.112	+0.014	+0.116	+0.008	+0.034
+1,	− 0.27	+ 9.03	+1.70	+ 0.91	+ 1.65	−7.35	+ 3.35	− 6.44	−0.003	−0.112	−0.042	−0.080	+0.040	−0.042
2,	−14.51	0	0.00	0.00	− 7.01	−0.21	− 7.01	− 0.21	−0.230	0	+0.112	−0.003	0	−0.029
3,	− 2.85	− 2.29	−0.42	− 0.19	− 0.97	+1.11	− 1.39	+ 0.92	−0.062	+0.050	+0.030	+0.020	−0.063	−0.054
4,	+ 0.43	− 4.23	+0.44	+ 3.67	+ 0.02	+0.22	+ 0.46	+ 3.89	+0.015	+0.146	−0.016	+0.134	−0.080	−0.014
5,	+ 0.80	− 0.99	+0.80	+ 0.99	+ 0.02	+0.02	+ 0.82	+ 1.01	+0.067	+0.083	−0.068	+0.084	+0.008	−0.004
2, −3	− 1.17	+ 0.86	+1.22	+ 0.97	+ 0.09	+0.33	+ 1.31	+ 1.30	−0.010	−0.008	−0.011	+0.012	+0.050	−0.046
3,	− 8.54	0	0.00	0.00	− 4.34	0	− 4.34	0.00	−0.090	0	+0.046	0	+0.004	−0.010
4,	− 2.08	− 1.65	−0.13	− 0.15	− 0.88	+0.85	− 1.01	+ 0.70	−0.026	+0.022	+0.013	+0.009	−0.012	−0.012
5,	+ 0.14	− 2.50	+0.24	+ 2.00	0.00	+0.22	+ 0.24	+ 2.22	+0.002	+0.041	−0.004	+0.037	−0.010	−0.002
6,	+ 0.55	− 0.66	+0.55	+ 0.66	+ 0.02	+0.02	+ 0.57	+ 0.68	+0.013	+0.015	−0.013	+0.016	+0.004	−0.001
3, −4	− 0.72	+ 0.47	+0.82	+ 0.78	+ 0.09	+0.32	+ 0.91	+ 1.10	−0.005	−0.003	−0.006	+0.008	+0.022	−0.020
4,	− 4.80	0	0.00	0.00	− 2.48	−0.02	− 2.48	− 0.02	−0.038	0	+0.020	0	+0.002	−0.005
5,	− 1.41	− 1.09	−0.01	− 0.10	− 0.66	+0.59	− 0.67	+ 0.49	−0.013	+0.010	+0.006	+0.004	−0.004	−0.003
6,	+ 0.12	− 1.3	+0.12	+ 1.0	− 0.02	+0.19	+ 0.10	+ 1.2	+0.001	+0.014	−0.001	+0.013	+0.004	+0.002
5, −5	− 2.64	0	0	0	− 1.35	+0.04	− 1.35	+ 0.04	−0.017	0	+0.008	0		

Secular terms $\begin{cases} \delta k = +\ 4''.77\ T \\ \delta\beta = -\ 4''.77\ T \cos v. \end{cases}$

CHAPTER III.

PERTURBATIONS PRODUCED BY NEPTUNE AND JUPITER.

THE perturbations of Uranus by Neptune were originally computed with elements of both planets quite different from those finally adopted. But the last computations, on which the concluded values of the perturbations depend, were made with the concluded elements of Neptune found in my investigation of the orbit of that planet.[1] They are as follows:

	°	′	″
π,	43	17	30
θ,	130	7	33
ε,	335	5	39
ϕ,	1	47	1.6
n,			7864.935
e,	0.0084962		
$\log a$,	1.478141		
Mass,	$\frac{1}{17000}$		

Hence follow the following functions of the elements of Neptune and Uranus:

$$\alpha = 0.638195$$
$$\omega = 12^\circ\ 44'\ 58''$$
$$\omega' = 247\ \ 45\ \ 20$$
$$\gamma = 1\ \ 30\ \ 29.6$$
$$\sigma = \sin\tfrac{1}{2}\gamma = 0.013161$$
$$M = 37.522 \text{ (in units of 6th place of decimals).}$$

From these values of the elements are obtained the following values of the various terms in the development of the perturbative function, and of ν. As the developments have been formed on the same principle as in the case of Saturn, it is deemed unnecessary to give the details of the process. It is only necessary to remark that the indices i' and i are the coefficients of l' and g respectively, the mean longitude of Neptune, or l', being counted from the perihelion of Uranus.

[1] Smithsonian Contributions to Knowledge, Vol. XV.

Action of Neptune.

		$R=\frac{m'}{a_1}\times$		$\frac{\partial R}{\partial v}=\frac{m'}{a_1}\times$		$\frac{\partial R}{\partial \rho}=\frac{m'}{a_1}\times$		$Q=\frac{m'}{a_1}\times$		ν
i'	i	h_c	h_s	v_s	v_c	cos	sin	k_c	k_s	
0,	0	+1135.63	0	0	+ 0.202	+368.26	0	+368.26	0	
	+ 1	− 18.07	− 1.25	+ 1.01	− 1.23	− 64.80	− 5.89	−100.94	− 8.39	+ 1.0
	+ 2	+ 0.41	− 0.03	− 0.6	− 0.1	+ 2.98	+ 0.12	+ 3.82	+ 0.06	+ 0.5
	+ 3	0	0	0	0	0	0	0	0	+ 0.33333
1,	− 3	+ 0.21	− 0.01	+ 0.7	− 0.2	+ 1.6	+ 0.2	+ 2.1	+ 0.1	− 0.40158
	− 2	− 5.21	− 0.62	− 4.90	+ 1.27	− 29 5	+ 0.6	− 43.5	− 1.1	− 0.67107
	− 1	+ 134.19	+ 0.30	+133.98	− 0.08	+509.94	+ 0.77	+1057.49	+ 1.99	− 2.04023
	0	− 25.45	−10 43	− 18.06	+ 0.02	− 84 57	−11.47	− 84.57	− 11.47	+ 1.96137
	+ 1	+ 1.12	+ 1.19	+ 0.5	− 0.1	+ 5.83	+ 0.82	+ 7.31	+ 1.07	+ 0.66232
	+ 2	− 0.05	+ 0.01	+ 0.02	+ 0.02	− 0.24	0	− 0.32	+ 0.02	+ 0.39843
2,	− 4	+ 0.85	− 0.03	+ 2.1	0	+ 2.5	+ 0.1	+ 4.8	0	− 0.33554
	− 3	+ 13.53	− 1.01	+ 27.7	+ 3.1	+ 18.7	− 2.4	+ 59.7	− 5.5	− 0.50497
	− 2	+ 375.92	+ 0.36	+751.59	− 0.76	+9 7.82	+ 1.25	+2491.69	+ 2.72	− 1.02009
	− 1	− 60.090	− 3.180	−117 903	+ 3.203	−171.79	−13.13	+5931.21	+309.82	+50.7820
	0	+ 3.31	+ 0 46	+ 5.9	− 0.4	+ 12.24	+ 1.84	+ 12.24	+ 1.84	+ 0.98069
	+ 1	− 0.09	− 0.01	− 0.09	+ 0.02	− 0.51	− 0.08	− 0.60	− 0.09	+ 0.49512
3,	− 5	+ 0.83	− 0.04	+ 2 7	+ 0.2	+ 3.0	− 0.1	+ 5.5	− 0.2	− 0.28814
	− 4	+ 12.09	− 0.96	+ 36.9	+ 3.8	+ 34.5	− 3.6	+ 73.6	− 6.7	− 0.40478
	− 3	+ 200.37	− 0.04	+601.13	− 0.42	+717.94	+ 0.81	+1535.51	+ 0.65	− 0.68006
	− 2	− 52 33	− 9.88	−150.02	+19.81	−190.33	−27.51	−635.23	−111.69	− 2.12559
	− 1	+ 5.05	+ 1.56	+ 13.8	− 3.1	+ 20.12	+ 4.65	+ 1.0	− 1.2	+ 1.88844
	0	− 0.24	− 0.07	− 0.58	+ 0.15	− 1.11	− 0.30	− 1.11	− 0.30	+ 0.65378
4,	− 6	+ 0.75	− 0.07	+ 3.2	+ 0.3	+ 3.4	− 0.2	+ 5.7	− 0.4	− 0.25249
	− 5	+ 9.47	− 0.79	+ 38.3	+ 4.0	+ 38.5	− 3.9	+ 70.4	− 6.6	− 0.33777
	− 4	+ 111.16	− 0.12	+444.7	− 0.18	+512.22	+ 0.35	+965.7	− 0.1	− 0.51004
	− 3	− 38.58	− 7.43	−149.10	+22.37	−178.15	−27.95	−419.12	− 74.36	− 1.04100
	− 2	+ 5.205	+ 1.896	+ 19.54	− 5.50	+ 25.30	+ 7.19	−503.3	−185.4	+25.3910
	− 1	− 0.36	− 0.17	− 1.28	+ 1.49	− 1.90	− 0.70	− 1.21	− 0.37	+ 0.96210
5,	− 7	+ 0.65	− 0.07	+ 3.5	+ 0.4	+ 3.5	− 0.3	+ 5.5	− 0.5	− 0.22468
	− 6	+ 6.97	− 0.59	+ 35.1	+ 3.4	+ 36.7	− 3.4	+ 60.9	− 5.5	− 0.28984
	− 5	+ 63.03	− 0.16	+315.3	+ 0.1	+354.8	− 0.1	+612.0	− 0.7	− 0.40804
	− 4	− 27.43	− 5.32	−133.4	+21.4	−153.6	−25.3	−304.8	− 54.6	− 0.68929
	− 3	+ 4.84	+ 1.81	+ 23.0	− 7.0	+ 27.82	+ 8.57	+ 92.2	+ 32.7	− 2.21843
	− 2	− 0.45	− 0.24	− 2.09	+ 0.90	− 2.77	− 1.14	+ 0.48	+ 0.57	+ 1.82073
	− 1	+ 0.03	+ 0.02	+ 0.12	− 0.06	+ 0.18	+ 0.09	+ 0.15	+ 0.07	+ 0.6455
6,	− 7	+ 4.93	− 0.41	+ 29 8	+ 2.7	+ 31.5	− 2.9	+ 49.0	− 4.4	− 0.2538
	− 6	+ 36.16	− 0.17	+217.1	+ 0.4	+239.9	− 0.4	+387.4	− 1.1	− 0.3400
	− 5	− 19.04	− 3 74	−111.6	+18.8	−125.7	−21.4	−223.8	− 40.6	− 0.5152
	− 4	+ 4.14	+ 1.59	+ 23.9	− 7.8	+ 27.76	+ 9.06	+ 63.0	+ 22.6	− 1.0628
	− 3	− 0.490	− 0.271	− 2.79	+ 1.30	− 3.42	− 1.55	+ 46.40	+ 26.01	+16.9273
	− 2	+ 0.036	+ 0.026	+ 0.20	− 0.12	+ 0.28	+ 0.16	+ 0.14	+ 0.06	+ 0.9443
7,	− 8	+ 3.41	− 0.28	+ 24.0	+ 2.1	+ 25.5	− 2.5	+ 37.8	− 3.5	− 0.2257
	− 7	+ 20.90	− 0.16	+146.4	+ 0.4	+160 0	− 0.6	+245.3	− 1.3	− 0.2914
	− 6	− 12.99	− 2.54	− 89 2	+15.4	− 98.9	−17.2	−163.1	− 29.8	− 0.4113
	− 5	+ 3.40	+ 1.31	+ 23.0	− 7.7	+ 26.0	+ 8.8	+ 49.8	+ 18.0	− 0.6988
	− 4	− 0.487	− 0.276	− 3.25	+ 1.61	− 3.79	− 1.87	− 12.83	− 6.99	− 2.3198
	− 3	+ 0.044	+ 0.034	+ 0.28	− 0.19	+ 0.37	+ 0.24	− 0.09	− 0.12	+ 1.7577
8,	− 9	+ 2.35	− 0.20	+ 18.8	+ 1.6	+ 19.5	− 2.1	+ 28.1	− 1.2	− 0.2032
	− 8	+ 12.20	− 0.13	+ 96.6	+ 0 3	+105.0	− 0.7	+154.8	− 1.3	− 0.2550
	− 7	− 8.8	− 1.71	− 68.9	+12.1	− 75.5	−13.1	−117.7	− 21.3	− 0.3423
	− 6	+ 2 69	+ 1.05	+ 21.0	− 7.1	+ 23.2	+ 8.1	+ 40.0	+ 14.7	− 0.5205
	− 5	− 0 45	− 0.26	− 3.5	+ 1.8	− 3.90	− 1.99	− 8.8	− 4.8	− 1.0855
	− 4	+ 0.049	+ 0.039	+ 0.37	− 0.26	+ 0.45	+ 0.31	− 4.50	− 3.69	+12.6955
9,	− 9	+ 7.0	− 0.1	+ 63.4	+ 0.1	+ 68.2	− 0.6	+ 96.8	− 1.0	− 0.2267
	− 8	− 5.9	− 1.13	− 51 9	+ 9.1	− 56 3	− 9.9	− 84.0	− 15.2	− 0.2931
	− 7	+ 2.07	+ 0.81	+ 18.1	− 6.4	+ 19.8	+ 7.0	+ 31.8	+ 11.7	− 0.4147
	− 6	− 0.39	− 0.25	− 3.4	+ 1.9	− 3.78	− 2.02	− 7.1	− 4.1	− 0.7085
	− 5	+ 0.050	+ 0.042	+ 0.43	− 0.32	+ 0.51	+ 0.36	+ 1.72	+ 1.38	− 2.4308
10,	−10	+ 4.1	− 0.1	+ 43.	0	+ 44.3	− 0.5	+ 61.	− 0.8	− 0.2040
	− 9	− 3.9	− 0.8	− 40.	+ 6.	− 42.0	− 7.0	− 60.	− 11.	− 0.2563
	− 8	+ 1.6	+ 0.6	+ 14.7	− 5.6	+ 16.1	+ 6.0	+ 25.1	+ 9.3	− 0.3446
	− 7	− 0.3	− 0.2	− 3.1	+ 1.9	− 3.5	− 1.9	− 5.8	− 3.4	− 0.5259
	− 6	+ 0.05	+ 0.04	+ 0.46	− 0.37	+ 0.53	+ 0.40	+ 1.16	+ 0.96	− 1.1092

The term of Long Period.

From the expressions for the perturbations of Uranus, subsequently given, it will be seen that several of the terms have very large coefficients, that of sin $(2l'-g)$ being nearly an entire degree. The magnitude of most of the terms in which i' is even arises from the near approach to commensurability in the mean motions of the two planets. Twice the mean annual motion of Neptune exceeds that of Uranus by only 303".8. The elements of the orbits of both planets will therefore, in consequence of their mutual action, be affected with a slow oscillation, having a period of about 4266 years. The employment of these large terms and the great inconveniences to which they will give rise, especially in the corrections of the elements of Uranus, may be avoided by the device employed in the theory of Neptune. The following are the essential features of this method:

First, all the perturbations arising from that portion of the perturbative function in which the coefficient of the time is $2n'-n$ or its multiples are considered and developed as perturbations of the elements.

Secondly, the arbitrary constants to be added to the integrals of these perturbations are so taken that the perturbations shall vanish at the epoch 1850.0.

In other words, the perturbations in question will be treated as producing secular variations of the elements of the orbit, only, instead of being developed in powers of the time, these variations will be retained in their rigorous form.

The formulæ for the computations of the perturbations in question, are as follows:

Let

$$\frac{m'}{a_1} h \cos(i'l' + il + j'\omega' + j\omega) = \frac{m'}{a_1} \cos N$$

be any term of the perturbative function, h being a function of α, e, and σ.

$$\sin\psi = e$$
$$g = \cos\psi \tan\tfrac{1}{2}\psi$$
$$\iota = -(i' + i + j' + j)$$
$$\nu = \frac{n}{i'n' + in}.$$

For each such term, compute

$$A = \quad 2ih$$
$$L = -3i\nu h - 2\frac{\partial h}{\partial \nu} + \frac{\partial h}{\partial e}$$
$$eW = \quad \cos\psi \frac{\partial h}{\partial e}$$
$$E = -h(ig + j\cot\psi)$$
$$T = \quad \tfrac{1}{2}\frac{\partial h}{\partial \sigma}$$
$$I = \quad \tfrac{1}{2}\iota\frac{h}{\sigma} + (i + j)\sigma h.$$

The corresponding perturbations of the elements may then be put into the form

$$\begin{aligned}
\delta \log a &= M\nu A \cos N + \delta \varpi_0, \\
\delta l &= M\nu L \sin N + \delta l_0, \\
e\delta\pi &= M\nu e W \sin N + e\delta\pi_0, \\
\delta e &= M\nu E \cos N + \delta e_0, \\
\delta\gamma &= M\nu I \cos N + \delta\gamma_0, \\
\tan\gamma\, \delta\tau &= M\nu T \sin N + \tan\gamma\delta\tau_0.
\end{aligned}$$

Here, $\delta\varpi_0$, δl_0, etc., are arbitrary constants so taken that $\delta \log a$, δl, etc., shall vanish at the fundamental epoch.

All the terms depending on the same values of i' and i are to be combined into a single one. And it will save labor to make this combination at as early a stage as possible in the computation; that is, to multiply the various values of h, $\frac{\partial h}{\partial \varpi}$, $\frac{\partial h}{\partial e}$, $\frac{\partial h}{\partial \sigma}$, and E by the sines and cosines of $j'\omega' + j\omega$, and afterward proceed with the sums of the products according to the proper modification of the formulæ.

Thus are obtained the following long period perturbations of the elements of Uranus:

$$\begin{aligned}
\delta l = & - 3474''.32 \sin (2l' - g) + 180''.10 \cos (2l' - g) \\
& + 146.72 \sin (4l' - 2g) - 54.10 \cos (4l' - 2g) \\
& - 8.97 \sin (6l' - 3g) + 5.03 \cos (6l' - 3g) \\
& + 0.64 \sin (8l' - 4g) - 0.53 \cos (8l' - 4g) \\
& + \text{constant} = 3320''.18.
\end{aligned}$$

$$\begin{aligned}
e\delta\pi = & - 484''.96 \sin (2l' - g) + 0''.73 \cos (2l' - g) \\
& + 38.06 \sin (4l' - 2g) - 7.06 \cos (4l' - 2g) \\
& - 3.61 \sin (6l' - 3g) + 1.38 \cos (6l' - 3g) \\
& + 0.33 \sin (8l' - 4g) - 0.15 \cos (8l' - 4g) \\
& + \text{constant} = 465''.23.
\end{aligned}$$

$$\begin{aligned}
= & - 484''.21 \cos (2l' - g) - 0''.29 \sin (2l' - g) \\
& + 38.21 \cos (4l' - 2g) + 7.16 \sin (4l' - 2g) \\
& - 3.61 \cos (6l' - 3g) - 1.40 \sin (6l' - 3g) \\
& + 0.33 \cos (8l' - 4g) + 0.15 \sin (8l' - 4g) \\
& - \text{constant} = 158''.59.
\end{aligned}$$

$$\begin{aligned}
\delta\varpi = & + 2277 \cos (2l' - g) + 120 \sin (2l' - g) \\
& - 198 \cos (4l' - 2g) - 78 \sin (4l' - 2g) \\
& + 18 \cos (6l' - 3g) + 7 \sin (6l' - 3g) \\
& + \text{constant} = 630.
\end{aligned}$$

The variations of the elements which fix the position of the plane of the orbit are here omitted, because their nature is such that it is indifferent in which form they are developed.

These expressions are reduced to perturbations of the co-ordinates by the follow-

ing formulæ. Express the usual developments of the longitude and logarithm of radius vector in the form

$$v = l + \Sigma V_i \sin ig;$$
$$\rho = \varpi + \Sigma R_i \cos ig.$$

Put also

$$V'_i = \frac{\partial V_i}{\partial e}$$

$$V''_i = \frac{i}{e} V_i$$

$$R'_i = \frac{\partial R_i}{\partial e}$$

$$R''_i = -\frac{i}{e} R_i.$$

Express any set of corresponding terms of the preceding perturbations in the form

$$\begin{aligned} \delta l &= L_s \sin N + L_c \cos N; \\ e\delta l - e\delta\pi &= F_s \sin N + F_c \cos N; \\ \delta e &= E_c \cos N + E_s \sin N; \\ \delta\varpi &= A_s \cos N + A_s \sin N. \end{aligned}$$

We shall then have

$$\begin{aligned} 2\delta v = {} & \Sigma (V''_i F_s + V'_i E_c) \sin (N + ig) + \Sigma (V''_i F_s - V'_i E_c) \sin (N - ig) \\ & + \Sigma (V''_i F_c - V'_i E_s) \cos (N + ig) + \Sigma (V''_i F_c + V'_i E_s) \cos (N - ig) \\ & + 2\delta l \end{aligned}$$

$$\begin{aligned} 2\delta\rho = {} & \Sigma (R'_i E_c - R''_i F_s) \cos (N + ig) + \Sigma (R'_i E_c + R''_i F_s) \cos (N - ig) \\ & + \Sigma (R'_i E_s + R''_i F_c) \sin (N + ig) + \Sigma (R'_i E_s - R''_i F_c) \sin (N - ig) \\ & + 2\delta\varpi \end{aligned}$$

The numerical values of V', V'', R', and R'' are as follows:

$V'_1 = 1.99835$	$V''_1 = 1.99945$
$V'_2 = 0.11713$	$V''_2 = 0.11722$
$V'_3 = 0.00714$	$V''_3 = 0.00714$
$V'_4 = 0.00044$	$V''_4 = 0.00044$
$R'_0 = +0.02348$	
$R'_1 = -0.99753$	$R''_1 = +0.99917$
$R'_2 = -0.07020$	$R''_2 = +0.07030$
$R'_3 = -0.00466$	$R''_3 = +0.00467$

The final results of the entire computations are given in the following table: In the columns δv_1, we have the complete perturbations of the longitude computed by the direct method from the values of $\frac{\partial R}{\partial v}$, $\frac{\partial R}{\partial \rho}$, Q, etc., already given. Next we have, under the caption δv_2, the perturbations of the true longitude deduced from the long period perturbations of the elements, as set forth in the last paragraph, omitting the constants added to the perturbations. Under δv_3 we have the

perturbations of the longitude deduced from all the remaining terms of the perturbations of the elements. The sum of the columns δv_2 and δv_3 shows the entire perturbations computed by the method of variation of elements. Thus, in δv_1 and $\delta v_2 + \delta v_3$ we have two complete sets of perturbations computed by methods entirely independent. The differences of the results, expressed in thousandths of a second, are given in the last two columns of the table.

This comparison gives rise to remarks similar to those suggested by the perturbations of Saturn computed by the same methods. The only terms in which the difference of results amounts to as much as one-tenth of a second are those of very long period, and those very nearly the period of Uranus, where a more accurate value is not at present of great importance, because the error will be compensated by the corrections of the element during several centuries.

Perturbations of the Longitude of Uranus produced by Neptune.

	δv_1		δv_2		δv_3		Discrepancy.	
l' g	sin	cos	sin	cos	sin	cos	sin	cos
	"	"						
0, 0		—0.4262 t						
—1	— .065 nt	—1.181 nt						
—2	— .004 nt	—0.069 nt						
0, 0								
—1	+ 0.697	— 0.088						
—2	+ 0.046	— 0.005						
—3	+ 0.003							
1,—3	+ 0.147	— 0.001	....	....	+ 0.146	—0.002	1	1
—2	+ 2.509	— 0.019	....	....	+ 2.509	—0.010	0	9
—1	+ 39.658	— 0.080	....	....	+39.673	—0.081	15	1
0	+ 4.257	— 0.511	....	....	+ 4 249	—0.478	8	33
—1	+ 0.280	— 0.032	....	....	+ 0.275	—0.032	5	0
—2	+ 0.017	— 0.002						
2,—4	+ 2.978	+ 0.027	+ 2.89	+ 0.03	+ 0.098	—0.002	10	1
—3	+ 49.015	+ 0.413	+ 47.22	+ 0.44	+ 1.797	—0.021	2	6
—2	+ 840.93	+ 7.388	+ 805.64	+ 7.43	+35.355	—0.028	65	14
—1	—3475.4	+ 180.36	—3474.32	+180.10	— 0.700	+0.095	380	165
0	— 162.07	+ 8.01	— 161.96	+ 8.01	— 0.067	+0.015	43	15
—1	— 9.447	+ 0.468	— 9.50	+ 0.47				
3,—5	— 0.076	0.000	....	....	— 0.077	—0.003	1	3
—4	— 1.162	0.000	....	....	— 1.153	—0.011	9	11
—3	— 17.286	+ 0.228	....	....	—17.285	+0.229	1	1
—2	— 22.085	+ 4.037	....	....	—22.077	+4.020	8	17
—1	— 0.673	+ 0.082	....	....	— 0.682	+0.079	9	3
0	— 0.037	+ 0.006						
4,—6	— 0.036	+ 0.002	— 0.015	+ 0.002	— 0.027	—0.003	6	3
—5	— 0.558	+ 0.037	— 0.25	+ 0.04	— 0.315	—0.007	7	4
—4	— 7.968	+ 0.750	— 4.08	+ 0.68	— 3.908	+0.059	20	11
—3	— 75.00	+ 12.832	— 69.55	+11.67	— 5.733	+1.067	283	95
—2	+ 146.78	— 54.218	+146.72	—54.10	+ 0.126	—0.079	66	39
—1	+ 6.960	— 2.579	+ 6.81	— 2.63				

Perturbations of the Longitude—*Continued.*

	δv_1		δv_2		δv_3		Discrepancy.	
l' g	sin	cos	sin	cos	sin	cos	sin	cos
	"	"						
5,— 7	—0.009	0.000	....	....	—0.015	—0.002	6	2
— 6	—0.103	+0.006	....	....	—0.113	—0.004	10	10
— 5	—0.986	—0.042	....	....	—0.994	—0.042	8	0
— 4	+3.366	—0.670	....	....	+3.370	—0.662	4	8
— 3	+3.210	—1.169	....	....	+3.227	—1.186	17	17
— 2	+0.077	—0.017	....	....	+0.075	—0.002	2	15
— 1	+0.005	—0.001						
6,— 7	—0.050	—0.004	0.00	0.00	—0.054	—0.002	4	2
— 6	—0.387	—0.020	+0.02	—0.01	—0.423	—0.013	16	3
— 5	+1.261	—0.320	+0.40	—0.14	+0.855	—0.169	6	11
— 4	+7.781	—2.825	+6.80	—2.54	+0.857	—0.318	124	33
— 3	—9.025	+5.073	—8.97	+5.03	+0.017	—0.022	72	65
— 2	—0.437	+0.246	—0.42	+0.26				
7,— 8	—0.027	—0.002						
— 7	—0.186	—0.003	....	....	—0.189	—0.009	3	6
— 6	+0.272	—0.046	....	....	+0.260	—0.047	12	1
— 5	—0.571	+0.217	....	....	—0.538	+0.202	33	15
— 4	—0.459	+0.250	....	....	—0.419	+0.255	40	5
— 3	—0.010	+0.005						
8,— 9	—0.013	0.000						
— 8	—0.084	—0.002	....	....	—0.093	—0.007	9	5
— 7	+0.125	—0.022	....	....	+0.115	—0.024	10	2
— 6	—0.194	+0.082	—0.038	+0.017	—0.145	+0.055	11	10
— 5	—0.839	+0.463	—0.66	+0.30	—0.110	+0.067	69	96
— 4	+0.647	—0.501	+0.64	—0.53				
9,— 9	—0.040	—0.001	....	....	—0.048	—0.005	8	4
— 8	+0.063	—0.011	....	....	+0.002	—0.011	61	0
— 7	—0.055	+0.020	....	....	—0.059	+0.025	4	5
— 6	+0.080	—0.049	....	....	+0.083	—0.054	3	5
— 5	+0.059	—0.050						
10—10	—0.020	0.000	....	....	—0.025	—0.005	5	5
— 9	+0.035	—0.005						
— 8	—0.027	—0.010						
— 7	+0.026	—0.017						
— 6	+0.092	—0.079						

The perturbations of the logarithms of the radius vector are given in a form similar to those of the longitude. Under $\delta\rho_1$ we have the complete perturbation. Under $\delta\rho_2$ the effect of the perturbations of long period. But under $\delta\rho_3$ we have only the difference between $\delta\rho_1$ and $\delta\rho_2$, it being deemed unnecessary to present in full the perturbations of the radius vector as computed by the other method. $\delta\rho_1$ being employed in computing δv_1 may, in fact, be regarded as completely checked by its affording a correct value of the latter.

In the last two columns $\delta\rho_3$ is reduced to common logarithms by multiplying the coefficients by the modulus 0.434294.

PERTURBATIONS OF THE LOGARITHM OF THE RADIUS VECTOR OF URANUS PRODUCED BY NEPTUNE.

l' g	$\delta\rho_1$		$\delta\rho_2$		$\delta\rho_3$		$M\delta\rho$	
	cos	sin	cos	sin	cos	sin	cos	sin
0, 0	+138	....	....	....	....	....	+ 60	0
—1								
—2								
1,—3	+ 4	0	....	....	....	....	+ 2	0
—2	+ 68	0	....	....	....	....	+ 30	0
—1	+523	+ 1	....	....	....	....	+227	0
0	— 68	— 6	....	....	....	....	— 30	— 3
+1	— 8	— 1	....	....	....	....	— 3	0
2,—4	+ 94	— 1	+92	— 1	+ 2	0	+ 1	0
—3	+1416	— 12	+ 1374	— 12	+ 42	0	+ 18	0
—2	+20025	—179	+19490	—180	+535	+ 1	+232	0
—1	+1663	+116	+ 1726	+120	— 63	— 4	— 27	— 2
0	+3912	+194	+ 3927	+194	— 15	0	— 7	0
3,—5	— 3	0	....	....	....	....	— 1	0
—4	— 38	0	....	....	....	....	— 17	0
—3	—527	— 6	....	....	....	....	—229	— 3
—2	—284	—54	....	....	....	....	—124	—23
—1	+ 16	+ 2	....	....	....	....	+ 7	+ 1
4,—6	— 1	0						
—5	— 19	— 1	— 8	....	— 11	— 1	— 5	0
—4	—254	— 22	— 118	— 20	—136	— 2	— 59	— 1
—3	—1759	—300	—1681	—282	— 78	—18	— 34	— 8
—2	—143	— 61	— 154	— 70	+ 11	+ 9	+ 5	+ 4
—1	—167	— 62	— 165	— 64	— 2	+ 2	— 1	+ 1
5,—6	— 4	0	....	....	....	....	— 2	0
—5	— 40	+ 1	....	....	....	....	— 17	0
—4	+103	+20	....	....	....	....	+ 45	+ 9
—3	+ 39	+15	....	....	....	....	+ 17	+ 6
6,—7	— 2	0	....	....	— 2	0	— 1	0
—6	— 17	+ 1	....	....	— 17	+ 1	— 7	0
—5	+ 42	+10	+ 10	+ 4	+ 32	+ 6	+ 14	+ 3
—4	+180	+65	+164	+ 61	+ 16	+ 4	+ 7	+ 2
—3	+ 13	+ 8	+ 14	+ 5	— 1	+ 3	0	+ 1
7,—7	— 8	0	....	....	....	....	— 3	0
—6	+11	+ 2	....	....	....	....	+ 5	+ 1
—5	—17	— 7	....	....	....	....	— 7	— 3
—4	— 5	— 3	....	....	....	....	— 2	— 1
8,—8	— 4	0	....	....	....	....	— 2	0
—7	+ 5	+ 1	....	....	....	....	+ 2	0
—6	— 7	— 3	....	....	....	....	— 3	— 1
—5	—19	—11	....	....	....	....	— 8	— 5
9,—9	— 2	0	....	....	....	....	— 1	0
—8	+ 3	0	....	....	....	....	+ 1	0
—7	— 2	— 1	....	....	....	....	— 1	0
—6	+ 2	+ 2	....	....	....	....	+ 1	+ 1

PERTURBATIONS OF THE LATITUDE PRODUCED BY NEPTUNE.

		$\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \times$		$2\frac{ih}{2\sigma}\sin N$		$2(i+j)\sigma \times h\sin N$		$\frac{d\eta}{dt} = m'an \times$		δk		$\delta\eta$		$\delta\beta$	
i'	i	cos	sin	sin	cos	sin	cos	sin	cos	sin	cos	cos	sin	sin	cos
										″	″	″	″	″	″
0	0	−21.65	0	0	0	0	0.00	0	0					0	−.012
	1	+ 1.08	+2.73	− 4.00	−1.66	+ .01	−0.01	− 3.99	−1.67	− 0.008	+0.021	+0.030	−0.012	+.080	+.020
	2	+19.32	−9.39	+19.32	+9.39	− .01	0	+19.31	+9.39	− 0.073	−0.036	−0.073	+0.036	+.016	−.002
1,	−3	+14.58	+7.0	−14.7	+7.0	0	0	−14 7	+7.0	+0.045	−0.022	−0.045	−0.022	+.007	−.002
	−2	− 1.16	−2.65	+ 4.04	−2.40	−0.07	+0.02	+ 3 97	−2.4	−0.006	+0.014	+0.020	+0.012	+.201	−.046
	−1	−34 26	0	0	0	+1.76	0	+ 1.76	0 ...	−0.540	0	+0.028	0	−.008	−.037
	0	+ 3.74	+2.32	− 2.80	−1.48	−0.24	0	− 3.04	−1.48	−0.057	+0.035	+0.046	−0.022	+.364	+.084
	+1	+15.77	−7.87	+16.25	+7.75	0	0	+16.25	+7.75	−0.081	−0.040	−0.083	+0.040	+.060	+.007
2,	−4	+10.52	+5.06	−10.6	+5.0	0	0	−10.6	+5.0	+0.027	−0.013	−0.027	−0.013	+.002	−.001
	−3	− 2.60	−2.38	+ 3.65	−2.28	+0.36	+0.04	+ 4.01	−2.24	−0.010	+0.009	+0.016	+0.009	+.062	−.014
	−2	−33.38	0	0	0	+9.88	−0.01	+ 9.88	−0.01	−0.264	0	+0.078	0	+.008	−.004
	−1	+ 5.81	+2.04	− 2.12	−1 04	−1.53	+0.04	− 3.65	−1.00	[−2.283]	[+0.802]	[+1.431]	[−0.393]	+.326	+.075
	0	+18.79	−9.52	+19.56	+9.32	+0.08	0	+19.64	+9.32	−0.143	−0.072	−0.049	+0.071	+.002	0
3,	−5	+ 7.4	+3.6	− 7.4	+3.5	0	0	− 7.4	+3.5	+0.017	−0.008	−0.017	−0.008	+.002	0
	−4	− 3.06	−2.00	+ 3.12	−2.0	+0.48	+0.05	+ 3.60	−2.0	−0.010	+0.006	+0.011	+0.006	+.023	−.004
	−3	−24.4	0	0	0	+7.87	0	+ 7.87	0	−0.128	0	+0.041	0	−.004	+.041
	−2	+ 6.58	+1.73	− 1.20	−0.54	−1.98	+0.26	− 3.18	−0.28	+0.108	−0.029	−0.053	+0.004	+.303	+.074
	−1	+13.66	−7.28	+14.68	+7.00	+0.18	−0.04	−14.86	+6.96	−0.199	−0.106	−0.217	+0.101	−.090	−.026
	0	− 1.51	+0.22	− 1.51	−0.22	0	0	− 1.51	−0.22	+0.008	+0.001	+0.008	−0.001	−.013	0
4,	−5	− 2.96	−1.63	+ 2.53	−1.7	+0.5	0	+ 3.0	−1.7	−0.008	+0.004	+0.008	+0.004	+.022	−.004
	−4	−17.4	0	0	0	+5.8	0	+ 5.8	0	−0.070	0	+0.023	0	−.008	+.006
	−3	+ 6.34	+1.34	− 0.59	−0.22	−1.97	+0.30	− 2.56	+0.08	+0.051	−0.011	−0.020	−0.001	+.048	+.010
	−2	+ 9.42	−5.37	+10 57	+5.04	+0.25	−0.07	+10.82	+4.97	[−1.852]	[−1.056]	[−2.127]	[+0.977]	−.047	0
	−1	− 1.72	+0.32	− 1.72	−0.32	−0.02	+0.02	− 1.74	−0.30	+0.013	+0.002	+0.013	−0.002	−.001	0
5,	−6	− 2.4	−1.40	+ 1.8	−1.4	+0.5	0	+ 2.3	−1.4	−0.005	+0.003	+0.005	+0.003	+.012	−.003
	−5	−12.0	0	0	0	+4.0	0	+ 4.0	0	−0.038	0	+0.013	0	−.012	0
	−4	+ 5.58	+1.01	− 0.22	−0.04	−1.75	+0.28	− 1.97	+0.24	+0.030	−0.005	−0.010	−0.001	−.109	−.028
	−3	+ 6.25	−3.89	+ 7.43	+3 54	+0.30	−0.09	+ 7.73	+3.45	+0.107	+0.067	+0.131	−0.059	−.038	+.001
	−2	− 1.66	+0.33	− 1.66	−0.33	−0.03	+0.01	− 1.69	−0.32	+0.024	+0.005	+0.025	−0.004	+.018	0
6,	−6	− 8.4	0	0	0	+2.8	0	+ 2.80	0	−0.022	0	+0.007	0	−.010	+.002
	−5	+ 4.63	+0.76	− 0.03	+0.04	−1.46	+0.24	− 1.49	+0.28	+0.019	−0.003	−0.006	−0.001	−.028	−.008
	−4	+ 4.03	−2.79	+ 5.15	+2.45	+0.31	−0.10	+ 5.46	+2 35	+0.033	+0.022	+0.045	−0.019	−.010	0
	−3	− 1.45	+0.31	− 1.45	−0.31	−0.04	+0.02	− 1.49	−0.29	[+0.190]	[+0.041]	[+0.195]	[−0.038]	+.015	0
7,	−7	− 5.6	0	0	0	+1.9	0	+ 1.9	0	−0.012	0	+0.004	0	−.006	+.001
	−6	+ 3.63	+0.61	0	0	−1.17	+0.20	− 1.17	+0.20	+0.012	−0.002	−0 004	−0.001	−.008	−,004
	−5	+ 2.49	−2.00	+ 3.52	+1.68	+0.30	−0.10	+ 3.82	+1.68	+0.013	+0.011	+0.020	−0.008	+.014	0
	−4	− 1.19	+0.27	− 1.19	−0.27	−0.04	+0.02	− 1.23	−0.25	−0.021	−0.005	−0.022	+0.004	+.002	0
8,	−8	− 4.0	0	0	0	+1.3	0	+ 1.3	0	−0.008	0	+0.003	0	−.003	+.002
	−7	+ 2.76	+0.50	0	0	−0.91	+0.16	− 0.91	+0.16	+0.007	−0.002	+0.002	0	−.003	−.002
	−6	+ 1.49	−1.41	+ 2.38	+1.14	+0.27	−0.09	+ 2.65	+1.05	+0.006	+0.006	+0.011	−0.004	+.004	0
	−5	− 0.97	+0.22	− 0.97	−0.22	0	+0.02	− 0.97	−0.20	−0.008	−0.002	−0.008	+0.002	+.002	0

Secular term, $\delta k = +1''.25\, T$
$\delta\beta = -1''.25\, T \cos \nu$

The terms δk and $\delta\eta$, which are inclosed in brackets, are of very long period, and are therefore omitted in forming the values of $\delta\beta$ in the last two columns

Perturbations produced by Jupiter.

The series in which these perturbations are expressed converge so rapidly that I deem it unnecessary to present the details of the computation. They have been computed by both methods, and the separate and independent results are given in the following table, where δv_1 represents the perturbations computed by the method developed in Chapter I, and δv_2 those computed by the method of variation of elements.

The apparently large discrepancy between the coefficients multiplied by the time arises from the circumstances that in the form of development the mean motion, and hence the mean anomaly, appears affected by the perturbation $31''.2t$. Accordingly when we enter the table which gives the true longitude in terms of the mean anomaly in the form

$$v = l + 2e \sin (l - \pi) + \text{etc.},$$

we may consider this quantity $31''.2t$ as a secular variation of $l - \pi$ producing in v the term

$$\delta v = 62''.4et \cos (l - \pi).$$

In δv_1 this term is left in its primitive form, while in δv_2 the value of l is supposed to include this term, and the secular terms are only those which arise from the secular variation of the eccentricity and perihelion.

It is also to be remarked that the terms which are independent of the mean longitude of Jupiter, or those in which $i' = 0$, are not comparable, as they correspond to slightly different elliptic elements in the two theories.

PERTURBATIONS OF URANUS BY JUPITER.

	δv_1		δv_2		Diff.		$\cos\varphi\,\delta\rho$		$M\delta\rho$	
g l'	sin	cos	sin	cos	sin	cos	cos	sin	cos	sin
	″	″	″	″						
0, 0	....	+31.2116t	....	+31.1982t						
1	— 0.160nt	+37.585nt	— 0.1622nt	— 1.5406nt	2					
2	— 0.010nt	+ 2.207nt	— 0.0095nt	— 0.0899nt	0					
3	....	+ 0.135nt	— 0.0006nt	— 0.0054nt	0					
0, 0	....	....	....	....	...	...	—10089	....	—4387.	0
1	+25.657	— 1.859	— 1.346	+ 1.361	...	...	—491.5	— 1.9	— 213.7	— 0.8
2	+ 1.397	— 0.087	— 0.030	+ 0.086	...	...	— 32.8	+ 1.1	— 14.2	+ 0.5
3	+ 0.072	— 0.005	....	....	...	...	— 1.8	0	— 0.8	0
—1,—1	+ 0.027	+ 0.009	+ 0.017	+ 0.011	10	2	0.0	— 0.5	0.0	— 0.2
0	+ 1.269	+ 0.002	+ 1.232	+ 0.001	37	1	— 58.8	— 0.5	— 25.6	— 0.2
1	—53.064	— 0.004	—53.084	— 0.002	20	2	+2585.7	+ 0.1	+1127.2	0
2	— 3.495	— 0.092	— 3.565	— 0.082	70	10	+195.3	+ 5.0	+ 82.8	+ 2.2
3	— 0.148	— 0.047	— 0.164	— 0.050	16	3	+ 17.1	+ 2.3	+ 7.4	+ 1.0
0,—2	— 0.027	— 0.011	— 0.031	— 0.014	4	3	+ 1.4	— 0.6	+ 0.6	— 0.3
1	+ 1.182	+ 0.515	+ 1.176	+ 0.515	6	0	— 56.5	+24.9	—24.6	+10.7
2	+ 0.277	+ 0.036	+ 0.263	+ 0.037	14	1	+ 8.5	+ 1.8	+ 3.7	+ 1.6
3	+ 0.074	— 0.005	+ 0.083	— 0.008	9	3	+ 4.2	+ 0.5	+ 1.8	+ 0.8
4	+ 0.015	— 0.003	+ 0.014	— 0.003	1	0	+ 0.9	+ 0.2	+ 0.4	+ 0.2
1,—3	— 0.032	— 0.034	— 0.025	— 0.025	7	9				
2	— 0.005	0	— 0.005	— 0.001	0	1				
3	+ 0.025	0	+ 0.015	0	10	0				
4	+ 0.011	— 0.001	+ 0.037	— 0.010	26	9				
5	+ 0.003	0	— 0.017	+ 0.004	20	4				

PERTURBATIONS OF THE LATITUDE.

g l'	δk		$\delta\eta$		$\delta\beta$	
i i'	sin	cos	cos	sin	cos	sin
	″	″	″	″	″	″
0 0	....	....	....	....	....	+.065
1	+0.071	—0.041	+0.030	+0.036	+.060	+.047
2	—0.012	—0.075	—0.012	+0.075	—.010	—.007
3	—0.003	—0.012	—0.003	+0.012	—.002	0
—2—1	—0.006	+0.042	+0.006	+0.042	+.005	—.004
—1	—0.297	+1.949	+0.297	+1.949	+.001	—.003
0	—0.161	—0.191	—0.189	—0.190	—.494	+.420
1	+2.611	0	+2.628	0	—.024	—.013
2	+0.070	+0.021	+0.066	—0.002	+.002	+.019
—3—2	+0.055	—0.088	—0.055	—0.088	+.010	—.003
0	+0.002	—0.011	—0.002	—0.011	+.017	—.004
1	—0.110	—0.050	—0.109	+0.046	—.009	+.002
2	—0.014	0	—0.011	0	—.002	0

Secular terms $\begin{cases} \delta k = \;\; 1.14''\ T \\ \delta\beta = -1.14\ T \cos v \end{cases}$

CHAPTER IV.

TERMS OF THE SECOND ORDER PRODUCED BY THE ACTION OF SATURN.

Preliminary Investigation of the Orbit of Saturn.

For the accurate determination of the perturbations of a planet it is essential that the functions of the time which are substituted for the co-ordinates of each planet in the expression of the disturbing forces should approximately represent the true places of the planet. The difference between the true place and that implicitly assumed in the investigation should be so small and of such a character that, when multiplied by the mass of the disturbing planet, and by the factors introduced by the process of integration, the result shall be insensible. If one of these factors is so large as to make a perturbation of an order of magnitude approximating that of the inequality which gives rise to it, it will represent an inequality of very long period in the elements, which, though apparently sensible, may be neglected for a great length of time.

The perturbations hitherto found have been computed on the hypothesis that the disturbing action of Saturn on Uranus is the same as if both planets moved in the elliptic orbits corresponding to the adopted elements. We have given formulæ for the computation of the corrected perturbations when, to the co-ordinates of the two planets corresponding to the adopted ellipse, we add corrections represented by δv, $\delta v'$, $\delta \rho$, etc. These corrections are now to be taken of such magnitude that when thus added they shall very nearly represent the actual motions of the planets.

Generally, it is considered sufficient to take for these corrections the perturbations of the first order. But this presupposes that the elliptic elements are nearly correct, which does not hold true in the case of the old elements of the outer planets. Bouvard's Tables of Saturn, the elements of which have been adopted, are subject to recurring errors amounting to 30″ or more. Moreover, when we substitute the new and more accurate perturbations for the old and imperfect ones adopted in the tables, the chances are that the errors will be increased. Desiring that the theory shall be as far as possible free from doubt, we begin with a preliminary investigation of the orbit of Saturn, the design of which will be to give the co-ordinates of that body in terms of the time with sufficient certainty and accuracy to serve for computing the perturbations both of Jupiter and Uranus. As usual, the first step in this investigation will be the determinations of the perturbations of the planet.

General Perturbations of Saturn.

The perturbations produced by Jupiter will be taken from the exhaustive prize memoir of Hansen.[1] As the perturbations required are those of the co-ordinates, it will be necessary to transform those of Hansen into the usual form. Hansen gives the true anomaly v in the form

$$v = g + n\delta z + e_1 \sin (g + n\delta z) + e_2 \sin 2 (g + n\delta z) + \text{etc.},$$

e_1, e_2, etc., being the coefficients of the multiples of the mean anomaly in the usual development of the elliptic true anomaly. Whence, neglecting the second power of $n\delta z$,

$$\delta v = n\delta z \, (1 + e_1 \cos g + 2e_2 \cos 2g + \text{etc.}).$$

To make the development sufficiently rigorous it is only necessary to increase g by $\frac{1}{2} n\delta z$ in this expression. In the same way, we have for the perturbations of log r,

$$\delta\rho = \delta\rho_0 + n\delta z \, (e^{(1)} \sin g + e^{(2)} \sin 2\, g + \text{etc.})$$

$\delta\rho_0$ being Hansen's perturbation, and $e^{(i)}$ the negative coefficient of cos ig in the development of the elliptic log r.

Hansen having adopted $\frac{1}{1070.5}$ as the mass of Jupiter, it will be necessary to multiply his perturbations by 1.0216 to reduce them to Bessel's mass. Thus the perturbations by Jupiter hereafter given have been obtained.

The perturbations by Uranus and Neptune have been computed by the preceding general method, and are given in the following table. In the table l' is the mean longitude of the disturbing planet, Uranus or Neptune, counted from the perihelion of Saturn. $\delta\rho$ is the perturbation of the Naperian logarithm, in units of the seventh place of decimals.

General perturbations of the Longitude in orbit and the Logarithm of the radius vector of Saturn.

Action of Uranus.					Action of Neptune.				
	δv		$\delta\rho$			δv		$\delta\rho$	
l' g	sin	cos	cos	sin	l' g	sin	cos	cos	sin
	″	″				″	″		
1, 0	+ 0.88	+ 0.92	— 12.7	+ 7.9	1,—0	+0.23	—0.05	— 2.2	—0.2
—1	+ 8.60	+ 0.21	+146.6	— 3.5	—1	+1.93	0.00	+39.9	0.0
2, 0	— 0.42	— 0.17	+ 10.2	— 3.7	2, 0	+0.44	+0.02	— 3.0	+0.1
—1	— 8.49	— 2.53	— 66.8	+21.4	—1	—1.11	+0.02	—18.5	—0.4
—2	—13.39	— 0.25	—392.5	+ 6.6	—2	—1.27	0.00	—41.1	0.0
3, 0	+ 0.06	+ 1.62	+ 0.5	+39.1	3,—1	+0.02	—0.03	+ 0.2	+0.4
—1	— 2.9	+28.1	+ 10.8	+33.0	—2	+0.10	—0.04	+ 3.2	+1.3
—2	—10.61	—20.88	—247.	+487.	—3	—0.10	0.00	— 3.9	0.0
—3	— 2.05	— 1.47	— 67.5	+42.8					
4,—1	+ 0.06	+ 0.05	— 0.1	+ 0.6					
—2	— 0.62	+ 0.80	— 10.5	—12.5					
—3	+ 0.30	+ 0.73	+ 10.5	—23.4					
—4	— 0.26	+ 0.03	— 10.6	— 0.8					

[1] Untersuchungen über die gegenseitigen Störungen des Jupiters und Saturns. Von P. A. Hansen. Berlin, 1831.

I have submitted these perturbations to such duplicate computations and other checks as lead me to believe that none of the terms can be in error by more than a small fraction of a second, but, as they are not intended to form the basis of a definitive theory of Saturn, I do not vouch for their absolute precision.

In this provisional correction of the orbit of Saturn only heliocentric longitudes have been employed. These were derived for a series of dates from Airy's reduction of the Greenwich observations, the modern Greenwich observations, and the Washington observations.

For these dates the value of $n\delta z$ for Saturn was computed from the formulæ found on pages 189 and 190 of the work of Hansen, already quoted, omitting all terms less than 1″, and including only tenths of seconds in the results. The dates, the resulting values of $n\delta z$, of the factor $e_1 \cos(g + \frac{1}{2} n\delta z) + 2e_2 \cos 2(g + \frac{1}{2} n\delta z)$, and of the concluded δv are as follows. The formulæ for δv is

$$\delta v = 1.0216\, n\delta z\, \{1 + e_1 \cos(g + \tfrac{1}{2} n\delta z) + 2e_2 \cos 2(g + \tfrac{1}{2} n\delta z)\}.$$

Date Gr. Mean Noon.	$n\delta z$	Factor.	δv
	″		″
1751 May 31	—1947.7	—.0990	—1792.7
1757 Aug. 7	—2134.2	—.0652	—2038.2
1758 Aug. 27	—2212.5	—.0474	—2153.2
1761 Oct. 6	—2546.2	+.0244	—2664.5
1763 Nov. 1	—2880.3	+.0729	—3157.1
1765 Nov. 23	—3095.1	+.1082	—3504.0
1773 Feb. 26	—3342.0	+.0419	—3557.2
1780 May 24	—2858.2	—.0956	—2640.7
1794 Nov. 16	—3321.1	+.1017	—3737.9
1802 Feb. 23	—3184.7	+.0529	—3425.5
1823 Nov. 13	—2716.3	+.0944	—3036.8
1831 Feb. 18	—3378.5	+.0639	—3671.7
1838 May 19	—2976.7	—.0866	—2777.9
1845 Aug. 17	—2342.7	—.0721	—2220.9
1852 Nov. 15	—2847.0	+.0863	—3159.3
1860 Feb. 14	—3161.4	+.0740	—3468.3
1867 May 15	—2373.1	—.0812	—2227.6

The perturbations by Uranus and Neptune were computed from the values of their terms just given. The principal terms, the sum of which make up the heliocentric longitude resulting from the adopted elements, are shown in the first of the following tables.

In the next table we have after the date the heliocentric longitude from Bouvard's Tables, as deduced from the longitudes given in Airy's reductions of the Greenwich Observations, from the *Astronomisches Jahrbuch* for 1831, and from the Nautical Almanac. Then follow the corrections, roughly deduced from observations made near the opposition. Adding these columns, we have the longitude

from observation. To the right of these are the equations of condition for the correction of the elements.

Long. of Perihelion.	Mean anomaly.	Equation of centre.	Perturbations by Jupiter.	Perturbations by Uranus.	Perturbations by Neptune.	Red. to Ecliptic.	Nutation.	True longitude.
° ′ ″	° ′ ″	° ′ ″	′ ″	″	″	′ ″	″	° ′ ″
88 41 27.5	159 55 50.1	+2 3 56.6	—29 52.7	—45.7	—2.7	+1 36.9	+15.8	250 12 25.8
88 46 41.2	236 13 7.1	—5 7 38.1	—33 58.2	—36.6	+1.0	—1 20.5	—11.8	319 16 4.1
88 47 31.4	248 25 53.1	—5 48 30.4	—35 53.2	—35.2	+1.7	—1 35.8	—15.0	330 46 36.6
88 50 7.6	286 26 29.8	—6 16 1.3	—44 24.6	— 3.3	+2.9	—0 43.5	—14.1	8 15 13.5
88 51 51 6	311 44 13.6	—5 0 58.2	—52 37.1	+11.9	+2.4	+0 43.3	— 4.9	34 43 22.6
88 53 35.1	336 55 56.1	—2 41 10.6	—58 24.0	+19.2	+1.9	+1 36.6	+ 6.7	62 12 1.0
88 59 39.6	65 40 2.3	+6 0 35.9	—59 17.2	—32.9	—0.8	—1 37.1	+ 4.8	159 38 54.6
89 5 43.4	154 8 4.8	+2 37 57.7	—44 0.7	—53.8	—1.7	+1 37.5	—14.5	245 8 13.2
89 17 50.7	331 6 10.4	—3 18 5.2	—62 17.9	+ 9.6	+3.5	+1 30.9	—14.1	56 5 7.9
89 23 55.7	59 56 17.9	+5 44 46.6	—57 5.5	—18.6	+2.5	—1 37.3	+ 1.4	154 6 2.7
89 42 7.0	325 22 26.0	—3 51 58.0	—50 36.8	—27.9	—3.0	+1 21.6	+14.7	50 23 3.6
89 48 11.9	54 10 33.1	+5 25 2.6	—61 11.7	+13.3	+2.5	—1 33.2	— 7.4	148 21 11.1
89 54 16.1	142 44 37.0	+3 40 38.1	—46 17.9	— 3.4	+2.1	+1 29.7	— 4.3	235 34 37.4
90 0 20.4	231 18 40.9	—4 47 36.6	—37 0.9	—35.5	+0.9	—1 11.8	+14.8	315 52 52.2
90 6 24.6	319 52 44.8	—4 21 53.0	—52 39.3	+ 4.7	—0.9	+1 8.8	—18.4	44 45 31.3
90 12 28.7	48 26 48.7	+5 1 46.4	—57 48.3	+30.3	—3.4	—1 25.1	+14.1	142 42 31.4
90 18 32.8	137 0 52.6	+4 9 31.2	—37 7.6	— 8.2	+1.3	+1 21.6	— 4.1	230 52 59.7

Date.	Tabular longitude.	Obs. cor.	Long. from observation.	Equations of Condition.				
		″		″				
1751 May 31	250 13 38.0	— 8.5	250 13 29.5	63.7 =	$+0.90\delta\varepsilon$	$-44\delta n$	$+0.60\delta e$	$+1.82e\delta\omega$
1757 Aug. 27	319 17 8.5	—18.0	319 16 50.5	46.4	0.94	—39	—1.53	+1.14
1758 Aug. 27	330 47 37.9	—14.3	330 47 23.6	47.0	0.96	—40	—1.76	+0.80
1761 Oct. 6	8 15 13.8	+ 0.2	8 15 14.0	0.5	1.03	—39	—1.99	—0.50
1763 Nov. 1	34 43 42.3	+15.3	34 43 57.6	35.0	1.07	—39	—1.63	—1.31
1765 Nov. 23	62 12 20.7	+20.4	62 12 41.1	40.1	1.11	—38	—0.88	—1.88
1773 Feb. 26	159 40 3.8	+18.8	159 40 22.6	88.0	1.04	—28	+1.93	—0.79
1780 May 24	245 9 17.6	+ 7.8	245 9 25.4	72.2	0.90	—18	+0.76	+1.74
1794 Nov. 16	56 5 38.8	+14.7	56 5 53.5	45.6	1.10	— 6	—1.08	—1.78
1802 Feb. 23	154 7 2.7	+10.8	154 7 13.5	70.8	1.05	+ 2	+1.85	—0.98
1823 Nov. 13	50 23 34.5	+21.3	50 23 55.8	52.2	1.09	+26	—1.27	—1.66
1831 Feb. 18	148 22 38.5	+ 3.0	148 22 41.5	90.4	1.07	+33	+1.75	—1.16
1838 May 19	235 36 11.2	+ 3.4	235 36 14.6	97.2	0.91	+35	+1.08	+1.56
1845 Aug. 17	315 53 42.2	+15.7	315 53 57.9	65.7	0.93	+42	—1.43	+1.28
1852 Nov. 15	44 46 15.4	+ 5.9	44 46 21.3	50.0	1.09	+57	—1.42	—1.53
1860 Feb. 14	142 44 16.2	—13.7	142 44 2.5	91.1	1.08	+65	+1.63	—1.35
1867 May 15	230 54 47.8	+ 5.1	230 54 52.9	113.2	+0.92	+62	+1.23	+1.45

A normal equation for $\delta\varepsilon$ is obtained by taking the sum of all the equations. That for δn is formed by subtracting the sum of the first seven from the sum of the last seven, and those for δe and $e\delta\omega$ by taking the sum of the equations in which the coefficients of δe or $e\delta\omega$ are greater than unity, after changing the signs of the equations in which they are negative. The normals thus obtained are

$$
\begin{array}{rrrrcr}
17.19\delta\varepsilon & +31\delta n & -2.16\delta e & -3.15e\delta\omega & = & +1069.3'' \\
0.04 & +587 & +6.83 & -0.69 & & +239.3 \\
-2.14 & +207 & +21.58 & +2.19 & & +208.1 \\
-2.11 & -60 & +3.61 & +19.66 & & +53.8
\end{array}
$$

These equations give

$$
\begin{aligned}
\delta\varepsilon &= +64.8'' \text{ (Epoch, 1800.)} \\
\delta n &= +\ 0.268 \\
\delta e &= +12.6 \\
e\delta\omega &= +\ 8.2
\end{aligned}
$$

—Substituting these values in the seventeen equations of condition we have the following residuals, or excesses of theoretical over observed longitudes:

1	$+7.2''$	10	$+10.7''$
2	-4.3	11	-6.4
3	-10.2	12	-1.1
4	$+21.7$	13	-0.4
5	-9.0	14	0.0
6	-7.1	15	$+3.7$
7	-11.3	16	$+4.1$
8	$+8.2$	17	-7.7
9	-6.4		

These residuals are much larger than they should be, and I scarcely know to what cause to attribute their magnitude. The results are however amply reliable for the purposes of the investigation, and lead to the following elements of Saturn:

	°	′	″
π,	90	6	26
ε,	14	50	3.2
θ,	112	20	0
ϕ,	2	29	39.2
n,	43996.395		
e,	.0560660		
$\log(a+\delta a)$,	0.979676		

Epoch, 1850, Jan. 0, Greenwich mean noon.

It will be seen that the adopted position of the plane of Saturn's orbit is retained. It was corrected from observations before the perturbations were finally computed.

Of the above corrections, those of the epoch and mean motion need not be taken account of in the corrections of the co-ordinates, since the mean longitude remains in the formulæ as an arbitrary quantity to the end. The effect of the correction of the mean distance is insensible. The corrections of eccentricity aud perihelion are therefore alone to be retained. They are allowed for by adding to δv and $\delta\varrho$ the terms

$$\delta v = 2\delta e \sin g - e\delta\omega \cos g$$
$$= +25''.2 \sin g - 16''.4 \cos g;$$

$$\delta \rho = -\delta e \cos g - e\delta\omega \sin g$$
$$= -12''.6 \cos g - 8''.2 \cos g.$$

Perturbations of Saturn and Uranus.

The following expressions include, with these corrections, all the perturbations of Saturn and Uranus which can produce any appreciable perturbations of the second order in their mutual action. In these expressions the initial letter of each planet is put for its mean longitude counted from the perihelion of Uranus.

PERTURBATIONS OF SATURN.

Argument.	$\delta v'$		$\delta \rho'$	
	sin	cos	cos	sin
	"	"	"	"
S	+ 25.2	+ 21.4	— 2.7	+ 9.0
S	+ 1.64t	— 1.48t	— 0.80t	— 0.74t
$S - J$	— 26.1	+ 6.2	+176.7	— 15.9
$2S - J$	—421.6	— 8.8	—115.8	— 9.1
$3S - J$	— 54.7	— 12.4	+ 12.3	— 16.6
$2S - 2J$	+ 32.6	— 1.0	+ 29.5	— 1.1
$3S - 2J$	+ 27.7	— 21.3	— 15.4	— 12.3
$4S - 2J$	+216.9	—769.2	+ 11.4	+403.0
$5S - 2J$	+2392.	—1692.	— 58.8	— 34.4
$6S - 2J$	+133.0	— 93.6	— 58.1	+ 56.9
$U - S$	+ 8.6	+ 0.2	+ 3.0	— 0.1
$2U - S$	+ 0.8	— 8.8	+ 0.1	+ 1.4
$2U - 2S$	— 13.4	+ 0.2	— 8.0	+ 0.1
$3U - S$	— 8.4	— 27.0	+ 0.1	— 0.7
$3U - 2S$	+ 18.3	— 14.6	+ 8.8	+ 7.0

PERTURBATIONS OF URANUS.

Argument.	δv		$\delta \rho$	
	sin	cos	cos	sin
	"	"	"	"
U	— 0.11t	— 0.28t	+ 0.05t	— 0.14t
$U - S$	—20.8	+ 8.5	+36.0	— 4.9
$2U - S$	—11.9	+143.5	— 2.0	—63.8
$3U - S$	+49.3	+115.9	— 4.7	+13.2
$2U - 2S$	+ 4.1	0.0	+ 4.2	+ 0.1
$3U - 2S$	+ 2.1	— 1.6	+ 1.8	+ 1.4
$4U - 2S$	+ 0.6	— 1.8	+ 0.4	+ 1.3
$5U - 2S$	+ 1.2	+ 3.0	+ 0.4	— 1.2

Let us now resume the equation

$$\delta Q = 2\int D'_t \delta R dt + \delta \frac{\partial R}{\partial \rho} - \frac{1}{\mu} D_t^2 (r_0^2 \delta\rho^2) + \frac{1}{2}\frac{\delta\rho^2}{r_0}.$$

Beginning with the last two terms of this expression, it may be shown at the outset that they are quite insensible. The effect of the constant terms in $\delta\rho$ and $\delta\rho'$ has already been included by correcting the logarithm of the mean distance by their amount; they are therefore omitted. The largest remaining term is 64″, the square of which is only 0″.02. In the product $r_1^2\delta\rho^2$ the largest terms are

$$\begin{aligned} &+ 0''.014 \\ &- 0.013 \sin g \\ &- 0.011 \sin (3g - 2l') \\ &- 0.011 \cos (4g - 2l') \end{aligned}$$

which may be entirely neglected.

We shall therefore only consider in δQ the terms

$$2\int D'_t \delta R dt + \delta \frac{\partial R}{\partial \rho}$$

As already remarked, R is rigorously a function only of V, ρ, and ρ', V being the angle made by the radii vectores of the two planets. But, in the analytical development of R, the quantity V is considered as a function of v, v′, and γ, so that we have

$$R = f(\rho, \rho', \mathrm{v}, \mathrm{v}', \gamma).$$

In the previous computation of the perturbations of Uranus, we have supposed R to be a function of ρ_0, ρ'_0, etc. The corrections to R and its derivatives with respect to v and ρ are now given by the equations (11), with the modifications shown on pages 24 to 27. The derivatives of R_0 which enter into these equations are formed as follows: If, in the value of R produced by the action of Saturn on Uranus, we consider any term of the form

$$\frac{m'h}{a_1} \cos N$$

where

$$N = i\lambda + i'\lambda' + j\omega + j'\omega'$$

the accented quantities always referring to Saturn, but a_1 being the corrected mean distance of Uranus, then we shall have the following terms in the derivatives of R.

$$\frac{\partial R}{\partial \mathrm{v}} = -\frac{m'h}{a_1}(i + j) \sin N$$

$$\frac{\partial R}{\partial \mathrm{v}'} = -\frac{m'h}{a_1}(i' + j') \sin N$$

$$\frac{\partial R}{\partial \rho} = -\frac{m'}{a_1}\left(h + \frac{\partial h}{\partial \mathfrak{v}}\right) \cos N$$

$$\frac{\partial R}{\partial \rho'} = \frac{m'}{a_1}\frac{\partial h}{\partial \mathfrak{v}} \cos N$$

$$\frac{\partial^2 R}{\partial \mathrm{v}^2} = -\frac{m'h}{a_1}(i + j)^2 \cos N$$

$$\frac{\partial^2 R}{\partial v \partial v'} = -\frac{m'h}{a_1}(i+j)(i'+j')\cos N$$

$$\frac{\partial^2 R}{\partial v \partial \rho} = \frac{m'}{a_1}\left(h+\frac{\partial h}{\partial \mathfrak{v}}\right)(i+j)\sin N$$

$$= -\frac{\partial R}{\partial v} - \frac{\partial^2 R}{\partial v \partial \rho'}$$

$$\frac{\partial^2 R}{\partial v \partial \rho'} = -\frac{m'}{a_1}\frac{\partial h}{\partial \mathfrak{v}}(i+j)\sin N$$

$$\frac{\partial^2 R}{\partial \rho \partial v'} = \frac{m'}{a_1}\left(h+\frac{\partial h}{\partial \mathfrak{v}}\right)(i'+j')\sin N$$

$$\frac{\partial^2 R}{\partial \rho^2} = \frac{m'}{a_1}\left(h+2\frac{\partial h}{\partial \mathfrak{v}}+\frac{\partial^2 h}{\partial \mathfrak{v}^2}\right)\cos N$$

$$= -\frac{\partial R}{\partial \rho} - \frac{\partial^2 R}{\partial \rho \partial \rho'}$$

$$\frac{\partial^2 R}{\partial \rho \partial \rho'} = -\frac{m'}{a_1}\left(\frac{\partial h}{\partial \mathfrak{v}}+\frac{\partial^2 h}{\partial \mathfrak{v}^2}\right)\cos N$$

All the numerical data necessary for the computation of these derivatives have been given in Chapter II. Combining the terms having the same argument, we find the following values, omitting those given in Chapter II, and those which are derived from the others by mere addition. The terms of $\frac{\partial^2 R}{\partial v^2}$ are also omitted, because they are sensibly the same with those of $\frac{\partial^2 R}{\partial v \partial v'}$, changing the algebraic sign.

	$\frac{a_1}{m'}\frac{\partial R}{\partial v'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial v \partial v'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial \rho \partial v'}$	
g l'	sin	cos	sin	cos	sin	cos
0,—1	+0.2874	0	0	+0.2872	+0.2691	0
—2	+0.0066	—0.0310	+0.0310	+0.0067	+0.0060	—0.0322
1, 0	—0.0102	—0.0491	+0.0490	—0.0101	+0.0249	+0.1194
—1	—3.4811	—0.0010	—0.0021	—3.4810	—5.2684	0
—2	—0.0968	+0.4304	—0.4304	—0.1039	—0.0792	+0.4583
—3	+0.0388	+0.0173	—0.0179	+0.0389	+0.0398	+0.0163
2, 0	—0.0045	—0.0035	+0.0021	—0.0080	+0.0155	+0.0066
—1	—0.0544	—0.0712	+0.1426	—0.0689	—0.1760	+0.2403
—2	+0.4104	+0.0120	—0.0171	+0.8233	—1.3515	+0.0014
—3	—0.0053	—0.0191	+0.0390	—0.0226	+0.0288	+0.0547
3,—1	—0.0070	—0.0093	+0.0171	—0.0210	+0.0197	+0.0304
—2	+0.0570	—0.0650	+0.1957	+0.1013	—0.1791	+0.2840
—3	+0.2542	0	—0.0065	+0.7624	—1.0884	—0.0071
4,—2	—0.0014	—0.0130	+0.0368	—0.0193	+0.0213	+0.0547
—3	+0.0515	—0.0499	+0.1987	+0.1450	—0.2144	+0.2647
—4	+0.1448	—0.0015	—0.0016	+0.5789	—0.7644	—0.0005

	$\frac{a_1}{m'}\frac{\partial R}{\partial \rho'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial v \partial \rho'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial \rho \partial \rho'}$	
g' l	sin	cos	sin	cos	sin	cos
0, 0	....	+0.172	....	+0.002	....	—0.645
—1	—0.026	—0.562	+0.556	0	+0.113	—0.610
1, 0	+0.070	+0.015	+0.015	+0.070	—0.227	—0.079
—1	—0.003	+8.749	—8.750	—0.001	+0.011	+6.189
—2	+0.889	+0.180	—0.176	+0.889	+0.912	+0.190
2,—1	+0.084	+0.248	—0.230	+0.169	—0.322	+0.020
—2	+0.018	+0.472	—0.941	+0.013	—0.006	—1.685
3,—1	+0.012	+0.004	+0.013	+0.021	—0.049	+0.028
—2	+0.073	+0.068	—0.122	+0.219	—0.344	—0.244
—3	0	+0.278	—0.834	—0.007	+0.010	—1.260
4,—2	+0.015	—0.001	+0.020	+0.042	—0.071	+0.019
—4	+0.001	+0.155	—0.619	0	+0.001	—0.854

The derivations with respect to γ and the node have been omitted because they are quite insensible. The terms of δR depending on these derivatives are given by equation (31). In the case of Uranus disturbed by Saturn the largest values of the coefficients

$$\tfrac{1}{2}\iota h \cot \tfrac{1}{2}\gamma;\ \tfrac{1}{2} h \tan \tfrac{1}{2}\gamma,\ \tfrac{1}{2}\frac{\partial h}{\partial \sigma}$$

are only about .05, while the largest coefficients in δk, $\delta k'$, and $\delta\gamma$ are less than 10″. Hence the largest terms in (31) will be of the order of magnitude 0″.5 multiplied by the mass of Saturn, and may therefore be omitted entirely. Omitting them the values of δR, $\delta \frac{\partial R}{\partial v}$ and $\delta \frac{\partial R}{\partial \rho}$ become

$$\delta R = \frac{\partial R}{\partial v}\delta v + \frac{\partial R}{\partial v'}\delta v' + \frac{\partial R}{\partial \rho}\delta \rho + \frac{\partial R}{\partial \rho'}\delta \rho'$$

$$\delta \frac{\partial R}{\partial v} = \frac{\partial^2 R}{\partial v^2}\delta v + \frac{\partial^2 R}{\partial v \partial v'}\delta v' + \frac{\partial^2 R}{\partial v \partial \rho}\delta \rho + \frac{\partial^2 R}{\partial v \partial \rho'}\delta \rho$$

$$\delta \frac{\partial R}{\partial \rho} = \frac{\partial^2 R}{\partial v \partial \rho}\delta v + \frac{\partial^2 R}{\partial v' \partial \rho}\delta v' + \frac{\partial^2 R}{\partial \rho^2}\delta \rho + \frac{\partial^2 R}{\partial \rho \partial \rho'}\delta \rho'$$

All the separate factors from which the second members of these equations are formed have already been given. Forming their products in the way described in Chapter II, we have the result given in the following tables.

The expressions for δR are arranged so that the value of $D'_t \delta R$ can be obtained from them by direct differentiation. This is done by distinguishing the time introduced into R by the co-ordinates of Uranus from that introduced by the co-ordinates of Saturn.

	$\frac{2a_1}{m'}\left\{\frac{\partial R}{\partial v'}\delta v' + \frac{\partial R}{\partial \rho'}\delta\rho'\right\}$	
U S	cos	sin
	"	"
1, 0	— 1.40*t*	— 1.33*t*
2, 0	+ 0.06*t*	— 0.28*t*
0,—1	— 0.24*t*	+ 0.20*t*
1,—1	+ 0.03*t*	+ 0.04*t*
2,—1	— 1.05*t*	— 0.95*t*
3,—1	+ 0.02*t*	— 0.30*t*
0,—2	+ 0.93*t*	— 0.83*t*
1,—2	—12.69*t*	+11.60*t*
2,—2	— 0.24*t*	+ 0.30*t*
3,—2	— 0.63*t*	— 0.59*t*
1,—3	— 1.60*t*	— 1.14*t*
2,—3	+ 0.25*t*	— 0.29*t*
3,—3	+ 0.08*t*	+ 0.02*t*
4,—3	— 0.36*t*	— 0.34*t*
3,—4	+ 0.19*t*	— 0.17*t*
4,—4	+ 0.15*t*	0.0
U U' S		
1, 0 0	+ 38″	+ 4″
1, 0—1	+ 2	+ 6
1, 0—2	—112	—153
1, 0—3	+ 14	— 18
1,+1—1	+ 56	— 2
1,—1+1	— 4	0
1,+2—1	+ 3	+ 43
1,—2+1	— 1	+ 19
1,+2—2	—117	+ 2
1,—2+2	— 23	0
1,+3—1	— 28	+ 88
1,—3+1	+ 30	+100
1,+3—2	+140	+112
1,—3+2	+ 14	— 10
U S J		
0, 1—1	+246	— 1
1,	—2469	— 50
2,	— 22	+ 4
3,	+ 3	+ 6
—2, 2—1	+ 42	— 16
—1,	+1638	— 88
0,	—136	+ 12
1,	— 89	— 89
2,	+ 2	+ 4
—3, 3—1	+ 6	— 8
—2,	+ 66	+ 24
—1,	+478	—280
0,		
—3, 4—1	+ 9	+ 28
—2,	—221	— 5
—1,	+297	—286

	$\frac{2a_1}{m'}\left\{\frac{\partial R}{\partial v'}\delta v' + \frac{\partial R}{\partial \rho'}\delta\rho'\right\}$		
U S J			
	"	"	
+1, 3—2	+375	+7336	
+2,	—990	—558	
+3,	— 73	—261	
+4,	+ 23	— 42	
0, 4—2	—647	—319	
1,	+7737	+5616	
2,	+153	—103	
3,	+ 24	— 26	
4,	0	0	
—2, 5—2	+ 90	+ 60	
—1,	—625	+739	
0,	+ 46	— 76	
1,	+ 61	+719	
2,	— 20	+ 6	
—4, 6—2	+27.5	— 3.9	
—3,	+94.1	+ 24.4	
—2,	+ 66	+243	
—1,	—8823	—6114	
0,	+699	+522	
—4, 7—2	+ 70	— 2	
—3,	+299	+147	
—2,	+937	+706	
—1,	—1978	+1117	

	$\frac{2a_1}{m'}\left\{\frac{\partial R}{\partial v}\delta v + \frac{\partial R}{\partial \rho}\delta\rho\right\}$		
U S' S	cos	sin	Fact. *nt*
	"	"	"
2,—1—1	—120	— 3	—0.852
3,	+ 59	+801	+0.148
4,	—139	+353	+1.448
3,—1—2	—122	+ 26	+0.148
4,	— 60	— 29	+1.148
5,	+ 24	— 60	+2.148
—0,—1+1	—250	0	—2.852
+1,	— 4	—170	—1.852
2,	+197	—450	—0.852
3,	— 15	+ 38	+0.148
—1,—1+2	— 8	+ 19	—3.852
0,	— 11	+134	—2.852
1,	— 80	+ 33	—1.852
1,—1 0	— 90	+ 12	—1.852
2,	+ 6	+158	—0.852
3,	+ 12	— 32	+0.148

	$2\frac{a_1}{m'n}\delta D'_t R$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial v}$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial \rho}$	
U S	sin	cos	sin	cos	cos	sin
	″	″	″	″	″	″
0, 0	......	....	0	0	+ 0.14t	
1, 0	+ 1.40t	— 1.33t	+ 1.40t	— 1.35t	+ 3.54t	+ 3.21t
2, 0	— 0.12t	— 0.56t	— 0.27t	— 0.43t	— 0.34t	+ 0.90t
0,—1	0	0	— 0.04t	— 0.06t	+ 1.06t	— 1.02t
1,	— 0.03t	+ 0.04t	— 0.06t	+ 0.03t	+ 0.07t	— 0.12t
2,	+ 2.10t	— 1.90t	+ 2.10t	— 1.89t	+ 3.54t	+ 3.22t
3,	— 0.06t	— 0.90t	— 0.19t	— 0.73t	— 0.19t	+ 1.18t
0,—2	0	0	— 0.91t	— 0.83t	+ 0.84t	— 0.94t
1,	+12.69t	+11.60t	+12.69t	+11.62t	—13.52t	+12.34t
2,	+ 0.48t	+ 0.60t	+ 0.21t	+ 0.37t	— 0.42t	+ 0.14t
3,	+ 1.89t	— 1.77t	+ 1.89t	— 1.75t	+ 2.78t	+ 2.52t
1,—3	+ 1.60t	— 1.14t	+ 1.61t	— 1.11t	— 1.64t	— 1.21t
2,	— 0.50t	— 0.58t	— 0.53t	— 0.56t	— 0.83t	+ 0.75t
3,	— 0.24t	+ 0.06t	— 0.19t	+ 0.07t	— 0.25t	— 0.10t
4,	+ 1.44t	— 1.36t	+ 1.43t	— 1.32t	+ 1.93t	+ 1.76t
3,—4	— 0.57t	— 0.51t	— 0.55t	— 0.53t	— 0.74t	+ 0.68t
4,	— 0.60t	0	— 0.52t	+ 0.10t	— 0.73t	— 0.10t
0, 0	....	....	....	+ 5	—192	
1, 0	— 45	+319	+ 9	+160	— 49	—709
2,	+168	+383	+199	+449	+278	—591
3,	+ 2	+ 5				
—1,—1	+148	— 61	+ 70	— 71	+ 18	+125
0,	+ 27	—381	— 16	—247	— 49	+347
1,	—139	— 70	— 4	— 32	+103	+ 47
2,	— 21	— 36	+ 3	— 12	+ 1	—215
3,	— 4	+ 38	+ 1	— 1	— 10	+ 38
4,	+ 28	+ 88	— 8	+ 4	— 19	— 9
1,—2	+ 87	—153	— 14	—119	— 10	+251
2,	— 88	— 7	+145	— 13	+ 84	+ 7
3,	+108	+121	— 60	+796	+ 83	+816
4,	+ 20	+517	+141	+352	—262	+617
5,—2	....	....	— 18	+ 14	— 45	+ 2
3,—3	+ 18	+ 4	+208	+ 16	+ 17	+ 34
4,	+ 69	—33	+ 82	— 67	— 35	+ 44
5,	— 52	—128	— 49	—130	— 92	+181
4,—4	....	....	+ 55	— 1	+110	+ 8
5,	....	....	+ 8	—104	+ 12	+ 52
6,	....	....	— 42	—103	— 61	+148

	$2\frac{a_1}{m'n}\delta D'_1 R$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial v}$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial \rho}$	
U S $\dot{J}$	sin	cos	sin	cos	cos	sin
	″	″	″	″	″	″
0, 1—1	0	0	—185	+ 2	— 84	+ 13
1,	+2469	— 50	+2477	— 50	—2947	— 69
2,	+ 44	+ 8	— 7	+ 13	—176	— 17
3,	— 9	+ 18	— 6	+ 16	— 17	— 20
—2, 2—1	+ 84	+ 32	+ 42	+ 29	+ 9	+ 49
—1,	+1638	+ 88	+1642	+ 88	+1231	—143
0,	0	0	—114	— 2	+ 41	+ 11
+1,	+ 89	— 89	+ 83	— 89	—196	— 62
+2,	— 4	+ 8	— 1	+ 8	— 11	— 18
—3, 3—1	+ 18	+ 24	+ 25	+ 28	— 42	+ 41
—2,	+132	— 48	+143	—54	—191	—104
—1,	+478	+280	+477	+279	+1529	—279
0,	0	0	— 57	— 8	— 60	+ 44
—3, 4—1	+ 27	— 84	+ 71	— 91	— 89	—134
—2,	—442	+ 10	—449	+ 7	+776	— 8
—1,	+297	+286	+310	+270	+362	—258
1, 3—2	—375	+7336	—393	+7343	+629	+7722
2,	+1980	—1116	+1987	—1271	+3304	+2544
3,	+219	—783	+ 51	—672	+151	+1041
4,	— 92	—168	—104	—102	—136	+158
0, 4—2	0	0	+669	—485	—601	—966
1,	—7737	+5616	—7733	+5606	+12136	+8698
2,	—306	—206	—235	—258	+273	+910
3,	— 72	— 78	+ 59	— 52	— 60	+109
—2, 5—2	+180	—120	+159	+ 6	—337	— 37
—1,	—625	—739	—620	—740	—1222	—1217
0,	0	0	+ 62	— 56	+167	— 60
+1,	— 61	+719	— 63	+725	+ 72	+1109
+2,	+ 40	+ 12	— 13	— 6	— 17	+ 67
—4, 6—2	+110.0	+ 15.6	+ 86.1	+ 32.9	—127.5	+ 39.4
—3,	+282.3	— 73.2	+248.8	— 14.5	—413.6	— 75.7
—2,	+132	—486	+237	—557	—1131	—1461
—1,	—8823	+6114	—8829	+6121	—12973	—9020
0,	0	0	+746	—493	+774	+406
—4, 7—2	+280	+ 8	+207	+ 45	—216	+ 48
—3,	+897	—441	+727	—632	—1128	—977
—2,	+1874	—1412	+1913	—1374	—3236	—2195
—1,	—1978	—1117	—1995	—1097	—2071	+904
0,	0	0	+139	+ 90	+144	— 71

In the terms of δR introduced by the perturbations of Saturn, namely, $\frac{\partial R}{\partial v'}\delta v' + \frac{\partial R}{\partial \rho'}\delta\rho'$, the differentiation represented by D'_t should be performed by considering $\delta v'$ and $\delta\rho'$ as constant, although they are expressed as a function of the mean longitude of Uranus, as well as of Saturn. The mean longitude of Uranus thus introduced is therefore represented by U', which is regarded as constant in taking $D'_t R$, and U only supposed to vary.

Again, in the terms $\frac{\partial R}{\partial v}\delta v + \frac{\partial R}{\partial \rho}\delta\rho$, since δv and $\delta\rho$ represent perturbations of Uranus, their complete derivatives, with respect to the time, are to be taken. But their expressions contain the mean longitude of Saturn as well as Uranus. The mean longitude of Saturn thus introduced is represented by S', and is to be considered variable in obtaining $D'_t\delta R$, while S is considered constant. The ratio of the coefficient of t to n in the various terms of this part of δR is given to the right of each corresponding term.

The value of $D'_t\delta R$ being once obtained, there is no longer any distinction necessary between U, U', or between S and S'. The similar terms are therefore combined by putting $S' = S$; $U' = U$.

From the above values of $2\delta D'_t R$ and $2\delta\frac{\partial R}{\partial \rho}$, we form the following value of

$$\frac{a_1}{m}\delta Q = \frac{2a_1}{m}\int \delta D'_t R dt + \frac{a_1}{m}\delta\frac{\partial R}{\partial \rho}$$

and of the other quantities which enter the perturbations of the co-ordinates. We shall begin with those terms which depend only on the mutual action of Saturn and Uranus, because they are few and small, and the only terms which are sensible are those in which the coefficient of the mean longitude of Saturn is —1. We shall therefore confine ourselves to these. And, instead of employing the condensed formulæ, we shall make the computation in full by (13).

	$\frac{a_1}{m'}\delta Q = 2\frac{a_1}{m}\int \delta D'_t R dt + \delta\frac{\partial R}{\partial \rho}$		$\frac{a_1}{m'}\xi\delta Q$		$\frac{a_1}{m'}\eta\delta Q$	
g l'	cos	sin	cos	sin	sin	cos
	″ ″	″ ″	″ ″	″ ″	″ ″	″ ″
−1, −1	+ 47	+ 78	− 10	+148	+ 7	+153
0	− 14 + 0.53t	+307 − 0.51t	+ 13 + 0.02t	+ 49 + 0.05t	+ 36 − 0.06t	− 8 + 0.01t
1	− 24 + 0.02t	+ 62 − 0.08t	− 41 + 2.37t	+138 + 1.62t	+ 29 − 1.85t	−164 + 2.11t
2	− 60 + 4.23t	− 28 + 3.84t	−270 − 0.13t	+157 − 3.05t	+250 − 0.13t	+ 85 − 2.70t
3	−525 + 0.31t	+240 − 5.49t	+ 7 + 2.09t	− 31 + 2.30t	− 30 + 2.11t	+ 14 − 1.92t
4	....	....	−262 + 0.15t	+120 − 2.74t	−262 + 0.15t	−120 + 2.74t

	$\cos\psi\delta\rho$		δv	
g l'	cos	sin	sin	cos
	″ ″	″ ″	″ ″	
0, −1	+.001 − .00001t	−.023 + .00002t	+0.002 + 0.0000t	+0.021 + 0.0000t
1,	+.024 + .00033t	−.036 + .00033t	+0.043 + 0.0006t	+0.058 + 0.0000t
2,	+.333 + .00437t	−.406 + .00432t	+0.620 + 0.0098t	+0.803 − 0.0097t
3,	−.142 − .00002t	+.079 − .00169t	+2.69 − 0.0011t	+0.830 − 0.0153t
4,	−.071	+.019	+0.132 0t	+0.040 + 0.0000t

The computation of these terms being extremely complex, a check upon their accuracy is desirable. In the case of the secular variations of the coefficients, the coefficients of the time are easily obtained by substituting in the integrated perturbations the variations of the eccentricity and perihelion of Saturn. Thus I have found

$$\delta v = +0''.0103\, t \sin(2g - l') - 0''.0094\, t \cos(2g - l')$$
$$+0.0027\, t \sin(3g - l') - 0.0138\, t \cos(3g - l')$$

The greatest discrepancy is found in the coefficient of sin $(3g - l')$, and it amounts to $0''.0038t$, or about $0''.4$ in a century. But, owing to the great period of this term, nearly 600 years, this difference, during any one century, will be nearly eliminated through the mean longitude and mean motion.

It may also be remarked that in this case the terms derived from the perturbations of the elements are undoubtedly the correct ones, and will therefore be employed.

The terms which the preceding integration fails to give, owing to the constant terms introduced into $\xi\delta Q$ and $\eta\delta Q$, are found by (22).

We thus have

$$n\Sigma p_u k_c^{(u)} = +0''.36$$
$$n\Sigma q_u k_s^{(u)} = +0.27$$
$$r_1^2\delta\rho = \tfrac{1}{4} Mnt^2 \{0''.36 \sin g - 0''.27 \cos g\}$$
$$\delta v = \tfrac{1}{2} Mnt^2 \{0''.36 \cos g + 0''.27 \sin g\}$$
$$= t^2 \{0''.0000038 \cos g + 0''.0000029 \sin g\}.$$

The greatest effect of these terms amounts to less than one-twentieth of a second in a century. They may therefore be neglected in the present theory. The other terms containing the square of the time are yet smaller.

Applying the terms of the second order thus found to the terms of the first order depending on the corresponding arguments, the perturbations of Uranus by Saturn become

	δv		cos $\psi\delta\rho$	
g l'	sin	cos	cos	sin
	$''$ $''$	$''$ $''$		
—1,—1	+ 0.036	+ 0.039	+ 0	— 2
0,—1	+ 1.284	+ 0.739	— 11	— 31
1,—1	—20.774 + 0.06 T	+ 8.580	+1753	—244
2,—1	—11.270 + 1.03 T	+144.265 — 0.94 T	— 87	—3114
3,—1	+51.99 + 0.27 T	+116.69 — 1.38 T	—237	+644
4,—1	+ 2.265	+ 5.656	— 55	+140
5,—1	+ 0.126	+ 0.329	— 4	+ 10

T here represents the time counted in centuries from 1850.0.

The other terms remain the same as given on page 50.

Perturbations depending on the product of the masses of Jupiter and Saturn.

The values of $\delta D'_t R$, $\delta \frac{\partial R}{\partial v}$, and $\delta \frac{\partial R}{\partial \rho}$, depending on the products of the masses

of Jupiter and Saturn, are given on page 74. The computation from these data being conducted in the same way as in the case of the terms of the first order, it is not necessary to give much more than the results. These are shown in the following table. The indices to the left represent the coefficients of the mean longitudes of Uranus, Saturn, and Jupiter, all counted from the perihelion of Uranus. Column ν gives the ratio of the mean motion of Uranus to the coefficient of the time in each argument. The perturbations of the common logarithm of the radius vector are expressed in units of the seventh place of decimals.

	ν	δv		$0.4343\ \delta\rho$	
U S J		sin	cos	cos	sin
		"	"		
0, 1,—1	—0.2364	+0.002	0	0	0
1,	—0.3095	—0.020	0	0	0
3,	—0.4480	+0.004	—0.003	0	0
4,	—0.8127	+0.016	—0.024	0	0
—2, 2,—1	—0.2960	—0.007	—0.001	0	0
—1,	—0.4204	—0.108	—0.007	— 2	0
0,	—0.7254	—0.014	—0.012	0	0
1,	—2.6420	+0.164	—0.267	+ 1	+ 1
2,	+1.6090	+0.005	—0.005	0	0
—3, 3,—1	—0.6551	+0.012	—0.002	0	0
—2,	—1.8997	+0.175	—0.015	+ 1	0
—1,	+2.1115	+0.078	+0.512	— 2	+ 3
0,	+0.6786	+0.007	+0.024	0	0
—3, 4,—1	+0.754	—0.030	+0.081	0	+ 1
—2,	+0.430	+0.043	+0.005	— 1	0
—1,	+0.301	+0.001	—0.003	0	0
1, 3,—2	—0.2170	—0.002	—0.032	0	— 1
2,	—0.2771	—0.051	+0.035	— 1	— 1
3,	—0.3833	—0.010	+0.050	0	— 1
4,	—0.6215	+0.032	+0.052	0	— 1
0, 4,—2	—0.3627	—0.010	—0.004	0	0
1,	—0.5692	+0.075	—0.154	— 5	— 2
2,	—1.3210	—0.349	—1.297	— 3	+11
3,	+4.1150	—0.510	—0.453	+ 2	— 2
—2, 5,—2	—0.5250	—0.253	—0.034	— 3	0
—1,	—1.1051	—4.433	—0.617	—43	+ 7
0,	+10.5152	—0.546	—0.032	+ 2	0
1,	+0.9132	+0.206	—3.254	— 2	—36
2,	+0.4773	+0.012	—0.192	0	— 2
—4, 6,—2	—0.9497	+1.824	—0.519	+19	+ 6
—3,	—18.9250	+40.650	-10.500	+32	+10
—2,	+1.0558	+6.237	—7.866	—63	—79
—1,	+0.5136	+0.467	—0.539	0	— 2
0,	+0.3393	+0.007	—0.017	0	0
—4, 7,—2	+0.5558	—0.050	—0.003	+ 1	0
—3,	+0.3573	—0.046	+0.032	+ 1	+ 1
—2,	+0.2632	—0.045	+0.032	+ 1	+ 1
—1,	+0.2084	+0.006	+0.007	0	0
—0,	+0.1724	0	0	0	0

CHAPTER V.

COLLECTION AND TRANSFORMATION OF THE PRECEDING PERTURBATIONS OF URANUS.

THE terms of the perturbations which neither contain the elements of the disturbing planets, nor depend on the secular variations of the eccentricity and perihelion, admit of being greatly simplified by a slight change in the arbitrary elements. These terms are as follows:

(1) In the longitude of Uranus

	$''$	$''$	$''$	$''$	$''$
Action of Jupiter,	$+31.2116t$	$+25.657 \sin g$	$+1.397 \sin 2g$	$-1.859 \cos g$	$-0.087 \cos 2g$
Action of Saturn,	$+10.9690t$	$+8.545 \sin g$	$+0.461 \sin 2g$	$-4.735 \cos g$	$-0.169 \cos 2g$
Action of Neptune,	$-0.4262t$	$+0.697 \sin g$	$+0.046 \sin 2g$	$-0.088 \cos g$	$-0.005 \cos 2g$
Total,	$+41.7544t$	$+34.899 \sin g$	$+1.904 \sin 2g$	$-6.682 \cos g$	$-0.261 \cos 2g$

(2) In the value of cos $\psi\delta\rho$, units of 7th place of decimals.

Action of Jupiter,	-10089	$-492 \cos g$	$-33 \cos 2g$	$-2 \sin g$	$+1 \sin 2g$
Action of Saturn,	-3543	$-184 \cos g$	$-15 \cos 2g$	$-15 \sin g$	
Action of Neptune,	$+138$	$-1 \cos g$	$-1 \cos 2g$	$+1 \sin g$	
Total,	-13494	$-677 \cos g$	$-49 \cos 2g$	$-16 \sin g$	$+1 \sin 2g$

Let us first consider the first or constant term in the perturbation of each co-ordinate. If we suppose a change of δn in the mean motion of a planet, the corresponding change in $\delta\rho$ will be

$$\delta\rho = -\frac{2}{3}\frac{\delta n}{n}.$$

If, then, we increase the mean motion of Uranus by $41''.754$, the corresponding change in $\delta\rho$ will be -18045, and in cos $\psi\delta\rho$, -18025. Subtracting these from the above perturbations, the secular term in the mean motion will disappear, and we shall have for the constant term of cos $\psi\delta\rho$

$$+4531$$

This same change in the mean motion will produce a secular term in the equation of the centre of the same nature with that produced by the secular variation of the perihelion. The differences of the values of the secular terms, found by the two methods employed in Chapters II. and III., proceeds from the fact that in the one case the effect of the above term in the mean motion is included, and in the other excluded.

If we subduct the effect in question when necessary, the remainder will be the effect of the secular variation of the longitude of the perihelion of Uranus, to which we shall revert presently.

Let us next introduce such a change in the eccentricity of Uranus as shall produce the term $34''.899 \sin g$, and ascertain its effect on the other terms. For this purpose we must determine δe by the condition

$$(2-\frac{3}{4}e^2)\,\delta e = 34''.899$$

which gives

$$\delta e = 17''.464 = .0000847.$$

A change of this amount in δe will introduce the following terms in δv and $\delta\rho$

$$\delta v = 34''.899 \sin g + 2''.048 \sin 2g$$
$$\cos\psi\delta\rho = 20 - 844 \cos g - 59 \cos 2g.$$

Subtracting these terms from the expressions previously found we have

$$\delta v = -0''.144 \sin 2g - 6''.682 \cos g - 0''.261 \cos 2g.$$
$$\cos\psi\delta\rho = +4511 + 167 \cos g + 10 \cos 2g - 16 \sin g + 1 \sin 2g.$$

Again, let us put

$$e\delta\pi = 3''.342 = .0000162,$$

we shall have the elliptic terms

$$\delta v = -6''.682 \cos g - 0''.391 \cos 2g$$
$$\cos\psi\delta\rho = -162 \sin g - 11 \sin 2g.$$

Subtracting these expressions the constant terms, independent of the mean longitude of the disturbing planets, are reduced to

$$\delta v = -0''.144 \sin 2g + 0''.130 \cos 2g.$$
$$\cos\psi\delta\rho = 4511 + 167 \cos g + 10 \cos 2g + 146 \sin g + 12 \sin 2g.$$
$$0.43429\ \delta\rho = 1969 + 73 \cos g + 4 \cos 2g + 63 \sin g + 5 \sin 2g.$$

In the last equation we have introduced the constant $+.0000008$ produced in $\delta\rho$ by the combined action of Venus, the Earth, and Mars. The effect of each planet is computed by the approximate formula

$$\delta\rho = \frac{1}{6} m'(b_{\frac{1}{2}}^{(0)} + \alpha D_\alpha b_{\frac{1}{2}}^{(0)}).$$

Secular Variations.

The following inequalities result from the secular variations of the eccentricity and longitude of perihelion produced by each of the disturbing planets, T being the time expressed in centuries.

From the variation of the eccentricity

	″	″	″
Action of Jupiter,	$\delta v = -1.216\,T \sin g$	$-0.072\,T \sin 2g$	$-0.005\,T \sin 3g$
Action of Saturn,	$-9.182\,T \sin g$	$-0.538\,T \sin 2g$	$-0.032\,T \sin 3g$
Action of Neptune,	$-0.502\,T \sin g$	$-0.030\,T \sin 2g$	$-0.002\,T \sin 3g$

	$''$	$''$
Action of Jupiter,	$M\delta\rho = +13\,T\cos g$	$+1\,T\cos 2g$
Action of Saturn,	$+98\,T\cos g$	$+7\,T\cos 2g$
Action of Neptune,	$+\ 6\,T\cos g$	

The secular variation of the longitude of the perihelion is

	$''$
Action of Jupiter,	$+122.1\,T$
Action of Saturn,	$+118.4\,T$
Action of Neptune,	$+\ 51.1\,T$
Total,	$\delta\pi = +291.6\,T$

The effect of this secular variation on the longitude and radius vector is

	$''$	$''$	$''$
Action of Jupiter,	$\delta v = -11.46\,T\cos g$	$-0.671\,T\cos 2g$	$-0.047\,T\cos 3g$
Action of Saturn,	$-11.11\,T\cos g$	$-0.651\,T\cos 2g$	$-0.039\,T\cos 3g$
Action of Neptune,	$-\ 4.80\,T\cos g$	$-0.281\,T\cos 2g$	$-0.016\,T\cos 3g$
Total,	$-27.37\,T\cos g$	$-1.603\,T\cos 2g$	$-0.102\,T\cos 3g$

	$''$	$''$
Action of Jupiter,	$M\delta\rho = -120\,T\sin g$	$-8\,T\sin 2g$
Action of Saturn,	$-117\,T\sin g$	$-8\,T\sin 2g$
Action of Neptune,	$-\ 50\,T\sin g$	$-3\,T\sin 2g$

For the purpose of conveniently tabulating the perturbations, we shall express them in a form similar to that adopted in the theory of Neptune. Let us select, from the terms of the periodic perturbations produced by any planet, all those in which the difference between the indices i and i' is the same. For example, in the perturbations of the longitude produced by Jupiter, let us consider the terms

$$\begin{aligned}
\delta v = &+ 1.269\sin(\quad - l) &&+ 0.002\cos(\quad - l)\\
&- 3.495\sin(2g - l) &&- 0.092\cos(2g - l)\\
&+ 1.182\sin(\ g - 2l) &&+ 0.515\cos(\ g - 2l)\\
&+ 0.074\sin(3g - 2l) &&- 0.005\cos(3g - 2l)\\
&- 0.005\sin(2g - 3l) &&\\
&+ 0.011\sin(4g - 3l) &&- 0.001\cos(4g - 3l)
\end{aligned}$$

These terms may be expressed in the form

$$\delta v = \sin g \times \left\{\begin{array}{ll} \overset{''}{+0.094}\sin\ (g-l) & \overset{''}{-4.764}\cos\ (g-l)\\ +0.520\sin 2(g-l) & -1.108\cos 2(g-l)\\ & +0.016\cos 3(g-l)\end{array}\right\}$$

$$+\cos g \times \left\{\begin{array}{ll} \overset{''}{-2.226}\sin\ (g-l) & -0.090\cos\ (g-l)\\ +1.256\sin 2(g-l) & +0.510\cos 2(g-l)\\ +0.006\sin 3(g-l) & \end{array}\right\}$$

In general, a series of terms of the form

$$\Sigma a_i \sin(iA + sg) + \Sigma b_i \cos(iA + sg)$$
$$+ \Sigma a'_i \sin(iA - sg) + \Sigma b'_i \cos(iA - sg),$$

may be put in the form

$$\{\Sigma(a_i - a'_i)\cos iA - \Sigma(b_i - b'_i)\sin iA\}\sin sg$$
$$+ \{\Sigma(a_i + a'_i)\sin iA + \Sigma(b_i + b'_i)\sin iA\}\cos sg.$$

All the periodic terms containing only g and l in the arguments may be put into this form by taking

$$A = g - l,$$

so that the coefficients of sin sg and cos sg may all be expressed as a function of the single variable argument A.

The perturbations of the elements may be reduced to perturbations of the co-ordinates expressed as the sum of several products of slowly varying functions into the sines and cosines of the multiples of g. We have, in fact,

$$\delta v = \delta l$$
$$+ \left(2 - \frac{3}{4}e^2\right)\delta e \times \sin g \qquad + \left(2 - \frac{1}{4}e^2\right)e\delta g \times \cos g$$
$$+ \left(\frac{5}{2}e - \frac{11}{6}e^3\right)\delta e \times \sin 2g \qquad + \left(\frac{5}{2}e - \frac{11}{12}e^3\right)e\delta g \times \cos 2g.$$
$$+ \text{etc.} \qquad + \text{etc.}$$

It appears, therefore, that all the perturbations in which the arguments contain the mean longitudes of only two planets may be put in the form

$$\delta v = (v.c.0) + (v.c.1)\cos g + (v.c.2)\cos 2g + \text{etc.}$$
$$+ (v.s.1)\sin g + (v.s.2)\sin 2g + \text{etc.}$$
$$M\delta\rho = (\rho.c.0) + (\rho.c.1)\cos g + (\rho.c.2)\cos 2g + \text{etc.}$$
$$+ (\rho.s.1)\sin g + (\rho.s.2)\sin 2g + \text{etc.}$$

We have next to reduce to the same form those terms which contain the mean longitudes of both Jupiter and Saturn, and which are given on page 78. We have here twenty-four terms, each greater than 0''.04. As most of these terms depend on three independent arguments, they cannot be included in a double entry table, while, if we include them as perturbations of the longitude in tables of single entry, we shall have to enter twenty-two tables with as many different arguments. But, by taking, for the argument A, the middle one in each series of arguments which depend on the same multiples of Jupiter and Saturn, and expressing the terms above and below it in each series as coefficients of sin g, cos g, sin $2g$, and

$\cos 2g$, we may reduce the number of arguments to eight, and the number of tables to seventeen. Consider, for instance, the terms of the second series,

$$\begin{array}{ll} -0''.108 \sin(-g+2S-J) & -0.007\cos(-g+2S-J) \\ -0.014\sin(\quad 2S-J) & -0.012\cos(\quad 2S-J) \\ +0.164\sin(\ g+2S-J) & -0.267\cos(\ g+2S-J). \end{array}$$

These terms may be allowed for by adding to $(v.c.0)$, $(v.s.1)$, $(v.c.1)$, the terms

$$\begin{aligned} (v.c.0) &= -0''.014\sin(2S-J) - 0''.012\cos(2S-J) \\ (v.s.1) &= +0.260\sin(2S-J) + 0.272\cos(2S-J) \\ (v.c.1) &= +0.056\sin(2S-J) - 0.274\cos(2S-J). \end{aligned}$$

From the perturbations of longitude and radius vector already given, we readily find the following values of $(v.c.0)$, $(v.s.1)$, etc.

Action of Jupiter.

$$A_1 = l' - g$$

$$\begin{aligned} (v.c.0) = &+53''.064\sin A_1 - 0''.004\cos A_1 \\ &- 0.277\sin 2A_1 + 0.036\cos 2A_1 \\ &- 0.025\sin 3A_1 \end{aligned}$$

$$\begin{aligned} (v.c.1) = &+ 2.226\sin A_1 - 0.090\cos A_1 \\ &- 1.256\sin 2A_1 + 0.510\cos 2A_1 \\ &- 0.006\sin 3A_1 \\ &-11''.46T \end{aligned} \qquad \begin{aligned} (v.s.1) = &-0''.094\sin A_1 - 4''.764\cos A_1 \\ &-0.520\sin 2A_1 - 1.108\cos 2A_1 \\ &+0.016\cos 3A_1 \\ &-1''.22T \end{aligned}$$

$$\begin{aligned} (v.c.2) = &+ 0.121\sin A_1 - 0.038\cos A_1 \\ &+ 0.012\sin 2A_1 - 0.014\cos 2A_1 \\ &+ 0.029\sin 3A_1 - 0.034\cos 3A_1 \\ &-0''.67T \end{aligned} \qquad \begin{aligned} (v.s.2) = &-0.056\sin A_1 - 0.175\cos A_1 \\ &+0.008\sin 2A_1 + 0.042\cos 2A_1 \\ &+0.034\sin 3A_1 + 0.035\cos 3A_1 \\ &-0''.07T \end{aligned}$$

$$(v.c.3) = -\ 0.04T \qquad (v.s.3) = -0.005T$$

$$\begin{aligned} (\rho.c.0) = &+1127\cos A_1 \\ &+ \quad 4\cos 2A_1 \end{aligned}$$

$$\begin{aligned} (\rho.c.1) = &- \quad 2\sin A_1 + 57\cos A_1 \\ &+ \quad 10\sin 2A_1 - 23\cos 2A_1 \\ &+13T \end{aligned} \qquad \begin{aligned} (\rho.s.1) = &+108\sin A_1 + \ 2\cos A_1 \\ &+ \ 26\sin 2A_1 + 12\cos 2A_1 \\ &-120T \end{aligned}$$

$$(\rho.c.2) = + \quad 7\cos A_1 + 1T \qquad (\rho.s.2) = + \quad 7\sin A_1 - 8T$$

Action of Saturn.

$$A_2 = l' - g$$

$$\begin{array}{lll}
 & {}'' & {}'' \\
(v.c.0)= & \left(\begin{array}{l}+20.774\\ -\ 0.06\,T\end{array}\right)\sin A_2 & +8.580\cos A_2 \\
 & -\ 4.110\sin 2A_2 & -0.009\cos 2A_2 \\
 & -\ 0.824\sin 3A_2 & -0.019\cos 3A_2 \\
 & -\ 0.228\sin 4A_2 & -0.008\cos 4A_2 \\
 & -\ 0.074\sin 5A_2 & -0.003\cos 5A_2 \\
 & -\ 0.025\sin 6A_2 &
\end{array}$$

$$\begin{array}{lll}
(v.c.1)= & \left(\begin{array}{l}+\ 9.986\\ -\ 1.03\,T\end{array}\right)\sin A_2 + & \left(\begin{array}{l}+145.005\\ -\quad 0.94\,T\end{array}\right)\cos A_2 \\
 & -\ 2.121\sin 2A_2 & -0.793\cos 2A_2 \\
 & -\ 0.392\sin 3A_2 & -0.308\cos 3A_2 \\
 & -\ 0.109\sin 4A_2 & -0.099\cos 4A_2 \\
 & -\ 0.038\sin 5A_2 & -0.027\cos 5A_2 \\
 & \multicolumn{2}{c}{-11''.11\,T}
\end{array}$$

$$\begin{array}{lll}
 & {}'' & {}'' \\
(v.s.1)= & \left(\begin{array}{l}+143.52\\ -\quad 0.94\,T\end{array}\right)\sin A_2 + & \left(\begin{array}{l}-12.554\\ +\ 1.03\,T\end{array}\right)\cos A_2 \\
 & -2.421\sin 2A_2 & +2.037\cos 2A_2 \\
 & -0.226\sin 3A_2 & +0.318\cos 3A_2 \\
 & -0.055\sin 4A_2 & +0.097\cos 4A_2 \\
 & -0.027\sin 5A_2 & +0.038\cos 5A_2 \\
 & \multicolumn{2}{c}{-9''.18\,T}
\end{array}$$

$$\begin{array}{lll}
(v.c.2)= & \left(\begin{array}{l}-52.026\\ -\ 0.27\,T\end{array}\right)\sin A_2 + & \left(\begin{array}{l}+116.73\\ -\quad 1.38\,T\end{array}\right)\cos A_2 \\
 & -\ 0.665\sin 2A_2 & -1.847\cos 2A_2 \\
 & -\ 0.081\sin 3A_2 & -0.153\cos 3A_2 \\
 & -\ 0.013\sin 4A_2 & -0.044\cos 4A_2 \\
 & -\ 0.005\sin 5A_2 & -0.016\cos 5A_2 \\
 & \multicolumn{2}{c}{-0''.65\,T}
\end{array}$$

$$\begin{array}{lll}
(v.s.2)= & \left(\begin{array}{l}+116.650\\ -\quad 1.38\,T\end{array}\right)\sin A_2 + & \left(\begin{array}{l}\ \ 51.954\\ +\ 0.27\,T\end{array}\right)\cos A_2 \\
 & -1.813\sin 2A_2 & +0.631\cos 2A_2 \\
 & -0.177\sin 3A_2 & +0.013\cos 3A_2 \\
 & -0.044\sin 4A_2 & +0.013\cos 4A_2 \\
 & -0.016\sin 5A_2 & +0.005\cos 5A_2 \\
 & \multicolumn{2}{c}{-0''.54\,T}
\end{array}$$

$$\begin{array}{lll}
(v.c.3)= & -\ 2.265\sin A_2 & +5.656\cos A_2 \\
 & -\ 1.163\sin 2A_2 & +2.956\cos 2A_2 \\
 & +\ 0.026\sin 3A_2 & -0.063\cos 3A_2 \\
 & +\ 0.005\sin 4A_2 & -0.013\cos 4A_2 \\
 & +\ 0.002\sin 5A_2 & -0.004\cos 5A_2 \\
 & \multicolumn{2}{c}{-0''.04\,T}
\end{array}$$

$$\begin{array}{lll}
(v.s.3)= & +5.656\sin A_2 & +2.265\cos A_2 \\
 & +2.956\sin 2A_2 & +1.163\cos 2A_2 \\
 & -0.063\sin 3A_2 & -0.026\cos 3A_2 \\
 & -0.013\sin 4A_2 & -0.005\cos 4A_2 \\
 & -0.004\sin 5A_2 & -0.002\cos 5A_2 \\
 & \multicolumn{2}{c}{-0''.03\,T}
\end{array}$$

$$\begin{array}{lll}
(v.c.4)= & -\ 0.126\sin A_2 & +0.329\cos A_2 \\
 & -\ 0.503\sin 2A_2 & +0.378\cos 2A_2 \\
 & +\ 0.053\sin 3A_2 & -0.032\cos 3A_2
\end{array}$$

$$\begin{array}{lll}
(v.s.4)= & +0.329\sin A_2 & +0.126\cos A_2 \\
 & +0.378\sin 2A_2 & +0.503\cos 2A_2 \\
 & -0.032\sin 3A_2 & -0.053\cos 3A_2
\end{array}$$

$$\begin{array}{lll}
(\rho.c.0)= & +\ 106\ \sin A_2 & +\ 761\cos A_2 \\
 & -\quad 2\ \sin 2A_2 & +\ \ 89\cos 2A_2 \\
 & & +\ \ 19\cos 3A_2 \\
 & & +\quad 5\cos 4A_2
\end{array}$$

$$\begin{array}{lll}
(\rho.c.1)= & +\ 1364\ \sin A_2 & -\ 43\cos A_2 \\
 & -\quad 46\ \sin 2A_2 & +\ 45\cos 2A_2 \\
 & -\quad\ 7\ \sin 3A_2 & +\quad 9\cos 3A_2 \\
 & -\quad\ 3\ \sin 4A_2 & +\quad 2\cos 4A_2 \\
 & \multicolumn{2}{c}{+98\,T}
\end{array}$$

$$\begin{array}{lll}
(\rho.s.1)= & -\ 33\sin A_2 & -\ 1338\cos A_2 \\
 & +\ 31\sin 2A_2 & +\quad 14\cos 2A_2 \\
 & +\quad 7\sin 3A_2 & +\quad\ 5\cos 3A_2 \\
 & +\quad 2\sin 4A_2 & +\quad\ 1\cos 4A_2 \\
 & \multicolumn{2}{c}{-117\,T}
\end{array}$$

$$\begin{array}{lll}
(\rho.c.2)= & -\ 279\ \sin A_2 & -\ 103\cos A_2 \\
 & -\quad 27\ \sin 2A_2 & +\quad 9\cos 2A_2 \\
 & -\quad\ 3\ \sin 3A_2 & \\
 & \multicolumn{2}{c}{+7\,T}
\end{array}$$

$$\begin{array}{lll}
(\rho.s.2)= & -\ 103\sin A_2 & +\ 281\cos A_2 \\
 & +\quad 9\sin 2A_2 & +\quad 27\cos 2A_2 \\
 & & +\quad\ 3\cos 3A_2 \\
 & \multicolumn{2}{c}{-8\,T}
\end{array}$$

$$\begin{array}{lll}
(\rho.c.3)= & -\ 61\ \sin A_2 & -\ 24\cos A_2 \\
 & +\ 24\ \sin 2A_2 & +\quad 9\cos 2A_2
\end{array}$$

$$\begin{array}{lll}
(\rho.s.3)= & -\ 24\sin A_2 & +\ 61\cos A_2 \\
 & +\quad 9\sin 2A_2 & -\ 24\cos 2A_2
\end{array}$$

Action of Neptune.

$$A_3 = g - l'$$

$$
\begin{aligned}
(v.c.0) = &-39''.66 \sin A_3 - 0''.08 \cos A_3 \\
&-35.36 \sin 2A_3 - 0.03 \cos 2A_3 \\
&+17.29 \sin 3A_3 + 0.23 \cos 3A_3 \\
&+\ 3.91 \sin 4A_3 + 0.06 \cos 4A_3 \\
&+\ 0.99 \sin 5A_3 - 0.04 \cos 5A_3 \\
&+\ 0.42 \sin 6A_3 - 0.01 \cos 6A_3 \\
&+\ 0.19 \sin 7A_3 \\
&+\ 0.09 \sin 8A_3 \\
&+\ 0.02 \sin 9A_3 \\
&+\delta l
\end{aligned}
$$

$$
\begin{aligned}
(v.c.1) = &-\ 6.77 \sin A_3 - 0.53 \cos A_3 \\
&-\ 1.10 \sin 2A_3 + 0.07 \cos 2A_3 \\
&+23.25 \sin 3A_3 + 4.04 \cos 3A_3 \\
&+\ 6.05 \sin 4A_3 + 1.06 \cos 4A_3 \\
&-\ 3.26 \sin 5A_3 - 0.66 \cos 5A_3 \\
&-\ 0.80 \sin 6A_3 - 0.17 \cos 6A_3 \\
&-\ 0.24 \sin 7A_3 - 0.05 \cos 7A_3 \\
&-\ 0.12 \sin 8A_3 - 0.02 \cos 8A_3 \\
&-\ 0.06 \sin 9A_3 - 0.01 \cos 9A_3 \\
&-\ 0.04 \sin 10A_3 \\
&+1.99945 e\delta g
\end{aligned}
$$

$$
\begin{aligned}
(v.s.1) = &-0''.49 \sin A_3 + 1''.75 \cos A_3 \\
&+0.12 \sin 2A_3 - 2.48 \cos 2A_3 \\
&+4.04 \sin 3A_3 - 20.92 \cos 3A_3 \\
&+1.07 \sin 4A_3 - 5.42 \cos 4A_3 \\
&-0.68 \sin 5A_3 + 3.47 \cos 5A_3 \\
&-0.17 \sin 6A_3 + 0.91 \cos 6A_3 \\
&-0.04 \sin 7A_3 + 0.30 \cos 7A_3 \\
&-0.02 \sin 8A_3 + 0.12 \cos 8A_3 \\
&-0.01 \sin 9A_3 + 0.06 \cos 9A_3 \\
&+ 0.04 \cos 10A_3 \\
&+1.99835 \delta e
\end{aligned}
$$

$$
\begin{aligned}
(v.c.2) = &-0.43 \sin A_3 - 0.03 \cos A_3 \\
&-0.03 \sin 2A_3 + 0.01 \cos 2A_3 \\
&+0.75 \sin 3A_3 + 0.08 \cos 3A_3 \\
&-0.10 \sin 4A_3 - 0.08 \cos 4A_3 \\
&-3.20 \sin 5A_3 - 1.17 \cos 5A_3 \\
&-0.83 \sin 6A_3 - 0.32 \cos 6A_3 \\
&+0.57 \sin 7A_3 + 0.22 \cos 7A_3 \\
&+0.14 \sin 8A_3 + 0.06 \cos 8A_3 \\
&+0.06 \sin 9A_3 + 0.02 \cos 9A_3 \\
&+0.03 \sin 10A_3 - 0.01 \cos 10A_3 \\
&+0.11722 e\delta g
\end{aligned}
$$

$$
\begin{aligned}
(v.s.2) = &-0.03 \sin A_3 + 0.13 \cos A_3 \\
&+0.02 \sin 2A_3 - 0.16 \cos 2A_3 \\
&+0.08 \sin 3A_3 - 0.60 \cos 3A_3 \\
&-0.08 \sin 4A_3 + 0.15 \cos 4A_3 \\
&-1.17 \sin 5A_3 + 3.22 \cos 5A_3 \\
&-0.32 \sin 6A_3 + 0.86 \cos 6A_3 \\
&+0.22 \sin 7A_3 - 0.57 \cos 7A_3 \\
&+0.06 \sin 8A_3 - 0.14 \cos 8A_3 \\
&+0.02 \sin 9A_3 - 0.06 \cos 9A_3 \\
&-0.01 \sin 10A_3 - 0.03 \cos 10A_3 \\
&+0.11713 \delta e
\end{aligned}
$$

$$
\begin{aligned}
(v.c.3) = &-0.02 \sin A_3 \\
&+0.04 \sin 3A_3 + 0.01 \cos 3A_3 \\
&-0.15 \sin 4A_3 - 0.05 \cos 4A_3 \\
&-0.08 \sin 5A_3 - 0.02 \cos 5A_3 \\
&-0.02 \sin 6A_3 - 0.02 \cos 6A_3 \\
&+0.46 \sin 7A_3 + 0.25 \cos 7A_3 \\
&+0.11 \sin 8A_3 + 0.07 \cos 8A_3 \\
&-0.08 \sin 9A_3 - 0.05 \cos 9A_3 \\
&-0.03 \sin 10A_3 - 0.02 \cos 10A_3 \\
&+0.00714 e\delta g
\end{aligned}
$$

$$
\begin{aligned}
(v.s.3) = &+0.02 \cos A_3 \\
&+0.01 \sin 3A_4 - 0.04 \cos 3A_3 \\
&-0.05 \sin 4A_3 + 0.15 \cos 4A_3 \\
&-0.02 \sin 5A_3 + 0.08 \cos 5A_3 \\
&-0.02 \sin 6A_3 + 0.02 \cos 6A_3 \\
&+0.25 \sin 7A_3 - 0.46 \cos 7A_3 \\
&+0.07 \sin 8A_3 - 0.11 \cos 8A_3 \\
&-0.05 \sin 9A_3 + 0.08 \cos 9A_3 \\
&-0.02 \sin 10A_3 + 0.03 \cos 10A_3 \\
&+0.00714 \delta e
\end{aligned}
$$

Action of Neptune.—Continued.

$$A_3 = g - l'$$

$$\begin{aligned}(v.c.4) = &-0.''06 \sin 9A_3 - 0.''05 \cos 9A_3 \\ &-0.09 \sin 10A_3 - 0.08 \cos 10A_3 \\ &+0.00044 e\delta g\end{aligned}$$

$$\begin{aligned}(v.s.4) = &-0.''05 \sin 9A_3 + 0.''06 \cos 9A_3 \\ &-0.08 \sin 10A_3 + 0.09 \cos 10A_3 \\ &+0.00044 \delta e\end{aligned}$$

$$\begin{array}{lrr}(\rho.c.0) = & & +227 \cos A_3 \\ & & +232 \cos 2A_3 \\ & +3 \sin 3A_3 & -229 \cos 3A_3 \\ & & -59 \cos 4A_3 \\ & & -17 \cos 5A_3 \\ & & -7 \cos 6A_3 \\ & & -3 \cos 7A_3 \\ & +0.01018\delta e & \end{array}$$

$$\begin{array}{lrr}(\rho.c.1) = & +3 \sin A_3 & \\ & +2 \sin 2A_3 & -9 \cos 2A_3 \\ & +23 \sin 3A_3 & -141 \cos 3A_3 \\ & +8 \sin 4A_3 & -39 \cos 4A_3 \\ & -9 \sin 5A_3 & +43 \cos 5A_3 \\ & -3 \sin 6A_3 & +13 \cos 6A_3 \\ & -1 \sin 7A_3 & +5 \cos 7A_3 \\ & & -0.43322\delta e \end{array}$$

$$\begin{array}{lrr}(\rho.s.1) = & -60 \sin A^3 & -3 \cos A_3 \\ & -45 \sin 2A_3 & -2 \cos 2A_3 \\ & -107 \sin 3A_3 & -23 \cos 3A_3 \\ & -29 \sin 4A_3 & -8 \cos 4A_3 \\ & +47 \sin 5A_3 & +9 \cos 5A_3 \\ & +15 \sin 6A_3 & +3 \cos 6A_3 \\ & +5 \sin 7A_3 & +1 \cos 7A_3 \\ & & +0.43394 e\delta g \end{array}$$

$$\begin{array}{lrr}(\rho.c.2) = & & -1 \cos A_3 \\ & & -6 \cos 2A_3 \\ & -1 \sin 3A_3 & +6 \cos 3A_3 \\ & -4 \sin 4A_3 & +5 \cos 4A_3 \\ & -6 \sin 5A_3 & +17 \cos 5A_3 \\ & -2 \sin 6A_3 & +7 \cos 6A_3 \\ & +3 \sin 7A_3 & -7 \cos 7A_3 \\ & & -0.03048\delta e \end{array}$$

$$\begin{array}{lrr}(\rho.s.2) = & -5 \sin A_3 & \\ & -8 \sin 2A_3 & \\ & +8 \sin 3A_3 & +1 \cos 3A_3 \\ & +5 \sin 4A_3 & +4 \cos 4A_3 \\ & +17 \sin 5A_3 & +6 \cos 5A_3 \\ & +7 \sin 6A_3 & +2 \cos 6A_3 \\ & -7 \sin 7A_3 & -3 \cos 7A_3 \\ & & +0.03053 e\delta g \end{array}$$

$$(\rho.c.3) = -0.000202\delta e \qquad (\rho.s.3) = +0.00203 e\delta g$$

Perturbations of the Latitude.

(The secular terms being omitted.)

Action of Jupiter.

$$\begin{aligned}(b.c.0) &= 0.''024 \sin A_1 - 0.''013 \cos A_1 \\ (b.s.1) &= -0.420 \sin A_1 + 0.494 \cos A_1 \\ (b.c.1) &= +0.494 \sin A_1 + 0.420 \cos A_1 \\ (b.s.2) &= +0.004 \sin 2A_1 - 0.017 \cos 2A_1 \\ (b.c.2) &= -0.017 \sin 2A_1 - 0.004 \cos 2A_1\end{aligned}$$

Action of Saturn.

$$
\begin{aligned}
(b.c.0) = &-0''.08 \sin A_2 - 0''.03 \cos A_2 \\
& - 0.03 \cos 2A_2 \\
& - 0.01 \cos 3A_2
\end{aligned}
$$

$$
\begin{aligned}
(b.s.1) = &+1''.34 \sin A_2 + 2''.88 \cos A_2 \\
&-0.01 \sin 2A_2 - 0.10 \cos 2A_2 \\
&+0.03 \sin 3A_2 - 0.06 \cos 3A_2 \\
&+0.02 \sin 4A_2 - 0.03 \cos 4A_2
\end{aligned}
\qquad
\begin{aligned}
(b.c.1) = &-1.56 \sin A_2 + 2.50 \cos A_2 \\
&+0.02 \sin 2A_2 - 0.10 \cos 2A_2 \\
&-0.04 \sin 3A_2 - 0.06 \cos 3A_2 \\
&-0.02 \sin 4A_2 - 0.02 \cos 4A_2
\end{aligned}
$$

$$
\begin{aligned}
(b.s.2) = &-0.09 \sin A_2 + 0.10 \cos A_2 \\
&-0.05 \sin 2A_2 - 0.09 \cos 2A_2 \\
& - 0.01 \cos 3A_2
\end{aligned}
\qquad
\begin{aligned}
(b.c.2) = &-0.09 \sin A_2 - 0.05 \cos A_2 \\
&+0.07 \sin 2A_2 + 0.02 \cos 2A_2 \\
&+0.01 \sin 3A_2
\end{aligned}
$$

Action of Neptune.

$$
\begin{aligned}
(b.c.0) = &+0''.01 \sin A_3 - 0''.04 \cos A_3 \\
&-0.01 \sin 2A_3 + 0.00 \cos 2A_3 \\
&-0.01 \sin 3A_3 + 0.04 \cos 3A_3 \\
&+0.01 \sin 4A_3 + 0.01 \cos 4A_3 \\
&+0.01 \sin 5A_3 \\
&+0.01 \sin 6A_3
\end{aligned}
$$

$$
\begin{aligned}
(b.s.1) = &+0''.13 \sin A_3 + 0''.16 \cos A_3 \\
&+0.09 \sin 2A_3 + 0.26 \cos 2A_3 \\
&+0.08 \sin 3A_3 + 0.28 \cos 3A_3 \\
&+0.01 \sin 4A_3 + 0.03 \cos 4A_3 \\
&-0.02 \sin 5A_3 - 0.12 \cos 5A_3 \\
&-0.01 \sin 6A_3 - 0.03 \cos 6A_3 \\
& - 0.01 \cos 7A_3
\end{aligned}
\qquad
\begin{aligned}
(b.c.1) = &-0.57 \sin A_3 + 0.04 \cos A_3 \\
&-0.39 \sin 2A_3 + 0.06 \cos 2A_3 \\
&-0.33 \sin 3A_3 + 0.07 \cos 3A_3 \\
&-0.07 \sin 4A_3 + 0.01 \cos 4A_3 \\
&+0.10 \sin 5A_3 - 0.03 \cos 5A_3 \\
&+0.03 \sin 6A_3 \\
&+0.01 \sin 7A_3
\end{aligned}
$$

$$
\begin{aligned}
(b.s.2) = &+0.01 \sin A_3 + 0.05 \cos A_3 \\
&-0.03 \sin 3A_3 - 0.09 \cos 3A_3 \\
& - 0.05 \cos 4A_3 \\
& - 0.04 \cos 5A_3 \\
& + 0.01 \cos 7A_3
\end{aligned}
\qquad
\begin{aligned}
(b.c.2) = &-0.07 \sin A_3 \\
&+0.09 \sin 3A_3 - 0.03 \cos 3A_3 \\
&+0.05 \sin 4A_3 \\
&+0.04 \sin 5A_3 \\
&+0.01 \sin 6A_3 \\
&-0.01 \sin 7A_3
\end{aligned}
$$

Action of Jupiter and Saturn.

(Terms multiplied by the product of their masses.)

$$
\begin{aligned}
N_1 &= 2S - J \\
N_2 &= -U + 3S - J \\
N_3 &= -2U + 4S - J \\
N_4 &= 3U + 3S - 2J \\
N_5 &= 2U + 4S - 2J \\
N_6 &= 5S - 2J \\
N_7 &= -3U + 6S - 2J \\
N_8 &= -3U + 7S - 2J
\end{aligned}
$$

Action of Jupiter and Saturn —Continued.

(Terms multiplied by the product of their masses.)

$$
\begin{aligned}
(v.c.0) = & + 0.08'' \sin N_2 + 0.51'' \cos N_2 \\
& + 0.04 \sin N_3 + 0.01 \cos N_3 \\
& - 0.01 \sin N_4 + 0.05 \cos N_4 \\
& - 0.35 \sin N_5 - 1.30 \cos N_5 \\
& \left\{ \begin{array}{l} - 0.55 \sin N_6 - 0.03 \cos N_6 \\ +40.65 \sin N_7 - 10.50 \cos N_7 \end{array} \right\} \\
& - 0.05 \sin N_8 + 0.03 \cos N_8
\end{aligned}
$$

$$
\begin{aligned}
(v.c.1) = & + 0.06 \sin N_1 - 0.27 \cos N_1 \\
& + 0.18 \sin N_2 + 0.01 \cos N_2 \\
& - 0.03 \sin N_3 + 0.08 \cos N_3 \\
& - 0.02 \sin N_4 + 0.09 \cos N_4 \\
& - 0.44 \sin N_5 - 0.61 \cos N_5 \\
& \left\{ \begin{array}{l} - 4.23 \sin N_6 - 3.87 \cos N_6 \\ + 8.06 \sin N_7 - 8.38 \cos N_7 \end{array} \right\} \\
& - 0.10 \sin N_8 + 0.03 \cos N_8
\end{aligned}
\qquad
\begin{aligned}
(v.s.1) = & +0.26'' \sin N_1 + 0.27'' \cos N_1 \\
& -0.04 \sin N_2 - 0.17 \cos N_2 \\
& +0.08 \sin N_3 + 0.03 \cos N_3 \\
& -0.02 \sin N_4 + 0.08 \cos N_4 \\
& +0.30 \sin N_5 - 0.58 \cos N_5 \\
& \left\{ \begin{array}{l} +2.64 \sin N_6 + 4.64 \cos N_6 \\ +7.35 \sin N_7 + 4.41 \cos N_7 \end{array} \right\} \\
& -0.04 \sin N_8 + 0.00 \cos N_8
\end{aligned}
$$

$$
(v.c.2) = \left\{ \begin{array}{l} -0.24 \sin N_6 - 0.22 \cos N_6 \\ +0.47 \sin N_7 - 0.54 \cos N_7 \end{array} \right\}
\qquad
(v.s,2) = \left\{ \begin{array}{l} +0.16 \sin N_6 + 0.26 \cos N_6 \\ +0.54 \sin N_7 + 0.47 \cos N_7 \end{array} \right\}
$$

$$(\rho.c.0) = +11 \sin N_5 \quad -3 \cos N_5$$

Two of these arguments, namely, $5S - 2J$, and $-3g + 6S - 2J$, are of very long period, that of the first being about 880, and that of the second about 1590 years. It will, therefore, be convenient to tabulate them both as functions of the time for the time during which the theory is to be used. To make their effect as small as possible during the period for which the provisional ephemeris is to be computed, we shall suppose the longitude of epoch, mean motion, and longitude of the perigee to be affected with the negative of the following corrections:

$$
\begin{aligned}
\delta\epsilon &= + 27.27'', \\
\delta\pi &= + 27.27, \\
\delta n &= - \quad 0.1172.
\end{aligned}
$$

Reducing these corrections to corrections of the co-ordinates, and adding them to the terms of long period in the true longitude and logarithm of radius vector, we shall have for these terms,

$$
\begin{aligned}
(v.c.0) = & - 0.546'' \sin N_6 - 0.032'' \cos N_6 \\
& + 40.650 \sin N - 10.500 \cos N_7 + 27.27 - 11.72\, T
\end{aligned}
$$

$$(v.s.1) = 2.63'' \sin N_6 + 4.64'' \cos N_6 + 7.35'' \sin N_7 + 4.42'' \cos N_7$$

$$(v.c.1) = - 4.22 \sin N_6 - 3.87 \cos N_6 + 8.06 \sin N_7 - 8.39 \cos N_7 - 1.10 T$$

$$(v.s.2) = +0''.16 \sin N_6 + 0''.26 \cos N_6 + 0''.54 \sin N_7 + 0''.47 \cos N_7$$
$$(v.c.2) = -0.24 \sin N_6 - 0.22 \cos N_6 + 0.47 \sin N_7 - 0.54 \cos N_7 - 0''.07 T$$
$$(\rho.c.0) = + \ 2 \cos N_6 + 10 \sin N_7 + 32 \cos N_7 + 22$$
$$(\rho.s.1) = -41 \sin N_6 - 43 \cos N_6 + 82 \sin N_7 - 85 \cos N_7 - 12 T$$
$$(\rho.c.1) = -29 \sin N_6 - 45 \cos N_6 - 73 \sin N_7 - 44 \cos N_7$$

The values of these and of the other secular terms and terms of long period for the period during which Uranus has been observed, are given in the following table:

(v.c.0)

	Neptune (long per.)	Jupiter and Saturn (long per.)	Sum.
	"	"	"
1700	+85.54	+4.86	+90.40
1750	+38.45	+1.90	+40.35
1760	31.25	1.48	32.73
1770	24.80	1.11	25.91
1780	19.09	0.80	19.89
1790	14.12	0.54	14.66
1800	9.89	+0.33	10.22
1810	6.42	0.17	6.59
1820	3.69	+0.07	3.76
1830	1.71	0.00	1.71
1840	+ 0.48	—0.02	+ 0.46
1850	0.00	0.00	0.00
1860	+ 0.27	+0.06	+ 0.33
1870	1.29	+0.16	1.45
1880	+ 3.06	+0.30	+ 3.36

VALUES OF (v.s.1)

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	"	"	"	"	"	"
1700	+1.82	+13.77	+0.75	—212.73	—10.18	—206.57
1750	+1.22	+ 9.18	+0.50	—141.97	— 9.11	—140.18
1760	1.10	8.26	0.45	—127.76	— 8.83	—126.78
1770	0.97	7.34	0.40	—113.55	— 8.54	—113.38
1780	0.85	6.43	0.35	— 99.33	— 8.24	— 99.94
1790	0.73	5.51	0.30	— 85.11	— 7.93	— 86.50
1800	0.61	4.59	0.25	— 70.90	— 7.61	— 73.06
1810	0.49	3.67	0.20	— 56.69	— 7.28	— 59.61
1820	0.36	2.76	0.15	— 42.49	— 6.94	— 46.16
1830	0.24	1.84	0.10	— 28.31	— 6.60	— 32.73
1840	+0.12	+ 0.92	+0.05	— 14.14	— 6.26	— 19.31
1850	0.00	0.00	0.00	0.00	— 5.91	— 5.91
1860	—0.12	— 0.92	—0.05	+ 14.12	— 5.57	+ 7.46
1870	—0.24	— 1.84	—0.10	28.23	— 5.23	20.82
1880	—0.36	— 2.76	—0.15	+ 42.32	— 4.89	+ 34.16

VALUES OF ($v.c.1$)

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	+17.19	+16.67	+7.20	+37.63	—1.98	+76.71
1750	+11.46	+11.11	+4.80	+28.67	—2.28	+53.76
1760	10.31	10.00	4.32	26.46	—2.36	48.73
1770	9.17	8.89	3.84	24.10	—2.45	43.55
1780	8.02	7.78	3.36	21.60	—2.53	38.23
1790	6.88	6.67	2.88	18.95	—2.61	32.77
1800	5.73	5.56	2.40	16.16	—2.69	27.16
1810	4.58	4.44	1.92	13.23	—2.76	21.41
1820	3.44	3.33	1.44	10.15	—2.83	15.53
1830	2.29	2.22	0.96	6.92	—2.90	9.49
1840	+ 1.15	+ 1.11	+0.48	+ 3.53	—2.97	+ 3.30
1850	0.00	0.00	0.00	0.00	—3.03	— 3.03
1860	— 1.15	— 1.11	—0.48	— 3.68	—3.08	— 9.50
1870	— 2.29	— 2.22	—0.96	— 7.51	—3.13	—16.11
1880	— 3.44	— 3.33	—1.44	—11.49	—3.16	—22.86

($v.s.2$) const. = — 0″.14.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+0.11	+0.81	+0.04	—12.46	—0.72	—12.36
1750	+0.07	+0.54	+0.03	—8.31	—0.70	—8.51
1760	0.06	0.49	0.03	—7.48	—0.69	—7.73
1770	0.06	0.43	0.02	—6.64	—0.68	—6.95
1780	0.05	0.38	0.02	—5.81	—0.66	—6.16
1790	0.04	0.32	0.02	—4.98	—0.65	—5.39
1800	0.04	0.27	0.02	—4.15	—0.64	—4.60
1810	0.03	0.22	0.01	—3.32	—0.63	—3.83
1820	0.02	0.16	0.01	—2.49	—0.61	—3.05
1830	0.01	0.11	+0.01	—1.66	—0.60	—2.27
1840	+0.01	+0.05	0.00	—0.83	—0.58	—1.49
1850	0.00	0.00	0.00	0.00	—0.57	—0.71
1860	—0.01	—0.05	0.00	+0.82	—0.56	+0.06
1870	—0.01	—0.11	—0.01	1.65	—0.54	0.84
1880	—0.02	—0.16	—0.01	+2.48	—0.53	+1.62

($v.c.2$) const. = + 0″.13.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+1.00	+0.98	+0.42	+2.20	—0.11	+4.62
1750	+0.67	+0.65	+0.28	+1.68	—0.12	+3.29
1760	+0.60	0.59	0.25	1.55	—0.12	3.00
1770	+0.54	0.52	0.22	1.41	—0.12	2.70
1780	+0.47	0.46	0.20	1.26	—0.13	2.39
1790	+0.40	0.39	0.17	1.11	—0.13	2.07
1800	+0.34	0.33	0.14	0.94	—0.13	1.75
1810	+0.27	0.26	0.11	0.77	—0.14	1.40
1820	+0.20	0.20	0.08	0.59	—0.14	1.06
1830	+0.13	0.13	0.06	0.39	—0.14	0.70
1840	+0.07	+0.07	+0.03	+0.20	—0.15	+0.35
1850	0.00	0.00	0.00	0.00	—0.15	—0.02
1860	—0.07	—0.07	—0.03	—0.21	—0.15	—0.40
1870	—0.13	—0.13	—0.06	—0.43	—0.16	—0.78
1880	—0.20	—0.20	—0.08	—0.68	—0.16	—1.19

(v.s.3)

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	"	"	"	"	"	"
1700	+0.01	+0.05	0.00	—0.76	0	—0.70
1750	0.00	+0.03	0	—0.51	0	—0.48
1760	0	0.03	0	—0.46	0	—0.43
1770	0	0.03	0	—0.40	0	—0.37
1780	0	0.02	0	—0.35	0	—0.33
1790	0	0.02	0	—0.30	0	—0.28
1800	0	0.02	0	—0.25	0	—0.23
1810	0	0.01	0	—0.20	0	—0.19
1820	0	0.01	0	—0.15	0	—0.14
1830	0	+0.01	0	—0.10	0	—0.09
1840	0	0.00	0	—0.05	0	—0.05
1850	0	0.00	0	0.00	0	0.00
1860	0	0.00	0	+0.05	0	+0.05
1870	0	—0.01	0	+0.10	0	+0.09
1880	0	—0.01	0	+0.15	0	+0.14

(v.c.3)

	(1)	(2)	(3)	(4)		Sum.
	"	"	"	"		"
1700	+0.06	+0.06	+0.02	+0.13		+0.27
1750	+0.04	+0.04	+0.02	+0.10		+0.20
1760	0.04	0.04	0.01	+0.09		+0.18
1770	0.03	0.03	0.01	+0.08		+0.15
1780	0.03	0.03	0.01	+0.07		+0.14
1790	0.02	0.02	0.01	+0.06		+0.12
1800	0.02	0.02	0.01	+0.06		+0.11
1810	0.02	0.02	+0.01	+0.05		+0.09
1820	0.01	0.01	0.00	+0.04		+0.06
1830	+0.01	+0.01	0.00	+0.02		+0.04
1840	0.00	0.00	0.00	+0.01		+0.02
1850	0.00	0.00	0.00	0.00		0.00
1860	0.00	0.00	0.00	—0.01		—0.02
1870	—0.01	—0.01	0.00	—0.02		—0.04
1880	—0.01	—0.01	0.00	—0.04		—0.06

For the radius vector. Values of ($\rho.c.0$)

	(1)	(2)	(3)	(4)	(5)	Sum.
	"	"	"	"	"	"
1700	+0.4	+3.5	—0.2	+164	+13	+181
1750	+0.3	+2.3	—0.1	+110	7	+120
1760	0.3	2.1	0	99	6	107
1770	0.2	1.9	0	88	5	95
1780	0.2	1.6	0	77	4	83
1790	0.2	1.4	0	67	3	72
1800	0.2	1.2	0	56	2	60
1810	0.1	0.9	0	45	2	48
1820	0.1	0.7	0	34	+ 1	36
1830	+0.1	0.5	0	22	0	23
1840	.0	+0.2	0	+ 11	0	+ 11
1850	.0	0.0	0	0	— 1	— 1
1860	.0	—0.2	0	— 11	— 2	— 13
1870	—0.1	—0.5	0	22	— 2	— 25
1880	—0.1	—0.7	0	— 33	— 3	— 37

(ρ.s.1) const. = + 63.

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	+180	+176	+75	+396	—21	+869
1750	+120	+117	+50	+302	—25	+627
1760	108	105	45	280	26	575
1770	96	94	40	255	28	520
1780	84	82	35	228	29	463
1790	72	70	30	200	30	405
1800	60	59	25	170	31	346
1810	48	47	20	139	32	285
1820	36	35	15	106	33	222
1830	24	23	10	72	34	158
1840	+ 12	+ 12	+ 5	+ 37	34	95
1850	0	0	0	0	35	+ 28
1860	— 12	— 12	— 5	— 39	36	— 41
1870	24	23	10	80	36	—110
1880	— 36	— 35	—15	—122	—37	—182

(ρ.c.1) const. = + 73.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	—20	—147	—9	+2241	+104	+2242
1750	—13	— 98	—6	+1496	+ 94	+1546
1760	—12	— 88	—5	1346	91	1405
1770	—10	— 78	—5	1197	88	1265
1780	— 9	— 69	—4	1047	85	1123
1790	— 8	— 59	—4	897	82	981
1800	— 6	— 49	—3	747	78	840
1810	— 5	— 39	—2	598	74	699
1820	— 4	— 29	—2	448	71	557
1830	— 3	— 20	—1	299	68	416
1840	— 1	— 10	—1	+ 149	64	274
1850	0	0	0	0	61	+ 134
1860	+ 1	+ 10	+1	— 149	58	— 6
1870	3	20	1	298	54	— 147
1880	4	29	2	— 447	+ 51	— 288

(ρ.s.2) const. = + 5.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+12	+12	+4	+28	0	+61
1750	+ 8	+ 8	+6	+21	0	+48
1760	7	7	5	19	0	43
1770	6	6	5	17	0	39
1780	6	6	4	15	0	36
1790	5	5	4	13	0	32
1800	4	4	3	11	0	27
1810	3	3	2	8	0	21
1820	2	2	2	6	0	17
1830	2	2	1	4	0	14
1840	+ 1	+ 1	+1	+ 2	0	10
1850	0	0	0	0	0	+ 5
1860	— 1	— 1	—1	— 2	0	0
1870	2	— 2	—1	4	0	— 4
1880	— 2	— 2	—2	— 6	0	— 7

(ρ.c.2) const. = + 4						
	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	"	"	"	"	"	"
1700	—1	—10	0	+157	0	+150
1750	—1	— 7	0	+105	0	+101
1760	—1	— 6	0	95	0	92
1770	—1	— 6	0	84	0	81
1780	—1	— 5	0	74	0	72
1790	—1	— 4	0	63	0	62
1800	0	— 3	0	52	0	53
1810	0	— 3	0	42	0	43
1820	0	— 2	0	31	0	33
1830	0	— 1	0	21	0	24
1840	0	— 1	0	+ 10	0	13
1850	0	0	0	0	0	+ 4
1860	0	+ 1	0	— 10	0	— 5
1870	0	1	0	21	0	—16
1880	0	+ 2	0	— 31	0	—25

Reduced Expressions for the Latitude of Uranus.

If we represent by V_1, V_2, V_3 the distances of Uranus from its descending nodes on the respective orbits of Jupiter, Saturn, and Uranus, we find the following perturbations of the latitude, which are independent of the mean longitude of the disturbing planets.

$$\begin{aligned} \delta\beta = &-0''.0114t\cos V_1 \\ &-0.0477t\cos V_2 \\ &-0.0125t\cos V_3 \\[1em] &+0''.245 \\ &+0.386\sin\ g + 0''.266\cos\ g \\ &-0.043\sin 2g + 0.006\cos 2g. \end{aligned}$$

To find how far the last five terms may be represented by simple corrections to the elliptic elements, we first represent the effect of minute corrections to the inclination and node of Uranus as a function of its mean anomaly. Putting u for the argument of latitude of Uranus, we have to a sufficient degree of approximation

$$\delta\beta = \sin u\delta\phi - \sin\phi\cos u\delta\theta$$
$$u = g + \omega + 2e\sin g$$

$$\begin{aligned} \sin u = &-e\sin\omega \\ &+\cos\omega\sin\ g + \sin\omega\cos\ g \\ &+e\cos\omega\sin 2g + e\sin\omega\cos 2g \end{aligned}$$

$$\begin{aligned}\cos u = & - e\cos\omega \\ & + \cos\omega\cos g - \sin\omega\sin g \\ & + e\cos\omega\cos 2g - e\sin\omega\sin 2g.\end{aligned}$$

Substituting these values of $\sin u$ and $\cos u$ in the expression for $\delta\beta$, and putting $\sin\phi\delta\theta = \delta'\theta$, we have

$$\begin{aligned}\delta\beta = & \ e\cos\omega\delta'\theta - e\sin\omega\,\delta\phi \\ & + (\cos\omega\delta\phi + \sin\omega\delta'\theta)\sin g + (\sin\omega\delta\phi - \cos\omega\delta'\theta)\cos g \\ & + (e\cos\omega\delta\phi + e\sin\omega\delta'\theta)\sin 2g + (e\sin\omega\delta\phi - e\cos\omega\delta'\theta)\cos 2g.\end{aligned}$$

To represent the numerical coefficients of $\sin g$ and $\cos g$ in $\delta\beta$ we must put

$$\begin{aligned}\cos\omega\delta\phi + \sin\omega\delta'\theta &= 0''.386 \\ \sin\omega\delta\phi - \cos\omega\delta'\theta &= 0\ .266.\end{aligned}$$

Since $\omega = 95° 3'$, this gives

$$\begin{aligned}\delta\phi &= \ \ 0''.231; \\ \delta'\theta &= \ \ 0.409; \\ \delta\beta &= - 0.013 \\ & \quad + 0''.386\sin g + 0.266\cos g \\ & \quad + 0.018\sin 2g + 0.013\cos 2g\end{aligned}$$

Subtracting this expression from the corresponding terms of $\delta\beta$, we have left

$$\delta\beta = + 0''.258 - 0''.061\sin 2g - 0''.007\cos 2g.$$

The first term of this expression shows that the mean orbit of Uranus at the present time is a small circle of the sphere one-quarter of a second north of its parallel great circle.

If we put

v = longitude of Uranus in its orbit, referred to the equinox and ecliptic of 1850, we have

$$\begin{aligned}V_1 &= v - 127° 37' \\ V_2 &= v - 126\ \ 45 \\ V_3 &= v - 155\ \ 32\end{aligned}$$

Substituting these values in the first three terms of $\delta\beta$, and multiplying the last term by the factor $(1 + \mu)$ by which the adopted mass of Neptune, $\frac{1}{17000}$, must be multiplied to obtain the true mass, we find

$$\delta\beta = (4''.69 + 1''.14\mu)\,T\cos v - (5''.24 + 0''.52\mu)\,T\sin v.$$

To these terms must be added those which arise from the motion of the ecliptic.

In the absence of any exhaustive investigation of the obliquity and motion of the ecliptic, I adopt the elements of Hansen, employed in his "*Tables du Soleil,*" because they are a mean between the results of others, and are very accordant with recent observations. The secular motion of the obliquity there employed is

$$- 46''.78.$$

Hansen mentions — 5″.39 as the corresponding motion at the equinox of 1850, found by Olufsen, but I cannot reproduce this result from the secular diminution with any masses of Mercury, Venus, and Mars, which seem to me probable. The expressions in terms of the masses given by Le Verrier are (*Annales de l'Observatoire Imperial de Paris*, tome ii, p. 101),

$$\begin{aligned}\text{Secular change} &= -\,47''.59 - 0''.52\nu - 28''.90\nu' - 0''.83\nu'''\\ \text{Mot. at equinox} &= +\;\;5.89 + 0.62\nu + \;\;7.57\nu' + 0.73\nu'''.\end{aligned}$$

In this expression the masses of Mercury, Venus, and Mars are represented by $\frac{1+\nu}{3{,}000{,}000}$, $\frac{1+\nu'}{401{,}847}$, and $\frac{1+\nu'''}{2{,}680{,}337}$, respectively. The influence of admissible changes in the masses of the other plants is insensible.

From the researches of Le Verrier on the motions of the four inner planets I conclude that the following are about the most probable distribution of the corrections of the masses necessary to produce the motion of the obliquity given by Hansen, namely,

$$\begin{aligned}\nu &= -\;\tfrac{2}{5}\\ \nu' &= -\,.018\\ \nu''' &= -\;\tfrac{1}{10}\end{aligned}$$

These values give for the motion at the equinox of 1850

$$+\,5''.43$$

Introducing the secular variation of these motions we have, for the change in the latitude of any celestial body near the ecliptic, arising from motion of the ecliptic,

$$\delta\beta = (5''.43T + 0''.19T^2)\cos v + (46''.78T - 0''.06\;T^2)\sin v.$$

Combining this with the change arising from the motion of the orbit of Uranus, we find

$$\begin{aligned}\delta\beta = &\{(10''.12 + 1''.14\mu)\;T + 0''.19\;T^2\}\cos v\\ &+ \{(41''.54 - 0''.52\mu)\;T - 0''.06\;T^2\}\sin v.\end{aligned}$$

We may represent these expressions in the usual way by secular variations of the inclination and node of Uranus. But, owing to the small inclination, and consequent rapid motion of the node, it will be necessary to include the coefficients of the second power of the time. On the other hand, no distinction between τ and θ is necessary. Putting ϕ for the inclination of the orbit, θ for the longitude of the node referred to the equinox of 1850, and

$$\begin{aligned}p &= \sin\phi\sin\theta,\\ q &= \sin\phi\cos\theta;\end{aligned}$$

we have

$$\begin{aligned}\sin\beta\;\;\; &= -\;\;p\cos v + \;q\sin v\\ \cos\beta\delta\beta &= -\,\delta p\cos v + \delta q\sin v.\end{aligned}$$

From the expressions for p and q we obtain

$$\cos\phi\, D_t\phi = \sin\theta\, D_t p + \cos\theta\, D_t q;$$
$$\sin\phi\, D_t\theta = \cos\theta\, D_t p - \sin\theta\, D_t q.$$

And, neglecting $(D_t\phi)^2 \times \sin\phi$, we have farther,

$$\cos\phi D^2_t\phi = \qquad \sin\phi\,(D_t\theta)^2 + \sin\theta D^2_t p + \cos\theta D^2_t q;$$
$$\sin\phi D^2_t\theta = -2\cos\phi D_t\theta D_t\phi + \cos\theta D^2_t p - \sin\theta L^2_t q.$$

Since ϕ is only 46′ we may put $\cos\phi$ and $\cos\beta$ both equal to unity in these expressions, while we have, for 1850,

$$\begin{aligned} \sin\theta &= \quad 0.9573 \\ \cos\theta &= \quad 0.2890 \\ D_t p &= -10''.12 - 1''.14\mu \\ D_t q &= +41.54 - 0.52\mu \\ L^2_t p &= - \quad 0.38 \\ D^2_t q &= - \quad 0.12 \\ \log\sin\phi &= \quad 8.129606. \end{aligned}$$

The above formulæ then give

$$\begin{aligned} D_t\phi &= +2''.31 \quad -1''.24\mu \\ D_t\theta &= -3167''.5 + 12''.6\mu \\ D^2_t\phi &= +0''.26 \\ D^2_t\theta &= +5''.6 \end{aligned}$$

$$\phi = \phi_0 + (2''.31 \quad - 1''.24\mu)\,T + 0''.13\,T^2$$
$$\theta = \theta_0 - (3167''.5 - 12''.6\mu)\,T + 2''.8\,T^2,$$

or, adding Struve's precession, we have when θ is counted from the mean equinox of date,

$$\theta = \theta_0 + (1857''.7 + 12''.6\mu)\,T + 3''.9\,T^2.$$

Using the values of ϕ and θ given by these expressions, the latitude, secular variation included, will be given by the expression

$$\sin\beta = \sin\phi\sin(v - \theta).$$

If we take from a table, as the principal term of the latitude, the value of $\sin\phi_0 \sin(v-\theta)$, the secular term to be added will be

$$\{(2''.31 - 1''.24\mu)\,T + 0''.13\,T^2\}\sin(v-\theta).$$

If we represent, as before, by ω the variable distance of the perihelion from the node, this term will be allowed for by adding to $(b.s.1)$, $(b.c.1)$, etc., the terms

$$
\begin{aligned}
(b.c.0) &= -e \sin \omega \delta\phi, \\
(b.s.1) &= \cos \omega \delta\phi, \\
(b.c.1) &= \sin \omega \delta\phi, \\
(b.s.2) &= e \cos \omega \delta\phi, \\
(b.c.2) &= e \sin \omega \delta\phi;
\end{aligned}
$$

where

$$\delta\phi = (2''.31 - 1''.24\mu)T + 0''.13T^2$$

Putting in the above expressions

$$
\begin{aligned}
\omega &= 95^\circ 3' + 3459'' T, \\
\cos \omega &= -.0880 - .0167T, \\
\sin \omega &= +.9961 - .0015T,
\end{aligned}
$$

we find

$$
\begin{aligned}
(b.c.0) &= -(0''.11 - 0''.06\mu)T \\
(b.s.1) &= -(0\ .20 - 0\ .11\mu)T - 0''.05T^2 \\
(b.c.1) &= (2\ .30 - 1\ .24\mu)T + 0\ .12T^2 \\
(b.s.2) &= -0\ .01T \\
(b.c.2) &= (0\ .11 - 0\ .06\mu)T.
\end{aligned}
$$

We have, finally, to consider the terms of long period in $\delta\eta$ and δk which have been omitted from the periodic perturbations produced by Neptune, in computing the terms of $\delta\beta$ on page 61, and which are as follows:

$$
\begin{aligned}
\delta\eta = {} & 1''.43 \cos(2l' - g) - 0''.39 \sin(2l' - g) \\
& - 2\ .12 \cos(4l' - 2g) + 1\ .00 \sin(4l' - 2g) \\
& + 0\ .20 \cos(6l' - 3g) - 0\ .04 \sin(6l' - 3g) \\
& + \text{constant} = 0''.00 \\
\delta k = {} & 0''.80 \cos(2l' - g) - 2''.28 \sin(2l' - g) \\
& - 1\ .06 \cos(4l' - 2g) - 1\ .85 \sin(4l' - 2g) \\
& + 0\ .04 \cos(6l' - 3g) + 0\ .19 \sin(6l' - 3g) \\
& + \text{constant} = 0''.364.
\end{aligned}
$$

For the period during which Uranus has been observed, these values of $\delta\eta$ and δk may be replaced by the following:

$$
\begin{aligned}
\delta\eta &= -0''.80T \\
\delta k &= +0\ .27T
\end{aligned}
$$

which are to be multiplied by the factor $1 + \mu$. The corresponding perturbation of the latitude will be

$$\delta\beta = \sin \mathrm{v}\,\delta\eta - \cos \mathrm{v}\,\delta k.$$

Putting for v its approximate value

$$\mathrm{v} = g + \omega + 2e \sin g$$

and developing to quantities of the first order with respect to the eccentricities, we have

$$
\begin{aligned}
\sin \mathrm{v} &= \sin(g + \omega) + e \sin(2g + \omega) - e \sin \omega \\
\cos \mathrm{v} &= \cos(g + \omega) + e \cos(2g + \omega) - e \cos \omega.
\end{aligned}
$$

Substituting for ω its value, 12°45′, and for $\delta\eta$ and δk their above values in the expression for $\delta\beta$, we find that the terms of $\delta\beta$ in question will add the following terms to $(b.c.0)$, $(b.s.1)$, etc.

$$
\begin{aligned}
(b.c.0) &= -.010\,\delta\eta + .046\,\delta k = +0''.02T(1+\mu)\\
(b.s.1) &= +.975\,\delta\eta + .221\,\delta k = -0\,.72T(1+\mu)\\
(b.c.1) &= +.221\,\delta\eta - .975\,\delta k = -0\,.44T(1+\mu)\\
(b.s.2) &= +.046\,\delta\eta + .011\,\delta k = -0\,.04T(1+\mu)\\
(b.c.2) &= -(b.c.0) \qquad\qquad\qquad = -0\,.02T(1+\mu)
\end{aligned}
$$

These values will be employed in the construction of the provisional ephemeris, but not in the tables.

Collecting all three classes of terms discussed in this section, we have the following constant and secular terms in $(b.c.0)$, $(b.s.1)$, etc.

$$
\begin{aligned}
(b.c.0) &= +0''.26 + (-0''.09 + 0.08\mu)T\\
(b.s.1) &= (-0''.92 - 0''.61\mu)T - 0''.05T^2\\
(b.c.1) &= (+1\,.86 - 1\,.68\mu)T + 0\,.12T^2\\
(b.s.2) &= -0\,.06 - 0\,.05T\\
(b.c.2) &= -0\,.01 + (0\,.09 - .08\mu)T
\end{aligned}
$$

Positions of Uranus resulting from the preceding theory.

The next step in order is the preparation of an ephemeris of the planet for comparison with observations. As this provisional theory is, for future use, superseded by the tables appended to the present work, it seems unnecessary to enter very fully into the details of the computation of the ephemeris. The perturbations of the longitude, logarithm of radius vector, and latitude, were first computed by the formulæ already given.

$$
\begin{aligned}
\delta v &= (v.c.0) + (v.c.1)\cos g + (v.c.2)\cos 2g + \text{etc.},\\
&\quad + (v.s.1)\sin g + (v.s.2)\sin 2g + \text{etc.},\\
M\delta\rho &= (\rho.c.0) + (\rho.c.1)\cos g + (\rho.c.2)\cos 2g + \text{etc.},\\
&\quad + (\rho.s.1)\sin g + (\rho.s.2)\sin 2g + \text{etc.},\\
\delta\beta &= (b.c.0) + (b.c.1)\cos g + (b.s.1)\sin g.
\end{aligned}
$$

Each coefficient $(v.c.0)$, $(v.c.1)$, etc., is composed at most of the following quantities:

1. The five classes of secular, long period, or constant terms, the separate values of which, with the sum of all, are given on pages 89 to 93.

2. Periodic terms due to the action of Jupiter, Saturn, and Neptune, given on pages 83 to 87.

3. Terms depending on the product of the masses of Jupiter and Saturn, given on page 88, omitting those depending on N_6 and N_7, because they are given in column 5 of the terms of the first class.

The sum of the perturbations thus computed is given in the third column of the following ephemeris.

An approximate value of the perturbations produced by Neptune alone is independently computed for every fourth date, and the result is given in the fourth

column. The secular and long period terms are here taken from columns (3) and (4) of the tables on pages 89 to 93.

The elliptic co-ordinates were then derived from the following elements, which are a little different from those employed in the computation of the perturbations.

Elements III. of Uranus.

π,	168° 15′ 12″.0
ε,	28 25 29 .5
θ,	73 11 58 .0
ϕ,	0 46 20 .0
e,	.0469436
e, (in sec.)	9682″.81
n,	15426.196
log a,	1.2828989
Red. to Ecliptic,	$-9''.37 \sin 2(v-\theta)$

The longitudes thus found are corrected for lunar, but not for solar nutation, and the results are given in the fifth column.

The column "correction" arises in this way: after the comparison of the ephemeris with observations was nearly completed, it was found that some errors had crept into the former, the most important of which was the employment of a mean anomaly, g, corrected for secular variation of the perihelion in the computation of the perturbations from the preceding formulæ. As a large portion of the computations on the provisional ephemeris had been made by assistants furnished by the Smithsonian Institution and Nautical Almanac, I deemed it prudent to make a careful recomputation of the perturbations for every sixth date during the entire period of the modern observations. The longitudes actually printed in the fifth column are the results of the original incorrect computation, while the numbers in the next column show the several corrections to be applied to obtain the results of my final revised computation.

During the period of the modern observations the ephemeris is computed for intervals of 120 days, and the selected dates are all exact multiples of that interval before or after the fundamental epoch, 1850, Jan. 0, Greenwich mean noon. For convenience of reference the dates are numbered from an epoch earlier by 212 intervals, and the number is given in the second column.

Between 1796 and 1801 no observations worth using were made on Uranus, the ephemeris has, therefore, not been extended over this interval.

Heliocentric Ephemeris of Uranus from the preceding Provisional Theory.

[The longitudes are corrected for lunar but not for solar nutation.]

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1690, Dec. 23		+4 28.5	+236.1	59 40 35.2		—10 7.8	1.2876828
Dec. 24		+4 28.5		59 41 16.5		—10 7.3	1.2876789
1712, April 2		+8 37.9		155 24 42.6		+45 58.7	1.2626650
April 3		+8 37.9		155 25 29.0		+45 58.8	1.2626640
1715, Mar. 4		+6 58.3	+136.2	169 10 21.4		+46 2.1	1.2623890
Mar. 5		+6 58.2		169 11 7.7		+46 2.1	1.2623893
Mar. 10		+6 57.5		169 15 0.0		+46 1.7	1.2623911
April 29		+6 51.1	+134.8	169 53 42.2		+45 58.0	1.2624066
April 30		+6 51.1		169 54 28.8		+45 57.9	1.2624102
1748, Oct. 21		—0 28.9	— 65.0	316 13 19.3		—41 27.6	1.3001855
Oct. 22		—0 28.8		316 13 58.8		—41 27.9	1.3001875
1750, Sept. 13		+0 6.3	— 51.3	323 44 14.9		—43 47.2	1.3013714
Sept. 14		+0 6.3		323 44 54.0		—43 47.3	1.3013731
Oct. 14		+0 6.6	— 50.8	324 4 22.1		—43 52.4	1.3014165
Oct. 15		+0 6.6		324 5 1.0		—43 52.5	1.3014181
Dec. 3		+0 8.0	— 49.8	324 36 49.5		—44 0.6	1.3014856
Dec. 4		+0 8.0		324 37 28.5		—44 0.8	1.3014872
1753, Dec. 3		—0 5.6	— 25.6	336 25 0.6		—46 1.1	1.3025789
Dec. 4		—0 5.6		336 25 39.3		—46 1.2	1.3025796
1756, Sept. 25		—0 49.5	— 0.4	347 25 34.9		—46 9.2	1.3029363
Sept. 26		—0 49.5		347 26 13.6		—46 9.1	1.3029363
1764, Jan. 15		+1 24.0		16 13 35.0		—38 36.9	1.3003508
Jan. 16		+1 24.0		16 14 14.2		—38 36.7	1.3003486
1768, Dec. 27		+1 33.1		36 3 35.2		—27 42.4	1.2955331
Dec. 29		+1 33.1	+106.9	36 4 44.1		—27 41.5	1.2955274
Dec. 31		+1 33.1		36 6 5.0		—27 40.7	1.2955216
1769, Jan. 15		+1 33.5		36 16 4.6		—27 34.2	1.2954753
Jan. 18		+1 33.5		36 18 4.5		—27 33.0	1.2954659
Jan. 21		+1 33.5	+106.9	36 20 4.5		—27 31.6	1.2954566
Jan. 24		+1 33.5		36 22 4.4		—27 30.3	1.2954476
1781, Jan. 1	1	+3 57.46	+178.9	86 36 13.23	+0.16	+11 3.63	1.2785972
May 1	2	+3 53.41		88 2 34.70		+12 11.14	1.2780883
Aug. 29	3	+3 48.58		89 29 7.41		+13 18.33	1.2775834
Dec. 27	4	+3 43.09		90 55 51.40		+14 25.14	1.2770827
1782, April 26	5	+3 37.18	+180.4	92 22 46.89		+15 31.53	1.2765870
Aug. 24	6	+3 30.97		93 49 53.89		+16 37.47	1.2760968
Dec. 22	7	+3 24.52		95 17 12.51	+0.18	+17 42.88	1.2756124
1783, April 21	8	+3 18.03		96 44 42.73		+18 47.73	1.2751345
Aug. 19	9	+3 11.87	+180.1	98 12 24.96		+19 51.92	1.2746633
Dec. 17	10	+3 6.01		99 40 19.00		+20 55.67	1.2742012
1784, April 15	11	+3 0.78		101 8 25.13		+21 58.69	1.2737412
Aug. 13	12	+2 56.24		102 36 43.29		+23 0.92	1.2732907
Dec. 11	13	+2 52.82	+178.6	104 5 13.79	+0.23	+24 2.46	1.2728469
1785, April 10	14	+2 50.58		105 33 56.61		+25 3.14	1.2724105
Aug. 8	15	+2 49.29		107 2 51.36		+26 3.05	1.2719806
Dec. 6	16	+2 49.35		108 31 58.18		+27 1.98	1.2715564
1786, April 5	17	+2 50.53	+175.5	110 1 16.88		+27 59.97	1.2711405
Aug. 3	18	+2 53.29		111 30 47.68		+28 57.02	1.2707299
Dec. 1	19	+2 57.31		113 0 30.11	+0.31	+29 53.03	1.2703265
1787, Mar. 31	20	+3 2.38		114 30 23.81		+30 47.92	1.2699294
July 29	21	+3 8.51	+171.0	116 0 28.62		+31 41.72	1.2695394
Nov. 26	22	+3 15.48		117 30 44.11		+32 34.33	1.2691551

HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″		′ ″	
1788, Mar. 25	23	+3 23.02		119 1 9.79		+33 25.72	1.2687775
July 23	24	3 31.36		120 31 45.79		34 15.81	1.2684065
Nov. 20	25	3 39.82	+165.1	122 2 31.13	+0.28	35 4.58	1.2680423
1789, Mar. 20	26	3 38.67		123 33 25.96		35 51.99	1.2676852
July 18	27	3 57.40		125 4 29.49		36 38.01	1.2673353
Nov. 15	28	4 5.97		126 35 41.50		37 22.50	1.2669932
1790, Mar. 15	29	4 14.35	+157.8	128 7 1.70		38 5.50	1.2666592
July 13	30	4 22.48		129 38 29.88		38 47.00	1.2663336
Nov. 10	31	4 30.00		131 10 5.37	+0.25	39 26.85	1.2660167
1791, Mar. 10	32	4 36.95		132 41 47.99		40 5.04	1.2657087
July 8	33	4 42.95	+149.1	134 13 37.15		40 41.58	1.2654082
Nov. 5	34	4 48.11		135 45 32.62		41 16.44	1.2651215
1792, Mar. 4	35	4 52.33		137 17 34.16		41 49.53	1.2648434
July 2	36	4 56.03		138 49 41.78		42 20.89	1.2645770
Oct. 30	37	4 58.32	+139.3	140 21 54.47	+0.46	42 50.35	1.2643210
1793, Feb. 27	38	4 59.50		141 54 12.11		43 18.00	1.2640766
June 27	39	4 59.85		143 26 34.79		43 43.81	1.2638448
Oct. 25	40	4 58.84		144 59 1.64		44 7.75	1.2636267
1794, Feb. 22	41	4 56.80	+128.6	146 31 32.77		44 29.74	1.2634226
June 22	42	4 53.48		148 4 7.47		44 49.85	1.2632327
Oct. 20	43	4 49.13	+122.9	149 36 45.84	+0.47	45 7.99	1.2630577
1795, Feb. 17	44	4 44.20		151 9 27.73		45 24.24	
June 17	45	4 38.00		152 42 12.55		45 38.44	
Oct. 15	46	4 31.04		154 15 0.15		45 50.64	
1801, Jan. 17	62	+3 22.92		179 3 50.23		+44 34.51	1.2627599
Mar. 17	63	3 26.59	+ 57.3	180 36 54.94		44 12.86	1.2628920
Sept. 14	64	3 31.06		182 9 57.40		43 49.24	1.2630362
1802, Jan. 12	65	3 36.26		183 42 57.23		43 23.66	1.2631922
Mar. 12	66	3 41.88		185 15 54.08		42 56.34	1.2633599
Sept. 9	67	3 47.78	+ 43.7	186 48 46.54	+0.35	42 26.97	1.2635387
1803, Jan. 7	68	3 54.01		188 21 35.11		41 55.86	1.2637281
Mar. 7	69	4 0.29		189 54 18.55		41 22.99	1.2639280
Sept. 4	70	4 6.43		191 26 56.80		40 48.28	1.2641386
1804, Jan. 2	71	4 12.95	+ 30.7	192 59 30.15		40 11.79	1.2643591
Mar. 1	72	4 18.74		194 31 56.83		39 33.63	1.2645891
Aug. 29	73	4 24.38		196 4 17.13	+0.14	38 53.72	1.2648287
Dec. 27	74	4 29.34		197 36 30.24		38 12.21	1.2650777
1805, April 26	75	4 33.60	+ 18.0	199 8 35.97		37 29.07	1.2653360
Aug. 24	76	4 37.42		200 40 33.91		36 44.40	1.2656037
Dec. 22	77	4 40.11		202 12 23.39		35 58.15	1.2658808
1806, April 21	78	4 41.83		203 44 4.25		35 10.48	1.2661674
Aug. 19	79	4 42.55	+ 5.9	205 15 36.11	+0.21	34 21.34	1.2664631
Dec. 17	80	4 42.19		206 46 58.67		33 30.88	1.2667689
1807, April 16	81	4 40.52		208 18 11.39		32 39.05	1.2670847
Aug. 14	82	4 37.43		209 49 13.98		31 45.96	1.2674103
Dec. 12	83	4 33.71	— 5.0	211 20 6.96		30 51.63	1.2677459
1808, April 10	84	4 28.30		212 50 49.02		29 56.12	1.2680922
Aug. 8	85	4 21.63		214 21 20.46	+0.25	28 59.51	1.2684491
Dec. 6	86	4 14.32		215 51 41.72		28 1.80	1.2688166
1809, April 5	87	4 5.84	— 15.4	217 21 51.97		27 3.03	1.2691945
Aug. 3	88	3 56.63		218 51 51.53		26 3.28	1.2695829
Dec. 1	89	3 47.07		220 21 40.62		25 2.66	1.2699816
1810, Mar. 31	90	3 37.16		221 51 19.10		24 1.10	1.2703905
July 29	91	3 27.15	— 24.3	223 20 47.07	+0.23	22 58.68	1.2708094
Nov. 26	92	+3 17.52		224 50 4.83		+21 55.38	1.2712377

HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1811, Mar. 26	93	+3 7.99		226 19 12.04		+20 51.52	1.2716752
July 24	94	2 59.12		227 48 9.06		19 46.95	1.2721215
Nov. 21	95	2 50.91	—32.1	229 16 55.87		18 41.69	1.2725760
1812, Mar. 20	96	2 43.45		230 45 32.41		17 35.80	1.2730381
July 18	97	2 37.26		232 13 59.13	+0.11	16 29.34	1.2735072
Nov. 15	98	2 31.98		233 42 15.55		15 22.35	1.2739838
1813, Mar. 15	99	2 27.86	—39.1	235 10 21.88		14 14.92	1.2744651
July 13	100	2 25.05		236 38 18.26		13 7.03	1.2749507
Nov. 10	101	2 22.98		238 6 3.95		11 58.79	1.2754409
1814, Mar. 10	102	2 22.35		239 33 39.61		10 50.21	1.2759367
July 8	103	2 22.54	—44.6	241 1 4.63	+0.22	9 41.32	1.2764360
Nov. 5	104	2 24.24		242 28 19.59		8 32.19	1.2769372
1815, Mar. 5	105	2 26.58		243 55 23.67		7 22.86	1.2774403
July 3	106	2 30.00		245 22 17.17		6 13.39	1.2779459
Oct. 31	107	2 34.17	—49.3	246 48 59.79		5 3.80	1.2784533
1816, Feb. 28	108	2 39.06		248 15 31.47		3 54.09	1.2789600
June 27	109	2 44.51		249 41 52.08	—0.01	2 44.46	1.2794688
Oct. 25	110	2 50.43		251 8 1.42		1 34.82	1.2799725
1817, Feb. 22	111	2 56.75	—54.4	252 33 59.49		+ 0 25.25	1.2804769
June 22	112	3 3.22		253 59 46.00		— 0 44.19	1.2809792
Oct. 20	113	3 10.03		255 25 21.18		1 53.44	1.2814788
1818, Feb. 17	114	3 16.63		256 50 44.48		3 2.48	1.2819759
June 17	115	3 22.96	—54.7	258 15 55.79	0.00	4 11.23	1.2824701
Oct. 15	116	3 28.90		259 40 55.14		5 19.73	1.2829599
1819, Feb. 12	117	3 34.28		261 5 42.25		6 27.83	1.2834467
June 12	118	3 38.77		262 30 17.04		7 35.52	1.2839317
Oct. 10	119	3 42.48	—56.1	263 54 39.42		8 42.77	1.2844143
1820, Feb. 7	120	3 45.03		265 18 49.19		9 49.52	1.2848927
June 6	121	3 46.37		266 42 46.29	—0.01	10 55.76	1.2853670
Oct. 4	122	3 46.38		268 6 30.67		12 1.40	1.2858391
1821, Feb. 1	123	3 45.07	—56.4	269 30 2.43		13 6.44	1.2863087
June 1	124	3 42.35		270 53 21.55		14 10.88	1.2867757
Sept. 29	125	3 38.08		272 16 27.96		15 14.57	1.2872402
1822, Jan. 27	126	3 32.86		273 39 22.29		16 17.58	1.2877030
May 27	127	3 26.18	—56.2	275 2 4.25	—0.10	17 19.84	1.2881632
Sept. 24	128	3 18.32		276 24 34.06		18 21.32	1.2886214
1823, Jan. 22	129	3 9.68		277 46 52.38		19 22.01	1.2890772
May 22	130	3 0.24		279 8 59.05		20 21.89	1.2895311
Sept. 19	131	2 50.36	—56.1	280 30 54.82		21 20.91	1.2899826
1824, Jan. 17	132	2 40.10		281 52 39.71		22 19.10	1.2904315
May 16	133	2 29.74		283 14 14.16	—0.07	23 16.40	1.2908776
Sept. 13	134	2 19.49		284 35 38.60		24 12.78	1.2913208
1825, Jan. 11	135	2 9.25	—55.5	285 56 52.92		25 8.22	1.2917606
May 11	136	1 59.46		287 17 57.80		26 2.71	1.2921967
Sept. 3	137	1 50.21		288 38 53.43		26 56.22	1.2926290
1826, Jan. 6	138	1 41.44		289 59 40.04		27 48.71	1.2930570
May 6	139	1 33.58	—54.5	291 20 18.03	—0.09	28 40.26	1.2934804
Sept. 3	140	1 26.44		292 40 47.50		29 30.76	1.2938987
1827, Jan. 1	141	1 20.21		294 1 8.83		30 20.16	1.2943116
May 1	142	1 14.89		295 21 22.15		31 8.60	1.2947186
Aug. 29	143	1 10.52	—54.0	296 41 27.72		31 55.92	1.2951196
Dec. 27	144	1 7.22		298 1 25.72		32 42.14	1.2955136
1828, April 25	145	1 4.79		299 21 16.45	—0.07	33 27.24	1.2959002
Aug. 23	146	1 3.51		300 41 0.14		34 11.25	1.2962790
Dec. 21	147	+1 3.11	—52.9	302 0 36.75		—34 54.11	1.2966496

HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1829, April 20	148	+1 3.68		303 20 6.73		—35 35.77	1.2970111
Aug. 18	149	1 5.15		304 39 30.08		36 16.26	1.2973630
Dec. 16	150	1 7.37		305 58 46.96		36 55.60	1.2977052
1830, April 15	151	1 10.14	—52.3	307 17 57.34	+0.08	37 33.68	1.2980368
Aug. 13	152	1 13.74		308 37 1.72		38 10.56	1.2983571
Dec. 11	153	1 17.63		309 55 59.75		38 46.13	1.2986661
1831, April 10	154	1 21.69		311 14 51.68		39 20.49	1.2989638
Aug. 8	155	1 26.06	—52.2	312 33 37.76		39 53.54	1.2992501
Dec. 6	156	1 30.26		313 52 17.74		40 25.29	1.2995247
1832, April 4	157	1 34.13		315 10 51.63	+0.05	40 55.74	1.2997873
Aug. 2	158	1 37.50		316 29 19.67		41 24.90	1.3000379
Nov. 30	159	1 40.39	—52.2	317 47 41.94		41 52.65	1.3002768
1833, Mar. 30	160	1 42.02		319 5 57.95		42 19.10	1.3005047
July 28	161	1 42.86		320 24 8.31		42 44.17	1.3007216
Nov. 25	162	1 42.45		321 42 12.89		43 7.88	1.3009280
1834, Mar. 25	163	1 40.92	—52.5	323 0 11.92	+0.15	43 30.24	1.3011231
July 23	164	1 38.01		324 18 5.47		43 51.18	1.3013090
Nov. 20	165	1 34.01		325 35 53.99		44 10.74	1.3014851
1835, Mar. 20	166	1 28.76		326 53 37.59		44 28.92	1.3016515
July 18	167	1 22.19	—52.9	328 11 16.27		44 45.69	1.3018084
Nov. 15	168	1 14.89		329 28 50.92		45 1.14	1.3019567
1836, Mar. 14	169	1 6.50		330 46 21.41	+0.19	45 15.12	1.3020960
July 12	170	0 57.31		332 3 48.20		45 27.71	1.3022265
Nov. 9	171	0 47.51	—53.9	333 21 11.76		45 38.98	1.3023482
1837, Mar. 9	172	0 37.27		334 38 32.35		45 48.84	1.3024612
July 7	173	0 26.69		315 55 50.40		45 57.28	1.3025656
Nov. 4	174	0 15.92		337 13 6.16		46 4.45	1.3026622
1838, Mar. 4	175	+0 5.23	—55.1	338 30 20.17	+0.17	46 10.04	1.3027503
July 2	176	—0 5.21		339 47 32.76		46 14.36	1.3028297
Oct. 30	177	0 15.76		341 4 43.81		46 17.29	1.3029008
1839, Feb. 27	178	0 25.58		342 21 54.35		46 18.83	1.3029634
June 27	179	0 34.82	—56.9	243 39 4.45		46 19.00	1.3030176
Oct. 25	180	0 43.68		344 56 14.08		46 17.84	1.3030630
1840, Feb. 22	181	0 51.42		346 13 24.23	+0.17	46 15.28	1.3030996
June 21	182	0 58.36		347 30 34.70		46 11.29	1.3031269
Oct. 19	183	1 4.34	—58.4	348 47 46.09		46 6.01	1.3031447
1841, Feb. 16	184	1 9.47		350 4 58.23		45 59.35	1.3031528
June 16	185	1 13.36		351 22 11.79		45 51.33	1.3031509
Oct. 14	186	1 16.11		352 39 26.97		45 41.91	1.3031392
1842, Feb. 11	187	1 17.73	—60.5	353 56 43.91	+0.15	45 31.07	1.3031164
June 11	188	1 18.13		355 14 2.89		45 18.90	1.3030817
Oct. 9	189	1 17.35		256 31 24.13		45 5.38	1.3030351
1843, Feb. 6	190	1 15.50		357 48 47.74		44 50.50	1.3029767
June 6	191	1 12.83	—66.7	359 6 13.65		44 34.22	1.3029060
Oct. 4	192	1 9.27		0 23 42.19		44 16.64	1.3028228
1844, Feb. 1	193	1 4.76		1 41 13.59	+0.07	43 57.59	1.3027267
May 31	194	0 59.91		2 58 47.55		43 37.23	1.3026173
Sept. 28	195	0 54.61	—64.9	4 16 24.34		43 15.53	1.3024947
1845, Jan. 26	196	0 49.19		5 34 3.83		42 52.43	1.3023588
May 26	197	0 43.90		6 51 46.06		42 27.99	1.3022095
Sept. 23	198	0 38.92		8 9 31.09		42 2.22	1.3020472
1846, Jan. 21	199	0 34.37	—67.7	9 27 19.04	+0.15	41 35.11	1.3018722
May 21	200	0 30.55		10 45 9.82		41 6.76	1.3016851
Sept. 18	201	—0 27.21		12 3 3.39		—40 37.01	1.3014861

HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1847, Jan. 16	202	—0 25.64		13 21 0.27		—40 6.03	1.3012758
May 16	203	0 24.73	—70.2	14 39 0.21		39 33.75	1.3010544
Sept. 13	204	0 24.95		15 57 3.61		39 0.23	1.3008219
1848, Jan. 11	205	0 26.43		17 15 10.46	+0.06	38 25.40	1.3005790
May 10	206	0 28.95		18 33 21.30		37 49.42	1.3003263
Sept. 7	207	0 32.75	—72.8	19 51 36.15		37 12.18	1.3000640
1849, Jan. 5	208	0 37.59		21 9 55.34		36 33.73	1.2997923
May 5	209	0 43.47		22 28 19.02		35 54.16	1.2995115
Sept. 2	210	0 50.27		23 46 48.13		35 13.36	1.2992224
Dec. 31	211	0 57.74	—75.3	25 5 22.56	—0.08	34 31.47	1.2989252
1850, April 30	212	1 6.18		26 24 2.38		33 48.40	1.2986203
Aug. 28	213	1 15.11		27 42 48.39		33 4.37	1.2983078
Dec. 26	214	1 24.38		29 1 40.87		32 19.06	1.2979881
1851, April 25	215	1 34.33	—77.9	30 20 39.78		31 32.79	1.2976615
Aug. 23	216	1 44.28		31 39 45.91		30 45.46	1.2973287
Dec. 21	217	1 54.21		32 58 59.55	—0.09	29 57.09	1.2969894
1852, April 19	218	2 4.12		34 18 20.89		29 7.71	1.2966436
Aug. 17	219	2 13.61	—80.5	35 37 50.43		28 17.33	1.2962916
Dec. 15	220	2 23.00		36 57 28.19		27 25.95	1.2959338
1853, April 14	221	2 31.54		38 17 15.06		26 33.62	1.2955701
Aug. 12	222	2 39.58		39 37 10.77		25 40.35	1.2952015
Dec. 10	223	2 46.62	—82.6	40 57 16.17	—0.07	24 46.17	1.2948270
1854, April 9	224	2 52.64		42 17 31.26		23 51.08	1.2944458
Aug. 7	225	2 57.77		43 37 56.24		22 55.08	1.2940583
Dec. 5	226	3 1.60		44 58 31.59		21 58.25	1.2936645
1855, April 4	227	3 4.03	—84.6	46 19 17.49		21 0.51	1.2932645
Aug. 2	228	3 5.35		47 40 13.93		20 1.97	1.2928582
Nov. 30	229	3 5.32		49 1 21.26	—0.11	19 2.66	1.2924449
1856, Mar. 29	230	3 3.95		50 22 39.57		18 2.55	1.2920239
July 27	231	3 1.24	—86.1	51 44 9.08		17 1.67	1.2915954
Nov. 24	232	2 57.62		53 5 49.32		16 0.03	1.2911592
1857, Mar. 24	233	2 52.95		54 27 40.66		14 57.72	1.2907155
July 22	234	2 47.58		55 49 42.98		13 54.71	1.2902643
Nov. 19	235	2 41.65	—87.9	57 11 56.10	—0.10	12 51.08	1.2898057
1858, Mar. 19	236	2 35.18		58 34 20.16		11 46.89	1.2893395
July 17	237	2 28.82		59 56 54.59		10 41.96	1.2888659
Nov. 14	238	2 22.46		61 19 39.71		9 36.91	1.2883853
1859, Mar. 14	239	2 16.35	—88.6	62 42 35.14		8 30.87	1.2878981
July 12	240	2 10.33		64 5 41.38		7 24.61	1.2874049
Nov. 9	241	2 5.17		65 28 57.59	—0.13	6 17.90	1.2869061
1860, Mar. 8	242	2 0.67		66 52 24.15		5 10.84	1.2864023
July 6	243	1 57.10	—89.1	68 16 0.90		4 3.42	1.2858941
Nov. 3	244	1 54.37		69 39 47.78		2 55.74	1.2853816
1861, Mar. 3	245	1 52.61		71 3 44.98		1 47.83	1.2848658
July 1	246	1 51.92		72 27 52.38		— 0 39.69	1.2843468
Oct. 29	247	1 52.43	—89.3	73 52 10.31	—0.19	+ 0 28.60	1.2838257
1862, Feb. 26	248	1 53.51		75 16 38.24		1 37.00	1.2833029
June 26	249	1 55.99		76 41 16.65		2 45.46	1.2827788
Oct. 24	250	1 59.54		78 6 5.46		3 53.95	1.2822542
1863, Feb. 21	251	2 3.98	—88.5	79 31 4.97		5 2.40	1.2817290
June 21	252	2 9.43		80 56 15.07		6 10.80	1.2812047
Oct. 19	253	2 15.63		82 21 36.01	—0.04	7 19.09	1.2806819
1864, Feb. 16	254	2 22.52		83 47 7.95		8 27.25	1.2801609
June 15	255	2 30.20	—87.2	85 12 50.83		9 35.22	1.2796421
Oct. 13	256	—2 38.39		86 38 44.88		+ 8 42.95	1.2791263

Heliocentric Ephemeris of Uranus.—*Concluded.*							
Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1865, Feb. 10	257	—2 47.06		88 4 50.26		+11 52.20	1.2786142
June 10	258	2 55.93		89 31 7.17		12 57.54	1.2781063
Oct. 8	259	3 5.01	—85.2	90 57 35.61	—0.09	14 4.34	1.2776029
1866, Feb. 5	260	3 13.91		92 24 16.00		15 10.74	1.2771044
June 5	261	3 22.61		93 51 8.39		16 16.70	1.2766112
Oct. 3	262	3 30.79		95 18 13.06		17 22.18	1.2761237
1867, Jan. 31	263	3 38.49	—82.5	96 45 29.91		18 27.15	1.2756419
May 31	264	3 45.25		98 12 59.44		19 31.54	1.2751656
Sept. 28	265	3 50.96		99 40 41.71	—0.08	20 35.37	1.2746948
1868, Jan. 26	266	3 55.67		101 8 36.54		21 38.51	1.2742299
May 25	267	3 59.06	—79.1	102 36 44.31		22 40.96	1.2737709
Sept. 22	268	4 1.02		104 5 5.00		23 42.75	1.2733175
1869, Jan. 20	269	4 1.71		105 33 38.41		24 43.69	1.2728693
May 20	270	4 0.81		107 2 24.67		25 43.81	1.2724264
Sept. 17	271	3 58.59	—75.1	108 31 23.55	—0.11	26 43.07	1.2719888
1870, Jan. 15	272	3 54.96		110 0 34.97		27 41.44	1.2715563
May 15	273	3 50.23		111 29 58.50		28 38.77	1.2711290
Sept. 12	274	3 44.34		112 59 33.91		29 35.16	1.2707070
1871, Jan. 10	275	3 37.90	—69.8	114 29 20.79		30 30.41	1.2702903
May 10	276	3 30.56		115 59 19.11		31 24.57	1.2698791
Sept. 7	277	3 22.61		117 29 28.39	—0.18	32 17.57	1.2694735
1872, Jan. 5	278	3 14.50		118 59 48.05		33 9.31	1.2690738
May 4	279	3 6.19	—64.2	120 30 17.95		33 59.79	1.2686804
Sept. 1	280	—2 57.65		122 0 57.87		+34 48.97	1.2682934

The next operation would be to interpolate these co-ordinates to intervals of time suitable for the computation of a geocentric ephemeris, to correct the longitudes for solar nutation, and then to compute the geocentric right ascension and declination. This operation has not, however, been completely carried out except for most of the observations before 1830, and for three of the oppositions observed since, the latter being computed only as a check upon the accuracy of the comparisons. As a general rule, it may be said that wherever a complete geocentric ephemeris, with the heliocentric ephemeris from which it was computed, were available, these ephemerides were made use of in a manner which will be more fully described hereafter, while, in all other cases, the geocentric places were computed directly.

It may also be stated here that Hansen's *Tables du Soleil* have been adopted as giving the places of the sun to be used in computing the geocentric places.

CHAPTER VI.

REDUCTION OF THE OBSERVATIONS OF URANUS, AND THEIR COMPARISON WITH THE PRECEDING THEORY.

THE observations of Uranus naturally divide themselves into two distinct classes. (1) The purely accidental ones, made previous to the recognition of the planet by Herschel in 1781, and therefore without any suspicion on the part of the observers that the object was not a fixed star, and (2) the systematic observations made since.

The first class are nearly all so uncertain in comparison with the second that I have hesitated over the question of employing them at all. If nothing but a determination of the elements of Uranus were called for, they would certainly not be worth using, since these elements may be determined with entire certainty from the observations which have been made during the entire revolution of the planet since 1781. But the mass of Neptune is also to be determined, and it is at least possible that these observations, uncertain though they are, may add materially to the weight of this determination. I have, therefore, determined to include them all, re-reducing them when there seemed to be good reason so to do.

The earliest observations are those of Flamstead, published in the Historiæ Cœlestis. The observations themselves, as printed, together with the principal elements for reduction, are given in the following tables.

The first column of the table gives the name of the star. The second gives the clock time of transit over the wire of the quadrant as given by Flamstead. The time, it will be seen, is only given to entire seconds. We must, therefore, expect to find a probable error, of which the mathematical minimum is $0^s.25$, and of which the minimum we can reasonably expect is much greater.

Next we have the apparent right ascensions of the stars as computed. For these data I am indebted to Prof. Coffin, Superintendent of the American Ephemeris. The mean places are mostly derived from the "Star Tables of the American Ephemeris," and from the two Greenwich Seven Year Catalogues, while the reduction to apparent place is made with the modern constants.

The fourth column gives the apparent clock correction for sidereal time, in which is included the effect of deviation of the instrument from the meridian.

The clock keeping mean time, the errors are in the next column reduced to those of sidereal time at the moment of the transit of Uranus.

The next two columns give the corrections for clock rate, and for deviation of the instrument from the meridian, as inferred from the observations themselves, both being referred to the time and position of the transit of Uranus.

In the last column we have the seconds of concluded correction for clock and instrument to be applied to the observed time of transit of Uranus.

1690, *December* 23. *Right Ascension.*

Star.	Time of Tr.	R. A. of star.	Clock.	C′.	R.	Dev.	C″.
	h. m. s.	h. m. s.	h. m. s.	m. s.	s.	s.	s.
α Arietis,	7 48 40	1 49 52.1	—5 58 47.9	58 29.3	—1.1	—1.2	31.6
π Arietis,	8 30 54	2 32 8.4	—5 58 45.6	58 33.9	—0.7	+1.7	32.9
σ Arietis,	8 33 17	2 34 31.0	—5 58 46.0	58 34.7	—0.7	+2.9	32.5
ε Arietis,	8 40 21	2 41 38.0	—5 58 43.0	58 33.0	—0.6	—0.2	33.8
δ Arietis,	8 52 44	2 54 3.0	—5 58 41.0	58 33.0	—0.5	+0.5	33.0
η Tauri,	9 27 47	3 29 13.6	—5 58 33.4	58 31.1	—0.1	—1.3	32.5
Uranus,	9 41 49						
A Tauri,	9 45 3	3 46 31.1	—5 58 31.9	58 32.4	0.0	—0.8	33.2
α Virginis,	19 4 15	13 8 58.2	—5 55 16.8	56 49.3			
α Bootis,	19 58 31½	14 1 34.0	—5 56 57.5	58 39.0			

Hourly rate of clock,	—0ˢ.6
Deviation of instrument for each degree of Z. D.,	—0.5
	h. m. s.
Transit of Uranus,	9 41 49.0
Correction for clock and instrument (mean),	—5 58 32.8
Observed R. A. of the planet,	3 43 16.2

1690, *December* 23. *Declination.*

	Z. D. observed.	Refraction.	Declination.	Eq. point.
α Arietis,	29° 29′ 10″	+0′ 33″	21° 58′ 53″	51° 28′ 36″
π Arietis,	35 18 55	+0 41	16 9 4	51 28 40
σ Arietis,	37 41 0	+0 44	13 46 58	51 28 42
ε Arietis,	31 23 15	+0 35	20 4 37	51 28 27
δ Arietis,	32 55 55	+0 37	18 31 40	51 28 12
η Tauri,	28 20 55	+0 31	23 6 53	51 28 19
Uranus,	31 52 35	+0 36		
A Tauri,	30 15 55	+0 34	21 12 3	51 28 32

Circle reading for Uranus, corrected for refraction,	31° 53′ 11″
Equatorial point on circle,	51 28 30
Declination of Uranus, from observation,	+19 35 19

1712, *April* 2. *Right Ascension.*

	h. m. s.	h. m. s	m. s.
Uranus,	9 35 19		
ε Virginis,	12 0 19	12 47 52.4	+47 33.4
e Virginis,	12 14 51	13 2 31.5	+47 40.5
ζ Virginis,	12 32 11	13 20 5.7	+47 54.7

The discordance of clock errors, and the time which intervened between the transit of the planet and that of the first star, seem to render an accurate reduction impossible.

1715, *March* 4. *Right Ascension.*

	T.	R. A.	C.	C′.
	h. m. s.	h. m. s.	h. m. s.	s.
d Leonis,	11 50 19	10 45 52.5	—1 4 26.5	20.5
Uranus,	12 27 1			
b Virginis,	12 49 41	11 45 23.6	—1 4 17.4	21.1

	h. m. s.
Clock time of transit of Uranus,	12 27 1
Correction for clock and instrument,	—1 4 20.8
Right ascension of Uranus from observation,	11 22 40.2

1715, *March* 4. *Declination.*

	Z. D.	R.	Dec.	Eq. point.
d Leonis,	46° 19′ 40″	+1′ 0″	+5° 8′ 6″	51° 28′ 46″
Uranus,	46 33 10	+1 1		
b Virginis,	46 13 20	+1 0	5 14 17	51 28 37

Circle reading for Uranus,	46° 34′ 11″
Equatorial point,	51 28 42
Observed declination of Uranus,	+4 54 31

1715, *March* 5. *Right Ascension.*

	T.	R. A.	C.	C′
	h. m. s.	h. m. s.	h. m. s.	s.
d Leonis,	11 46 24	10 45 52.5	—1 0 31.5	25.5
Uranus,	12 22 59 : :			
b Virginis,	12 45 49	11 45 23.6	—1 0 25.4	29.1

	h. m. s.
Transit of Uranus,	12 22 59
Correction for clock and instrument,	—1 0 27.7
Observed right ascension of Uranus,	11 22 31.5

The large apparent clock rate, and the colons after the time of transit, both throw doubt on this observation.

Declination.

The circle readings for the stars are the same as on the day preceding, while that for Uranus is 50″ less. The declination is therefore 50″ greater, or

+4° 55′ 21″.

1715, *March* 10. *Right Ascension.*

	T.	R. A.	C.	C′.
	h. m. s.	h. m. s.	h. m. s.	s.
d Leonis,	11 25 58	10 45 52.5	—0 40 5.5	59.5
p^3 Leonis,	11 32 28	10 52 25.1	—0 40 2.9	58.1
Uranus,	12 1 42			
b Virginis,	12 25 18	11 45 23.6	—0 39 54.4	58.2

	h. m. s.
Clock time of transit of Uranus,	12 1 42
Correction for clock and instrument,	—0 39 58.6
Observed right ascension of Uranus,	11 21 43.4

Declination.

	Z. D.	R.	Dec.	Eq. Pt.
d Leonis,	46° 19′ 35″	+1′ 1″	5° 8′ 6″	51° 28′ 42″
p^3 Leonis,	47 58 35	+1 3	3 29 25	51 28 63
Uranus,	46 27 0	+1 1		
b Virginis,	46 13 25	+1 0	5 14 17	51 28 42
Circle reading for Uranus,				46 28 1
Equatorial point on circle,				51 28 49
Observed declination of Uranus,				+5 0 48

1715. *April* 29. *Right Ascension.*

	T.	R. A.	C.	C′.
	h. m. s.	h. m. s.	h. m. s.	s.
σ Leonis,	8 42 11	11 6 28.3	+2 24 17.3	18.7
Uranus,	8 50 44			
ν Virginis,	9 6 55	11 31 14.1	+2 24 19.1	16.4
17 Virginis,	9 43 38 : :	12 8 6.5	+2 24 28.5	19.8
κ Virginis,	11 32 48	13 57 47.6	+2 24 59.6	33.1

The discordance of the clock corrections makes a satisfactory determination of the right ascension very difficult. I deem it best to reject the doubtful observation of 17 Virginis, and the discordant one of κ Virginis. The result will then be

	h. m. s.
Observed transit of Uranus,	8 50 44
Correction for clock and instrument,	2 24 17.6
Observed right ascension of Uranus,	11 15 1.6

Declination.

	Z. D.	R.	Dec.	Eq. Pt.
σ Leonis,	43° 52′ 40″	+0′ 55″	7° 34′ 51″	51° 28′ 26″
Uranus,	45 45 30	+0 59		
ν Virginis,	43 20 20	+0 54	8 7 11	51 28 25
17 Virginis,	44 34 10	+0 56	6 53 33	51 28 39
κ Virginis,	60 23 5	+1 41	−8 55 42	51 28 64
Circle reading for Uranus,				45° 46′ 29″
Mean equatorial point,				51 28 38
Observed declination of Uranus,				+5 42 9

The next observations in the order of time are two by Bradley, discovered by Mr. Hugh Breen, but still unpublished. The following are the results as given by Mr. Breen in the *Astronomische Nachrichten*, No. 1463.

	Mean Time.	R. A.	N. P. D.
	h. m. s.	h. m. s.	° ′ ″
1748, October 21,	7 6 18.4	21 4 37.93	107 29
1750, September 13,	10 8 57.8	21 40 0.23	104 42 33.9

Mr. Breen remarks: "The right ascensions are very accurate. It has been assumed that the N. P. D., on 1750, September 13, is identical with μ Capricorni, with which it was compared. The first observation was by the transit instrument, and the second by the quadrant."

No ground is given for the above assumption respecting the N. P. D. for the second observation; it may, therefore, be omitted as valueless.

In the year 1750 we have also two observations by Le Monnier at Paris. For these, and all the other observations by the same observer, I shall adopt the results given by Bouvard in the *Connaissance des Temps*, for 1821, p. 341, with the corrections indicated by Le Verrier, in *Connaissance des Temps*, for 1849, pp. 125 and 126. The necessary uncertainty of the observations is such that, considering that Bouvard reduced them with the star positions of the "*Fundamenta*," scarcely anything will be gained by a new reduction.

1753, December 3, we have another observation of right ascension by Bradley. I adopt the result kindly communicated by my distinguished friend, Dr. Auwers.

1753, December 3, 5 h. 33 m. R. A. = 22 h. 23 m. 21.59 s.

1756, September 25. Observation by Mayer, at Gottingen. I adopt the result given by Bessel, in *Fundamenta Astronomiæ*, p. 284.

1756, September 25, 10 h. 12 m. R. A. = 348° 0′ 54.5″
Dec. = —6 1 49.4

The following is a tabular summary of the preceding results, with their comparison with the provisional theory. In the computation of the geocentric place the places of the sun were derived from Hansen's Tables. I am indebted to Professor Coffin for a duplicate computation of the geocentric places from the provisional ephemeris, which was executed by Mr. Joseph A. Rogers.

Date.	Right Ascension.		Declination.		Correction to theory.				
	Observation.	Theory.	Observation.	Theory.	R. A.	Dec.	Long.	$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	h m s	s	° ′ ″	″	″	″	″		
1690, Dec. 23	3 43 16.2	14.7	+19 35 19	7	+22	+12	+24	1.04	+.027
1715, Mar. 4	11 22 40.2	38.7	4 54 31	48	+22	—17	+28	1.06	0
Mar. 5	11 22 31.5	29.1	4 55 21	49	+36	—28	+44	1.06	0
Mar. 10	11 21 43.4	41.0	5 0 48	56	+36	— 8	+36	1.06	0
Apr. 29	11 15 1.6	1.5	+ 5 42 9	11	+ 1	— 2	+ 2	1.04	+.036
1748, Oct. 21	21 4 37.93	35.42	....	...	+37.6	....	+39.3	1.015	+.050
1750, Sept. 13	21 40 0.23	57.90	....	...	+35.0	....	+35.7	1.05	+.022
	° ′ ″	″							
Oct. 14	324 15 24.6	47.6	—15 1 40.4	47.0	+37.0	+ 6.6	+35.9	1.03	+.045

Date.	Right Ascension. Observation.	Right Ascension. Theory.	Declination. Observation.	Declination. Theory.	Correction to theory. R. A.	Dec.	Long.	$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	° ′ ″	″	° ′ ″	′	″	″	″		
1750, Dec. 3	324 34 53.5	15.4	—14 53 20.2	32.4	+38.1	+12.2	+38.8	0.98	+.047
	h m s	s							
1753, Dec. 3	22 23 21.60	19.34			+33.8		+35.6	1.00	+.048
	° ′ ″	″							
1756, Sept. 25	348 0 54.5	25.0	— 6 1 49.4	46.0	+29.5	—3.4	+25.7	1.05	+.013
1764, Jan. 15	12 37 39.0	56.4	+ 4 43 47.2	56.2	(—17.4)	(—9.0)			
1768, Dec. 27	31 26 52.0	32.6	12 15 35.0	30.6	+19.4	+4.4			
Dec. 30	31 26 45.8	38.0	12 14 55.4	55.9	+ 7.8	—0.5	+13.0	1.02	+.045
1769, Jan. 15	31 22 7.7	:55.8	12 14 26.0:	29.7	+11.9:	—3.7:			
Jan. 16	31 22 23.4	11.1	12 14 36.3	37.0	+12.3	—0.7			
Jan. 20	31 24 6.6	:43.7	12 15 19.0	18.8	+22.9:	+0.2			
Jan. 21	31 24 33.8	14.1	12 15 31.8	30.8	+19.7	+1.0	+12.5	1.01	+.049
Jan. 22	31 25 4.7	:47.7	12 15 45.7	44.7	+17.0:	+1.0			
Jan. 23	31 25 28.5	24.2	12 16 7.5	:59.6	+ 4.3:	+7.9			

Where no declination has been observed the observed corrections in right ascension have been changed to corrections in longitude on the hypothesis that the theoretical latitude is correct. The approximate formula is

$$\delta l = \frac{\delta \alpha \cos \delta}{\sin E}, \text{ where}$$

$$\cos E = \sin \varepsilon \cos \alpha, \; \varepsilon \text{ being the obliquity.}$$

DISCUSSION OF THE MODERN OBSERVATIONS.

Reduction of the Published Results of Observations to a Uniform System.

We have now to discuss the great mass of observations made at the principal observatories of the world since the discovery of the planet by Herschel, in 1781. To make all the data of reduction rigorously homogeneous and uniform, it would be necessary to completely re-reduce the greater part of the observations made before 1850, using the modern values of the constants of reduction, and to compare each observation separately with the geocentric place deduced from the provisional theory. Such a reduction and comparison would be extremely desirable. Their execution would, however, involve an amount of labor far greater than it is now possible for the author to bestow upon the problem. We must, therefore, adopt the reductions which have been already made, applying such systematic corrections for reduction to a uniform system of star places as we have the means readily to determine. No reduced places are employed unless we can find data for some more or less accurate determination of these corrections, a rule which necessitates the rejection of a great mass of observations made at the minor observatories of the European continent, and published in the *Astronomischen Nachrichten.* We still have the following rich collection of materials at our disposal:

1. Observations at Greenwich, 1781 to 1872.
2. . . . . . . Paris, 1802 to 1827, and 1837 to 1869.
3. . . . . . . Königsberg, 1813 to 1835.
4. . . . . . . Vienna, 1822, and 1827 to 1839.
5. . . . . . . Speier, 1827–29.
6. . . . . . . Cambridge, 1828 to 1842.
7. . . . . . . Edinburgh, 1836 to 1844.
8. . . . . . . Berlin, 1838 to 1842.
9. . . . . . . Pulkowa, 1841 and 1842.
10. . . . . . . Washington, 1861 to 1872.
11. . . . . . . Leiden, 1863 to 1871.
12. . . . . . . Santiago, 1854 and 1855.

As to the general distribution of these observations in time, we may remark that during the first three or four years the planet was zealously observed at Greenwich. Observations then began gradually to fall off until 1798, in which year we find but one. From this time until 1814 only one or two observations were made at each opposition. They become a little more numerous, until 1829, when there is a sudden increase. Few interruptions have occurred since. With regard to the other observatories it may be said that from 1802 until 1830 there is a gradual increase in the number of observations, and that since the latter year the number of observations is entirely satisfactory.

A great number of the observations were reduced with the star places of the *Tabulæ Regiomontanæ*, and the entire Paris series are reduced with the star positions of Le Verrier, given in his "*Annales de l'Observatoire Imperial de Paris*," Tome II. As a preliminary to the discussion of the systematic corrections to the principal published reductions, I have prepared the following table, showing the corrections which must be applied to the places of the equatorial fundamental stars in the above catalogues to reduce them to the adopted standard, namely, Dr. Gould's coast survey list in right ascension, and Auwers' standard in declination.

In the table of right ascensions the first column after name of the star gives the annual variation of that co-ordinate for the epoch 1860.0, as derived from Le Verrier's tables of right ascensions just cited. Next we have the correction to this annual variation, expressed in units of the fourth place of decimals, to reduce it to that given in the "*Star Tables of the American Ephemeris*," the positions in which are founded on Dr. Gould's Catalogue. The fourth column gives the correction to the right ascensions of Le Verrier for 1860, in hundredths of a second of time. Subtracting from this column sixth-tenths of the preceding, we have the corresponding corrections for 1800. The last four columns give the corresponding numbers for the right ascensions of the *Tabulæ Regiomontanæ*.

The table of declinations shows, for different epochs, the corrections necessary to reduce the tabular positions to those given by Auwers in his paper on the declinations of the fundamental stars

I. Right Ascensions.

Date.	Le Verrier's ann. var. 1860.	Corrections to			Ann. var. of Tab. Reg. 1860.	Corrections to		
		Ann. var.	R. A. 1860.	R. A. 1800.		Ann. var.	R. A. 1860.	R. A. 1800.
	s				s			
α Andromedæ,	+3.0844	+ 14	+ 2	— 6	3.0840	+18	+ 8	— 3
γ Pegasi,	3.0801	+ 15	+ 3	— 6	3.0824	— 8	— 9	— 4
α Arietis,	3.3644	+ 12	0	— 7	3.3636	+20	+ 6	— 6
α Ceti,	3.1266	+ 10	+ 5	— 1	3.1267	+ 9	+ 7	+ 2
α Tauri,	3.4346	— 1	— 1	0	3.4335	+10	+ 7	+ 1
β Orionis,	2.8797	+ 6	+ 2	— 2	2.8800	+ 3	+ 5	+ 3
β Tauri,	3.7871	— 4	+ 1	+ 3	3.7888	—21	— 4	+ 9
α Orionis,	3.2460	+ 5	+ 4	+ 1	3.2464	+ 1	+ 3	+ 2
α Geminorum,	3.8409	— 2	0	+ 1	3.8386	+21	+18	+ 5
α Canis Min.	3.1462	+ 5	+ 9	+ 6	3.1455	+12	+ 7	0
β Geminorum,	3.6828	+ 5	+ 3	0	3.6807	+26	+13	— 3
α Hydræ,	2.9485	+ 12	+ 6	— 1	2.9469	+28	+16	— 1
α Leonis,	3.2030	+ 15	+ 6	— 3	3.2014	+31	+13	— 6
β Leonis,	3.0654	+ 11	+ 4	— 3	3.0640	+25	+12	— 3
α Virginis,	3.1495	+ 20	+ 5	— 7	3.1497	+18	+ 2	— 9
α Bootis,	2.7325	+ 16	+ 3	— 7	2.7327	+14	+ 5	— 3
α² Libræ,	3.3044	+ 6	+ 3	— 1	3.3074	—24	— 6	+ 8
α Coronæ,	2.5378	+ 12	+ 1	— 6	2.5373	+17	+ 5	— 5
α Serpentis,	2.9488	+ 8	+ 4	— 1	2.9513	—17	— 7	+ 3
α Scorpii,	3.6654	+ 16	+ 4	— 6	3.6672	— 2	— 4	— 3
α Herculis,	2.7322	+ 7	+ 2	— 2	2.7319	+ 9	0	— 5
α Ophiuchi,	2.7808	+ 10	0	— 6	2.7783	+34	+15	— 5
α Lyræ,	2.0312	+ 2	— 1	— 2	2.0305	+ 9	+ 1	— 4
γ Aquilæ,	2.8520	+ 9	+ 2	— 3	2.8546	—17	— 8	+ 2
α Aquilæ,	2.9281	+ 4	+ 1	— 1	2.9281	+ 4	— 3	— 5
β Aquilæ,	2.9466	+ 5	+ 3	0	2.9496	—25	—12	+ 3
α² Capriconii,	3.3338	+ 4	+ 2	0	3.3349	— 7	— 6	— 2
α Aquarii,	3.0829	+ 13	+ 4	— 4	3.0822	+20	+ 4	— 8
α Piscis Aust.	3.3311	+ 8	+ 4	— 1	3.3326	— 7	—16	—12
α Pegasi,	2.9828	+ 11	0	— 7	2.9830	+ 9	— 1	— 6
Sum,		+254	+81	—72		+210	+71	—55
Mean,		+8.5	+.027	—.024		+7.0	+.024	—.018

II. DECLINATIONS.

	Corrections to Tabulæ Regiomontanæ.				Corrections to Le Verrier.	
	1780.	1800.	1820.	1840.	1820.	1840.
	″	″	″	″	″	″
α Andromedæ,	+0.3	+0.2	+0.1	0.0	+0.2	0.0
γ Pegasi,	—0.2	+0.1	+0.4	+0.8	+0.1	+0.3
α Arietis,	—0.3	0.0	+0.3	+0.5	+0.2	+0.3
α Ceti,	—1.0	—0.1	+0.7	+1.6	+0.1	+0.7
α Tauri,	0.0	+0.1	+0.1	+0.2	+0.1	0.0
β Orionis,	—0.5	+0.1	+0.6	+1.2	+0.3	+0.7
β Tauri,	—0.5	0.0	+0.6	+1.1	+0.2	+0.5
α Orionis,	—1.2	—0.6	0.0	+0.6	—0.2	+0.3
α Geminorum,	—0.8	—0.5	—0.1	+0.3	+0.5	+1.0
α Canis Min.	—0.2	0.0	+0.3	+0.6	+0.2	+0.5
β Geminorum,	—0.2	0.0	+0.3	+0.6	+0.2	+0.5
α Hydræ,	—0.2	+0.3	+0.7	+1.2	+0.3	+0.7
α Leonis,	+0.3	+0.4	+0.4	+0.5	+0.4	+0.5
β Leonis,	+0.2	+0.2	+0.2	+0.3	+0.2	+0.3
α Virginis,	+0.2	+0.6	+1.1	+1.5	+0.5	+0.8
α Bootis,	+0.4	+0.4	+0.3	+0.2	+0.4	+0.4
α^2 Libræ,	+0.5	+0.5	+0.6	+0.6	+0.5	+0.6
α Coronæ,	+0.7	+0.6	+0.4	+0.3	+0.4	+0.4
α Serpentis,	+0.2	+0.6	+1.1	+1.5	+0.5	+0.9
α Scorpii,	+0.1	+0.6	+1.0	+1.4	+0.4	+0.7
α Herculis,	+0.6	+0.8	+1.0	+1.3	+0.5	+0.7
α Ophiuchi,	+0.4	+0.5	+0.5	+0.6	+0.4	+0.6
α Lyræ,	+0.7	+0.8	+0.9	+1.0	+0.4	+0.4
γ Aquilæ,	0.0	+0.3	+0.6	+1.0	+0.2	+0.4
α Aquilæ,	+0.1	+0.4	+0.7	+1.0	+0.1	+0.4
β Aquilæ,	+0.1	+0.6	+1.2	+1.7	+0.5	+0.8
α^2 Capriconii,	+0.2	+0.9	+1.6	+2.3	+0.6	+1.1
α Aquarii,	—0.5	+0.1	+0.8	+1.5	+0.5	+1.0
α Piscis Aust.	+1.0	+2.4	+3.9	+5.3	+1.7	+2.3
α Pegasi,	+0.4	+0.3	+0.2	+0.1	+0.3	+0.2
Mean	+0.03	+0.36	+0.69	+1.03	+0.36	+0.60

The correction to the reductions to apparent place given in the Tabulæ Regiomontanæ on account of the correction to the constant of Nutation is;—

In right ascension:

$$-0''.46 \sin \Omega - 0''.18 \sin \Omega \sin \alpha \tan \delta - 0''.24 \sin \Omega \cos \alpha \tan \delta.$$

In declination:

$$-0''18 \sin \Omega \cos \alpha + 0.24 \cos \Omega \sin \alpha.$$

The terms which contain tan δ as a factor may be entirely neglected, as they are small, periodic, and contain tan δ as a factor which is sometimes positive and sometimes negative. I shall also neglect the corrections in declination, as their sum is sensibly

$$0''.21 \sin (\alpha - \Omega)$$

the effect of which will generally be confounded with the accidental errors of observation.

The only correction we shall apply on account of nutation is, therefore,

$$\delta\alpha = -0^s.030 \sin \Omega.$$

The values of this expression at the dates when it is zero, a maximum, or a minimum, are as follows:—

y.	s.	y.	s.
1778.5	—.03	1820.3	.00
1783.1	.00	1825.0	+.03
1787.7	+.03	1829.6	.00
1792.4	.00	1834.3	—.03
1797.0	—.03	1838.9	.00
1801.7	.00	1843.6	+.03
1806.3	+.03	1848.2	.00
1811.0	.00	1852.9	—.03
1815.6	—.03	1857.5	.00
1820.3	.00		

Having adopted this system of standard positions, we may adopt two ways of reducing the observations to it. One is to compare the positions of the stars adopted in the published reductions with the standard, and apply the mean difference to the reduced place of the planet. Another is to make a similar comparison of the standard catalogue with the positions of the fundamental stars which have been deduced from the observations by a system of reduction uniform with that employed in reducing the observations of the planet, and to regard the mean difference as a correction applicable to all the positions of the planet. If the standard catalogue and the observations are both free from systematic error, the results obtained in these two ways should be substantially identical. These are, however, conditions which we cannot expect to find fulfilled. In the following discussions I have sometimes used one, sometimes the other, and sometimes

combined both, the choice being determined by circumstances. We shall consider the different series of observations in succession.

Greenwich Observations from 1781 *to* 1830.

These observations are completely reduced by Airy and compared with Bouvard's Tables, in the work *Reduction of the Observations of Planets made at the Royal Observatory, Greenwich, from* 1750 *to* 1830. London, 1845. The concluded positions given in this work depend mainly on the star places of the Tabulæ Regiomontanæ, both in right ascension and declination. If we consider the first four oppositions—1781–1785—as forming a single group of which the mean epoch is 1783, we find that the general correction to the Tabulæ Regiomontanæ for this epoch is

In right ascension, $-0^s.030$;
In declination, $+0''.08$.

If, on the other hand, we consider only the particular stars compared with Uranus, the result will be a little different. The number of times each of the fundamental stars has been compared with Uranus, and the correction in right ascension corresponding to each star, are nearly as follows:—

α Arietis,	$N =$ 2	Cor. $= -0^s.09$	$N \times C = -\ ^s.18$
α Tauri,	2	$-.01$	$-\ .02$
γ Pegasi,	2	$-.03$	$-\ .06$
β Tauri,	19	$+.13$	$+2.47$
α Orionis,	33	$+.02$	$+0.66$
α Canis Minoris,	33	$-.02$	-0.66
β Geminorum,	34	$-.07$	-2.38
α Leonis,	7	$-.11$	$-\ .77$
β Leonis,	2	$-.07$	$-\ .14$

The mean correction from these data comes out $-0^s.008$, differing by $0^s.022$ from the general mean correction. Our choice between the two corrections depends on whether we are to consider the relative positions of the Tabulæ Regiomontanæ, or those of the standard catalogue, as nearest the truth at the epoch 1783, and particularly upon whether we are to consider the large correction to the proper motion of β Tauri as real. In the absence of exact data for settling this question, the mean of the two results, or $-0^s.020$, has been adopted.

A similar anomaly is exhibited by the declinations. It is probable that the declinations of Uranus during this period mainly depend on stars in the first twelve hours in right ascension, for which the mean correction is about $-0''.30$ instead of $+0''.08$. I have adopted $-0''.16$. Changing these corrections to longitude and latitude, we have, during the period 1781–1786:—

Correction to observed longitude, $= -0''.30$;
Correction to observed latitude, $-0\ .19$.

During the years 1788–1798 the above systematic difference in right ascension does not appear. The most probable correction seems to be

$$\Delta\alpha = -0^s.025; \qquad \Delta\delta = 0''.00.$$

Whence $\qquad \Delta \text{ long.} = -0''.34; \qquad \Delta \text{ lat.} = -0''.10.$

Between the years 1800 and 1823 the stars used for comparison are so widely scattered that I consider it safe to apply only the general mean correction for the epoch 1813, which is

$$\Delta\alpha = -{}^s.005; \qquad \Delta\delta = +0''.66.$$

Whence $\qquad \Delta \text{ long.} = 0''.00; \qquad \Delta \text{ lat.} = +0.66.$

From 1825 to 1830 more than half the weight of the right ascension comes upon the stars α, β, and γ Aquilæ, the mean correction to which, during this interval, is $-0^s.035$. The general mean correction at this epoch is $+0^s.002$. I think the right ascensions of these three stars in the Tabulæ Regiomontanæ are really too great at this epoch by the entire difference of these results. We may, in fact, hereafter regard the positions of the standard catalogue as sufficiently accurate. The mean corrections to be applied will then be

$$\Delta\alpha = -0^s.017; \qquad \Delta\delta = +0''.83.$$

Whence $\qquad \Delta \text{ long.} = -0''.05; \qquad \Delta \text{ lat.} = +0''.86.$

From the year 1831 until the present time the Greenwich observations are regularly reduced in the several annual volumes of observations. But a reduction of the observations from 1831 to 1835, executed by Mr. Hugh Breen, is given in an appendix to the volume for the year 1864. The results here given differ from those published by Pond in the several annual volumes for the same interval. The right ascensions are altered only by applying the constant correction $-0^s.030$, which is found necessary to reduce Pond's right ascensions to those of the Tabulæ Regiomontanæ. This correction I have verified. The mean correction to reduce the right ascensions of the Tabulæ Regiomontanæ to our standard is at this time $+0^s.005$. On the other hand, when we compare the concluded right ascensions of stars within six hours of Uranus, as given by Pond in the Greenwich observations for 1834, with our standard, we find a mean correction of $-{}^s.034$ to reduce his positions to the standard, which implies a correction $-{}^s.004$ to Breen's reduction. The two results being $+{}^s.005$ and $-{}^s.004$, I have applied no correction whatever.

In the paper in question the declinations are completely re-reduced, using improved data of reduction, but, so far as I see, making no changes in Pond's method. The results differ strikingly from those of Pond, and suggest the desirableness of a complete re-examination of all Pond's determinations of declination. Having no catalogue of observed declinations of standard stars reduced in this same way, we cannot directly determine the systematic correction to the declinations. I therefore proceed as follows: A comparison of Pond's observed declinations of standard stars with Auwers' normal catalogue show that the former require the following corrections near the parallel of Uranus:

In 1831 — 1″.42;
1834 — 2 .10.

Then comparing Airy's reduced declinations of Uranus with Pond's, we find the following mean differences:

In 1831, Airy — Pond = — 3″.18
1834, — 3 .50.

To reduce Airy to Auwers we must there apply to the declinations

In 1831 + 1″.76
1834 + 1 .40.

I have regarded the correction + 1″.60 as applicable throughout the period in question.

1836-72.

During this interval the corrections in right ascension have been derived by the following two sets of comparisons: (1) A comparison of the several collected six and seven year catalogues with Gould's standard, from which it appears that they require the following general corrections in right ascension:

Six year catalogue of 1840	+ 0ˢ.047
Six year catalogue of 1845	+ 0 .002
Seven year catalogue of 1860	+ 0 .003
Seven year catalogue of 1864	+ 0 .022

(2) A comparison of the corrections applied to the right ascensions of the individual years to reduce them to the standard of the catalogue, as given in the introduction to each catalogue. The sum of these two numbers gives the corrections for each year.

A slightly different method is to regard the above correction for each catalogue as applicable to all right ascensions which depend fundamentally upon that catalogue. I have sometimes combined both methods so as to derive what seemed to be the most probable result, and sometimes used but one.

The corrections to the declinations during the interval in question have been derived from Auwers' "*Tafeln zur Reduction der Declinationen verschiedener Sternverzeichnisse auf ein Fundamentalsystem*," *Astronomische Nachrichten*, No. 1536. These tables include the Greenwich seven year catalogue for 1860, when the correction corresponding to the declination of Uranus is about + 0″.45. The corrections for the previous catalogues vary between 0″.35 and 0″.68. The correction corresponding to the interval 1861–67 has been derived by a direct comparison with Auwers' declinations, and the result is + 0″.44, agreeing with the two preceding catalogues. But, on making a similar comparison with the annual catalogue for 1869, a considerable change was found, the correction being — 0″.17, a change of more than half a second. I shall use this correction for and after the beginning of 1868, as the change is probably due to the introduction of a new constant of refraction in the reduction of the observations for 1868 and subsequent years.

Cambridge.

An extended series of planetary observations was commenced here by Professor Airy, in 1827. The series was continued by him and Professor Challis, his successor, until 1842. During the first three or four years the combined right ascensions depend on a few special stars, and mainly on α^2 Capricorni. Taking the mean correction to the adopted right ascensions of the stars actually compared as they are given in the introduction to each annual volume, giving to each star a weight proportional to the number of comparisons, the following corrections are deduced:

1828	$-0^s.10$
1829–31	$-0\ .16$
1832–37	$-0\ .19$.

In the introduction to the volume for 1838 it is stated that the adopted right ascensions are diminished by the average amount of $0^s.083$, which would still leave a correction of $-0^s.107$. Actual comparisons in two subsequent years give

$$1840,\ \Delta\alpha = -0^s.087$$
$$1842,\qquad -0\ .069.$$

Although the positions deduced from each year's work were adopted for clock correction the year following, without any change of equinox, it seems that there was, effectively, a progressive change of about $0^s.01$ annually in the equinox as adopted.

No declinations were observed until 1830. On comparing the declinations deduced from several years' work with Auwers, it was evident that the correction increased with the polar distance of the star. The law of increase could be well enough represented by supposing the correction proportional to N. P. D. Thus the following corrections were deduced in three different years.

$$1834,\quad \delta\,\text{dec.} = -\frac{1''.78 \times \text{N.P.D. in degrees}}{100}$$

$$1840,\qquad -\frac{1''.00 \times \text{N.P.D. in degrees}}{100}$$

$$1842,\qquad -\frac{1''.03 \times \text{N.P.D. in degrees}}{100}.$$

From which the correction for other years was deduced by interpolation. But, on applying these corrections, the results were found systematically different from those of other observatories, and on referring to Auwers' corrections to Airy's Cambridge Catalogue, it appeared that the mural circle required a large correction near the declination of Uranus during this period. The above results were therefore altered so as to conform as nearly as practicable to Auwers' law.

Edinburgh.

In reducing the observations of 1836 Henderson uses the right ascensions of the Tabulæ Regiomontanæ, to which the general correction is at this epoch $+^s.007$.

But, if we take only the stars near Uranus, with which the latter was necessarily most frequently compared, the corrections will be negative. Comparing the concluded positions of the stars from α Serpentis through 0^h to β Orionis, we find the following mean corrections:

$$\text{In right ascension, } -0^s.012; \text{ in declination, } -0''.09.$$

In subsequent years it is stated that the adopted positions of clock stars used each year are derived from the right ascensions observed at Greenwich, Cambridge, and Edinburgh, during the year or the two years preceding, without any statement whether corrections were applied for difference of equinoxes. In some subsequent years the following corrections are deduced, sometimes from the adopted and sometimes from the concluded positions:

$$\begin{array}{llll} 1837, & \Delta\alpha = 0^s.000; & & \\ 1840, & \Delta\alpha = +0.015; & \Delta\,\text{Dec.} = & 0''.00; \\ 1844, & \Delta\alpha = +0.070; & \Delta\,\text{Dec.} = & +0.49. \end{array}$$

Paris.

All the positions of planets given by Le Verrier, in his "*Annales de l'Observatoire Imperial de Paris: Observations*" depend both in right ascension and N. P. D. on his adopted positions of fundamental stars, the corrections to which have already been given. As the corrections to the individual star places used by Le Verrier are not generally of a systematic character, the general mean correction is employed, which is:—

$$\begin{array}{ll} \text{In right ascension} & -\,0^s.024 + 0^s.085\,T, \\ \text{In declination} & +\,0''.12 \;\; + 1''.20\,T, \end{array}$$

T being the fraction of a century after 1800.

In 1854 a new and larger catalogue was introduced, and for this and the following years the correction in declination is derived from Auwers' tables.

A summary of the adopted corrections after 1830, as deduced from the preceding comparisons and discussions, is given in the following table:—

TABLE OF ADOPTED SYSTEMATIC CORRECTIONS.

Year.	Greenwich.		Paris.		Königsberg.		Berlin.		Cambridge.		Edinburgh.	
	$\Delta\alpha$ s	$\Delta\delta$ ''	$\Delta\alpha$ s	$\Delta\delta$ ''	$\Delta\alpha$ s	$\Delta\delta$ ''	$\Delta\alpha$ s	$\Delta\delta$ ''	$\Delta\alpha$ s	$\Delta\delta$ ''	$\Delta\alpha$ s	$\Delta\delta$ ''
1830	....	....	....	....	.00	....	....	+0.9	—.16	—1.7	.00	....
1831	.00	+1.8	.00	+0.5	—.02	+0.9	....	+0.9	—.16	—1.6	....	....
1832	....	+1.7	....	....	—.02	+0.9	....	....	—.19	—1.5	....	....
1833	....	+1.6	....	....	....	....	—.04	....	....	—1.4	....	....
1834	....	+1.4	....	....	....	....	—.05	....	....	—1.3	....	....
1835	....	+1.2	....	....	....	....	—.05	....	....	—1.2	....	....
1836	—.04	+1.0	+.01	+0.6	—.02	+1.0	—.04	+1.0	....	—1.0	—.01	0
1837	—.03	+0.9	....	....	....	....	—.03	....	....	—0.8	.00	0
1838	—.04	+0.8	....	....	....	....	—.02	....	—.11	—0.5	+.01	0
1839	....	+0.7	....	....	....	+1.0	—.01	....	—.10	—0.3	+.01	0
1840	....	+0.6	....	....	....	....	.00	....	—.09	—0.2	+.02	+0.1
1841	....	+0.5	....	....	....	....	+.01	....	—.08	—0.2	—.04	+0.2

TABLE OF ADOPTED SYSTEMATIC CORRECTIONS.—*Continued.*

Year.	Greenwich.		Paris.		Königsberg.		Berlin.		Cambridge.		Edinburgh.	
	$\Delta\alpha$ s	$\Delta\delta$ ″	$\Delta\alpha$ s	$\Delta\delta$ ″	$\Delta\alpha$ s	$\Delta\delta$ ″	$\Delta\alpha$ s	$\Delta\delta$ ″	$\Delta\alpha$ s	$\Delta\delta$ ″	$\Delta\alpha$ s	$\Delta\delta$ ″
1842	—.02	+0.4	+.01	+0.6	....	....	+.03	+1.1	—.07	—0.3	—.02	+0.3
1843	+.07	+0.4	+.01	+0.6	....	....	+.04	+1.1	....	....	+.07	+0.4
1844	+.06	+0.4	+.01	+0.6	....	....	+.04	+1.1	....	....	+.07	+0.5
1845	+.08	+0.4	+.01	+0.6	....	....	+.04	+1.1	....	....		
1846	+.04	+0.4	+.02	+0.6	....	....	+.03	+1.1	....	....		
1847	+.05	+0.4	+.02	+0 6	....	....	+.02	+1.1	....	....		
1848	+.05	+0.4	+.02	+0.6	....	....	+.01	+1.2	....	....		
1849–53	.00	+0.4	+.02	+0.6	....	....	.00	+1.2	Santiago			
1854–55	.00	+0.5	+.02	+0.2	....	....	—.01	+1.2	.00	+0.6		
1856–60	—.01	+0.5	+.02	+0.2	....	....	....	....	....	....	Washington	
1861	—.01	+0.4	+.03	+0.2	....	....	....	....	....	....		
1862–65	.00	+0.4	+.03	+0.2	....	....	....	....	....	....	.00	—0.5
1866	.00	+0.4	+.03	+0.2	....	....	....	....	....	....	.00	+1.1
1867	.00	+0.4	+.03	+0.2	....	....	....	....	....	....	.00	+1.1
1868	.00	—0.2	+.03	+0.2	....	....	....	....	....	....	.00	+1.2
1869	.00	—0.2	+.03	+0.2	....	....	....	....	....	....	.00	+0.6
1870	+.01	—0.2	....	....	....	....	....	....	....	....	.00	+0.4
1871	+.01	—0.2	....	....	....	....	....	....	....	....	.00	+0.4
1872	+.02	—0.2	....	....	....	....	....	....	....	....	.00	+0.4

Applying the preceding corrections to the positions of the planet as originally reduced and published, we have a series of observed positions as nearly homogeneous as it is possible to make them with the means now at our command. The next step in order will be the computation of the geocentric place of the planet from the provisional theory for the moment of every observation, to be compared with the results of the latter. The complete execution of this labor, *ab initio*, is, however, at present impracticable, and it is proposed to diminish it by making use of the published comparisons with the older tables. This can be done without danger of serious error, and with all the more ease that owing to the great distance of Uranus the errors of the solar tables are, for the most part, without appreciable effect upon the computed geocentric place of the planet. The method of making the comparison is different with different series of observations, and each series must therefore be described and discussed separately. The general plan, however, has been to replace observed and computed absolute positions by observed and computed corrections to the geocentric positions deduced from Bouvard's Tables. To carry out this plan it is necessary to have at our disposal an ephemeris both of the heliocentric and geocentric positions derived from these tables. The corrections to the latter given by the observations are then given by direct comparison. To obtain the corrections given by the provisional theory, the heliocentric longitudes, latitudes, and radii vectores given by that theory are interpolated to the dates of the heliocentric ephemeris from Bouvard's Tables, and compared with that ephemeris. The differences are then changed to differences of geocentric place by the usual differential formulæ, and thus the corrections given by theory are derived. The difference between the two sets of corrections is the difference

between the provisional theory and observation. A condensed summary of the results for each of the principal series of observations is here presented.

Greenwich, 1781–1830.

In Airy's reductions, already referred to, we have given for the moment of each individual observation a heliocentric place computed from Bouvard's Tables, and the geocentric longitudes and latitudes thence deduced. The observed right ascensions and declinations are then changed to longitudes and latitudes, and the apparent error of the tables thence deduced. The means of these errors are taken for groups of observations, and expressed in terms of the errors of heliocentric longitude, radius vector, and latitude. The mode in which these means have been treated is fully shown in the following table. The first column gives the mean date of each individual group of observations. The next three give the mean excesses of the co-ordinates interpolated from the heliocentric ephemeris, p. 100, and corrected for solar nutation, over those printed in the "Computations of tabular place, etc.," in the Greenwich reductions. In the fifth column these corrections are changed to corrections of geocentric longitude. In the next two columns we have the mean corrections to Bouvard's geocentric places given by observation. It is the negative of the mean error of tabular place printed in the "Reductions," corrected by the numbers already given to reduce the star places to a uniform system. Then we have the difference between these two sets of corrections, or, the mean correction to the geocentric place of the provisional theory as given by observation. Lastly, we have the differential coefficients for expressing the errors of geocentric in terms of the errors of heliocentric co-ordinates taken without change from the Greenwich volume.

Mean Date.	From Provisional Theory.				From Observations.			Correction to Prov. Theory.		$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	Correction to tabular position in Greenwich Reductions.				Correction to						
	Long.	Log. R. V.	Lat.	Geoc. long.	Geoc. long.	Hel. lat.	No. of Obs.	Geoc. long.	Hel. lat.		
	″		″		″	″		″	″		
1781, Oct. 10	— 7.8	....	+1.1	— 7.5	— 6.4	+ 2.6	4	+1.1	+ 1.5	1.01	— 9
Nov. 13	— 7.5	....	+1.1	— 7.5	— 4.9	— 1.6	4	+2.6	— 2.7	1.04	— 6
Dec. 27	— 7.3	— 159	+1.0	— 7.7	— 5.6	— 2.3	5	+2.1	— 3.3	1.05	+ 1
1782, Jan. 31	— 7.4	....	+0.9	— 8.0	— 4.9	— 1.1	4	+3.1	— 2.0	1.04	+ 6
Mar. 4	— 6.6	....	+0.9	— 7:1	— 4.6	— 0.6	4	+2.5	— 1.5	1.01	+ 9
Oct. 10	— 5.6	....	+0.4	— 5.3	— 3.6	— 4.0	4	+1.7	— 4.4	1.01	— 9
Nov. 26	— 5.3	....	+0.3	— 5.3	— 6.0	— 5.0	3	—0.7	— 5.3	1.04	— 5
1783, Jan. 11	— 5.0	....	+0.3	— 5.4	— 3.9	— 3.7	3	+1.4	— 4.0	1.05	+ 3
Feb. 24	— 4.4	....	+0.2	— 4 8	— 2.2	— 2.4	3	+2.6	— 2.6	1.02	+ 9
Oct. 10	— 3.5	....	—0.3	— 3.2	— 0.3	— 1.0	2	+2.9	— 0.7	1.00	—10
Nov. 1	— 3.2	....	—0.3	— 3.0	— 1.1	— 2.3	2	+1.9	— 2.0	1.02	— 9
Dec. 15	— 3.3	— 121	—0.4	— 3.4	— 2.2	— 3.8	3	+1.2	— 3.4	1.05	— 3
1784, Jan. 29	— 3.1	....	—0.4	— 3.4	— 2.4	— 2.4	3	+1.0	— 2.0	1.05	+ 5
Mar. 12	— 2.6	....	—0.5	— 2.9	— 0.3	— 4.3	3	+2.6	— 3.8	1.01	+ 9
Oct. 30	— 1.8	....	—0.8	— 1.5	— 0.7	+ 1.4	2	+0.8	+ 2.2	1.02	— 9
Dec. 14	— 1.5	— 128	—0.9	— 1.5	— 2.4	+ 2.7	2	—0.9	+ 3.6	1.05	— 4

Mean Date.	From Provisional Theory.				From Observations.			Correction to Prov. Theory.		$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	Correction to tabular position in Greenwich observations.				Correction to						
	Long.	Log. R. V.	Lat.	Geoc. long.	Geoc. long.	Hel. lat.	No. of Obs.	Geoc. long.	Hel. lat.		
	"		"		"	"		"	"		
1785, Jan. 16	— 1.2	....	—1.0	— 1.3	— 1.4	— 1.4	2	—0.1	— 0.4	1.05	+ 2
Feb. 13	— 1.0	....	—1.1	— 1.3	— 1.2	— 1.1	2	+0.1	0.0	1.04	+ 7
Mar. 20	— 0.6	....	—1.1	— 0.9	— 2.1	— 5.2	2	—1.2	— 4.1	1.01	+10
Nov. 8	+ 0.2	....	—1.5	+ 0.5	+ 1.2	— 1.7	2	+0.7	— 0.2	1.02	— 9
1788, Mar. 13	+ 2.1	— 187	—3.2	+ 1.8	+ 4.4	— 4.9	3	+2.6	— 1.7	1.03	+ 9
Oct. 26	+ 3.4	....	—3.6	+ 3.9	+ 3.0	— 2.2	2	—0.9	+ 1.4	1.00	—10
1789, Jan. 18	+ 4.2	— 224	—3.7	+ 4.5	+10.4	—14.5	1	(+5.9)	(—10.8)	1.06	— 1
Apr. 8	+ 4.7	....	—3.8	+ 4.2	+ 9.4	— 3.7	4	+5.2	+ 0.1	1.01	+10
Oct. 31	+ 4.8	....	—4.2	+ 5.4	+ 4.7	— 1.1	2	—0.7	+ 3.1	1.00	—10
1790, Jan. 24	+ 4.9	....	—4.4	+ 5.2	+ 3.8	— 2.6	2	—1.4	+ 1.8	1.06	— 1
Nov. 5	+ 5.1	— 254	—4.8	+ 5.8	+ 5.0	— 2.3	3	—0.8	+ 2.5	1.00	—10
1791, Jan. 29	+ 5.1	— 258	—5.0	+ 5.4	+ 4.5	— 3.8	3	—0.9	+ 1.2	1.06	0
Apr. 14	+ 5.0	....	—5.2	+ 4.5	+ 2.7	— 4.8	1	—1.8	+ 0.4	1.01	+10
Nov. 10	+ 5.0	— 234	—5.4	+ 5.6	+ 5.7	— 4.0	2	+0.1	+ 1.4	1.00	—10
1792, Feb. 5	+ 5.2	....	—5.6	+ 5.5	+ 4.2	— 4.0	1	—1.3	+ 1.6	1.06	0
Nov. 15	+ 5.6	....	—6.0	+ 6.2	+ 3.7	— 2.5	3	—2.5	+ 3.5	1.00	—10
1793, Feb. 8	+ 5.4	....	—6.1	+ 5.7	+ 8.5	— 5.0	2	+2.8	+ 1.1	1.06	0
Nov. 14	+ 5.6	— 135	—6.4	+ 5.9	+10.4	— 6.3	1	+4.5	+ 0.1	0.99	—10
1794, Feb. 15	+ 5.7	— 100	—6.5	+ 6.0	+ 7.3	— 6.3	2	+1.3	+ 0.2	1.06	0
Nov. 19	+ 5.4	— 67	—6.7	+ 5.6	+ 4.1	— 5.1	4	—1.5	+ 1.6	1.00	—10
1795, Feb. 20	+ 5.9	— 46	—6.9	+ 6.3	+ 6.0	— 8.0	3	—0.3	— 1.1	1.06	0
Nov. 29	+ 5.4	— 13	—7.0	+ 5.5	+ 3.9	— 3.4	2	—1.6	+ 3.6	1.00	—10
1796, Feb. 24	+ 5.3	— 8	—7.2	+ 5.6	+ 4.5	— 4.5	2	—1.1	+ 2.7	1.06	0
1797, Feb. 27	+ 5.1	+ 62	—7.5	+ 5.4	+ 4.7	— 4.2	3	—0.7	+ 3.3	1.06	0
1800, Mar. 14	+ 4.6	+ 224	—8.4	+ 4.9	+ 4.9	— 8.7	2	0.0	— 0.3	1.06	0
1814, May 22	— 1.2	+ 217	—2.2	— 1.3	+ 1.1	+ 0.2	2	+2.4	+ 2.4	1.06	0
1815, May 25	— 1.1	+ 214	—1.5	— 1.2	+ 1.4	— 1.0	4	+2.6	+ 0.5	1.06	0
1818, June 10	— 0.5	+ 413	—0.8	— 0.5	— 1.1	+ 4.8	2	—0.6	+ 5.6	1.06	0
1819, June 14	— 0.4	+ 483	+1.5	— 0.4	— 1.4	+ 3.7	4	—1.0	+ 2.2	1.06	0
1820, June 16	— 0 1	+ 498	+2.2	— 0.1	— 2.6	+ 4.0	2	—2.5	+ 1.8	1.06	0
1823, July 1	0.0	+ 538	+4.4	0.0	+ 0.2	+ 5.0	4	+0.2	+ 0.6	1.05	0
1825, July 11	— 2.4	+ 567	+5.8	— 2.5	— 4.8	+ 7.8	2	—2.3	+ 2.0	1.05	0
1826, July 16	— 4.5	+ 582	+6.6	— 4.7	— 3.6	+ 8.3	4	+1.1	+ 1.7	1.05	0
1827, July 20	— 6.6	+ 626	+7.2	— 6.9	— 7.2	+ 9.6	6	—0.3	+ 2.4	1.05	0
1828, July 23	— 9.8	+ 710	+7.9	—10.3	— 6.9	+ 6.9	3	+3.4	— 1.0	1.05	0
1829, Aug. 7	—13.4	+ 835	+8.5	—13.7	—16.3	+10.0	14	—2.6	+ 1.5	1.08	+ 2
Oct. 4	—14.2	+ 865	+8.6	—12.5	—13.0	+ 9.9	8	—0.5	+ 1.3	1.02	+ 8
1830, July 30	—17.8	+ 970	+9.0	—18.8	—21.4	+11.1	3	—2.6	+ 2.1		
Aug. 29	—18.2	+ 982	+9.1	—17.9	—20.0	+10.2	5	—2.1	+ 1.1		
Sept. 20	—18.5	+ 992	+9.1	—17.2	—19.1	+ 9.1	12	—1.9	0.0		
Oct. 14	—18.9	+1009	+9.1	—16.8	—17.3	+ 9.9	11	—0.5	+ 0.8		
Nov. 13	—19.3	+1009	+9.2	—16.8	—17.8	+ 9.3	8	—1.0	+ 0.1		

Paris, 1801-1827.

A complete reduction of this series is found in Le Verrier's *Annales de l'Observatoire Imperial de Paris, Observations*, tome 1. No comparison with any ephemeris is given here, nor is there any complete ephemeris to compare them with. A complete geocentric ephemeris was therefore computed from the provisional theory for the principal groups of the Paris observations. The individual observations being compared with it, the resulting mean corrections are given in the following table:

Mean date.	$\Delta\alpha$	$\Delta\delta$	*N.*	Mean date.	$\Delta\alpha$	$\Delta\delta$	*N.*
1801, March 24,	$-^s.02$	$+1''.2$	2	1813, May 20,	$+^s.19$	$+1''.8$	6
1802, April 1,	+ .08	+0 .6	13	1814, May 27,	+ .21	+0 .8	4
1805, April 22,	+ .10	+2 .2	13	1815, May 24,	— .02	+2 .2	5
1806, April 17,	— .01	—1 .6	5	1816, June 1,	— .01	+0 .8	7
1807, April 28,	+ .17	+0 .4	16	1817, June 5,	— .08	+1 .6	5
1808, April 28,	+ .02	+1 .4	6	1818, June 7,	+ .12	+2 .2	9
1809, May 5,	+ .20	+0 .1	9	1819, June 18,	— .07	+1 .8	7
1810, April 30,	+ .22	+2 .6	16	1820, June 20,	— .20	—2 .4	8.5
1811, Febr'y 18,	+ .21	+2 .2	3	1821, June 22,	+ .05	+1 .0	5
1811, May 17,	+ .14	+2 .6	8	1823, July 18,	+ .02	+1 .8	5
1812, Febr'y 16,	+ .28		2	1824, July 13,	+ .04	0 .0	7
1812, May 10,	+ .16	+3 .0	6	1827, July 25,	— .05	+0 .6	5
1813, Febr'y 25,	+ .44		3				

Total number of observations in right ascension, 175.

The observations in this series exhibit numbers of discordances of that class which leave the astronomer in doubt whether the observation should be retained or rejected. This remark applies more especially to the declinations. If we determine the probable error of an observation in declination by the condition that it is that amount which the error falls short of as often as it exceeds, it is found to be about 2″. Then, if the errors followed the commonly assumed law of probability, only about one in six of the errors should exceed 4″, and one in twenty-three 6″. But errors of these magnitudes are much more numerous, the deviations often amounting to six or eight seconds. I have rejected only a few in which the discordances approached 10″.

Bessel's Königsberg Observations, 1814–1835.

I have made a complete re-reduction of the right ascensions of this important series, and of most of the declinations. In order to avoid the necessity of applying systematic corrections, Dr. Gould's right ascensions and Dr. Auwers' declinations were used throughout in these reductions. In this work a selection of the fundamental stars observed by Bessel was made for each observation of the planet, to be used for clock error. These were chosen so that the mean of their right ascensions and declinations should be as near as practicable to those of Uranus, a condition, however, which could not generally be fulfilled for the declinations, owing to the southern position of the planet. Bessel's instrumental cor-

rections were applied to his observed times of transit over the mean wire, and the resulting time was employed as that of transit. Each time, compared with the computed right ascension of the star gave a value of the clock correction, which was reduced to the time of transit of the planet by the known daily rate. If the instrumental errors were always accurately determined, the mean of these clock corrections would be used to obtain the right ascension of Uranus. But it was frequently found that the clock error varied systematically with the declination of the star, so that it was deemed advisable to add to the clock correction a term varying as the simple declination, which was deduced from all the stars, and used to reduce the correction to the parallel of Uranus.

It was intended to give the results of this reduction for each observation, but on comparing the results with those of Fleming in the *Astronomische Nachrichten*, Band 30, it appeared that the results were not materially better than his. It does not, therefore, seem necessary to give more than the mean results for each opposition.

From Bessel's declinations, with the old Cary circle, I was unable to obtain any satisfactory results, owing, apparently, to a want of knowledge of some peculiarity of the instrument. Fleming's reductions were therefore adopted. They are designated by the letter F in the following list.

Mean Corrections to the Provisional Ephemeris given by Bessel's Observations at Königsberg, 1814–1829.

Mean date.	$\Delta\alpha$	$\Delta\delta$	$N.$	Mean date.	$\Delta\alpha$	$\Delta\delta$	$N.$
1814, May 22,	$+^s.11$	$+2''.5F$	9	1822, June 24,	$+^s.10$	$+1''.8$	7
1815, May 25,	$+.13$	$+1.8F$	11	1823, July 4,	$-.05$	$-1.6F$	2
1816, May 27,	$+.06$	$+1.2F$	11	1824, July 6,	$+.01$	$-1.0F$	5
1817, June 6,	$+.13$	$+2.3F$	8	1825, July 16,	$+.01$	$-2.4F$	5
1818, June 8,	$+.02$	$+4.3F$	13	1826, July 18,	$-.01$	$-3.0F$	7
1820, June 21,	$+.02$	$+4.1$	4	1828, July 25,	$-.15$	$-3.5F$	7
1821, June 23,	$+.12$	$+1.5$	5	1829, Aug. 1,	$-.10$	$-1.0F$	9

Total numbers of observations, 103.

Results of Observations at various Observatories, from 1827 *to* 1829 *inclusive.*

During these three years we have, besides the observations already quoted, the following:—

1. Observations by Schwerd, at Speier, of which the originals are given in *Astronomische Beobachtungen angestellt auf der Sternwarte des Königl. Lyzeums in Speyer von F. M. Schwerd,* Speyer, 1829–30, and of which the reduced results are found in the Astronomische Nachrichten, Band 8, S. 264.

2. The series by Airy, at Cambridge, commenced in 1828, and found in the Cambridge Observations.

3. Littrow's Vienna Observations, found in the first series of *Annalen der K. K. Sternwarte in Wien.*

The mean corrections to the provisional ephemeris given by these series are shown in the following table. The observations have been divided in the usual

way into groups of about a month each, and the mean date and mean correction found for each group. The Paris and Königsberg results are repeated for the sake of clearness. The small figures show, as usual, the number of observations employed in forming the mean.

Date.		Observatory.	$\Delta\alpha$ Original.	$\Delta\alpha$ Corrected.	$\Delta\delta$
1827,	July 22,	Speier,	$-0^s.16_6$	$-0^s.14$	
	July 25,	Paris,	-0.03_5	-0.05	$+0''.5_3$
	September 15,	Vienna,	-0.11_{11}	-0.10	0.0_{14}
	October 14,	Vienna,	-0.18_6	-0.17	-2.2_6
1828,	July 25,	Königsberg,	-0.15_7	-0.15	-3.5_7
	July 29,	Vienna,	-0.24_2	-0.20	-1.4_2
	August 14,	Vienna,	-0.13_{10}	-0.09	$+1.1_{10}$
	August 27,	Speier,	-0.10_6	-0.09	
	September 18,	Vienna,	-0.03_9	$+0.01$	$+1.0_9$
	September 25,	Cambridge,	-0.05_6	-0.16	
	October 17,	Vienna,	-0.13_{14}	-0.09	0.0_{14}
	October 17,	Cambridge,	-0.02_{10}	-0.12	
1829,	August 1,	Königsberg,	-0.10_9	-0.10	-1.0
	August 6,	Cambridge,	$+0.11_8$	-0.08	-1.1_{17}
	August 28,	Speier,	-0.04_5	-0.04	
	September 23,	Cambridge,	$+0.21_{10}$	$+0.05$	
	November 6,	Cambridge,	$+0.25_6$	$+0.09$	

Observations from 1830 *to* 1872.

Since the year 1830 heliocentric and geocentric ephemerides of Uranus computed from Bouvard's Tables are at our disposal. We make use of those in the Berlin Astronomisches Jahrbuch for the years 1830 to 1833, and of those in the Nautical Almanac from 1834 forward. The system of comparison is the same as that already explained. That is to say, we deduce separately:

(1) Mean corrections to the geocentric longitude and latitude of Uranus in the ephemeris as derived from observation.

(2) Mean corrections to the same, given by the provisional theory, as derived from a comparison of the heliocentric positions of that theory with the heliocentric positions in the ephemeris.

Then (1) — (2) is the correction to the provisional theory given by observation. The process of forming (1) and (2) is shown quite fully in the following pages. Each individual printed observation was first compared with the printed ephemeris, and a correction to the latter was thence deduced. When this correction was given with the observations themselves, it was of course not recomputed, unless in some doubtful cases. The observations were then divided into groups, usually of about a month each, and coinciding in time with the grouping of the Greenwich results. The mean of the dates and the mean of the corrections were then taken separately for each group and each observatory. The separate results are shown

in the proper columns of the following table, under the head "Mean dates," Mean cor. in R. A., and Mean cor. in Dec. These means are those given by the observations as printed, without the application of the systematic corrections on pages 120 and 121. In the columns "Corrected mean" these corrections are applied; this column would therefore exhibit no systematic differences between the results of the different observatories, unless the observations of Uranus were affected by errors different from those which affect the positions of the fundamental stars. A careful comparison of the differences in various parts of the table shows that this is unfortunately the case. A weight is next assigned to each individual result depending on the number of observations, the general sufficiency of the data of reduction, the mean discordance of the individual observations, and the quality of the instruments. The critical reader will notice a lack of homogeneity among the weights assigned, of which I shall speak presently. The mean of the separate group-results is then taken with regard to these weights, and also the mean of the mean dates, using for the latter the relative weights adopted for the several right ascensions. Thus, we have a mean result derived from all the observations for each month, or other group-period, which is written under the horizontal lines.

These corrections to right ascension and declination are next changed to corrections of longitude and latitude, using for this purpose the following table, which is computed from the formulæ of Gauss:

$$\cos E = \sin \varepsilon \cos \alpha \sec b = \sin \varepsilon \cos l \sec \delta$$

$$\Delta l = \frac{\sin E \cos \delta}{\cos b} \Delta \alpha + \frac{\cos E}{\cos b} \Delta \delta$$

$$\Delta b = - \cos E \cos \delta \, \Delta \alpha + \sin E \, \Delta \delta.$$

The differential coefficients in this table are expressed as a function of the right ascension of Uranus only, which may be done because, owing to the small inclination and great distance of the planet, its geocentric position on the celestial sphere is never more than about 2′ from some point of the projection of its heliocentric orbit. The coefficients of $\Delta\alpha$ are multiplied by 15, that the right ascension may be expressed in time.

To Convert Errors of Right Ascension and Declination of Uranus into errors of Longitude and Latitude.

When the Right Ascension exceeds 12^h, enter with R. A. -12^h, and change the signs of the quantities $\frac{\partial b}{\partial \alpha}$ and $\frac{\partial l}{\partial \delta}$.

R. A.	Logarithms of $\frac{\partial l}{\partial \alpha}$	Logarithms of $\frac{\partial b}{\partial \alpha}$		Logarithms of $\frac{\partial l}{\partial \delta}$		Logarithms of $\frac{\partial b}{\partial \delta}$	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$	$\frac{\partial l}{\partial \delta}$	$\frac{\partial b}{\partial \delta}$
$0^h\ 0^m$	1.1386	−0.7761	− 4	+9.6000	− 4	9.9626	+13.8+	−5 97+	+0.40−	+0.92+
10	1.1387	−0.7757	− 14	+9.5996	− 13	9.9626		5.96	0.40	
20	1.1388	−0.7743	− 23	+9.5983	− 20	9.9628		5 95	0.40	
30	1.1389	−0.7720	− 33	+9.5963	− 29	9.9632		5.92	0.40	
40	1.1390	−0.7687	− 44	+9.5934	− 38	9.9637		5.87	0.39	
50	1.1391	−0.7643	− 55	+9.5896	− 47	9.9645		5.81	0.38	
1 0	1.1392	−0.7588	− 63	+9.5849	− 55	9.9653	+13.8+	−5.74+	+0.38−	+0.92+
10	1.1393	−0.7525	− 74	+9.5794	− 64	9.9662		5.66	0.38	
20	1.1394	−0.7451	− 86	+9.5730	− 74	9.9673		5.56	0.37	
30	1.1394	−0.7365	− 95	+9.5656	− 83	9.9685		5.45	0.37	
40	1.1395	−0.7270	−108	+9.5573	− 94	9.9698		5.33	0.36	
50	1.1395	−0.7162	−118	+9.5479	−104	9.9711		5.20	0.35	

To Convert Errors of Right Ascension and Declination.—*Continued.*

R. A.	Logarithms of									
	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$		$\frac{\partial l}{\partial \delta}$		$\frac{\partial b}{\partial \delta}$	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$	$\frac{\partial l}{\partial \delta}$	$\frac{\partial b}{\partial \delta}$
2h 0m	1.1395	−0.7044	−129	+9.5375	−115	9.9725	+13.8+	−5.06+	+0.34−	+0.93+
10	1.1395	−0.6915	−142	+9.5260	−126	9.9740		4.91	0.33	
20	1.1395	−0.6773	−155	+9.5134	−139	9.9756		4.75	0.32	
30	1.1395	−0.6618	−168	+9.4995	−152	9.9772		4.58	0.31	
40	1.1395	−0.6450	−184	+0.4843	−167	9.9788		4.41	0.30	
50	1.1394	−0.6266	−197	+9.4676	−181	9.9804		4.23	0.29	
3 0	1.1394	−0.6069	−215	+9.4495	−198	9.9821	+13.8+	−4.05+	+0.28−	+0.96+
10	1.1394	−0.5854	−233	+9.4297	−216	9.9837		3.86	0.27	
20	1.1393	−0.5621	−253	+9.4081	−237	9.9853		3.66	0.26	
30	1.1393	−0.5368	−274	+9.3844	−258	9.9869		3.45	0.24	
40	1.1392	−0.5094	−299	+9.3586	−284	9.9883		3.24	0.23	
50	1.1391	−0.4795	−327	+9.3302	−312	9.9898		3.02	0.21	
4 0	1.1390	−0.4468	−359	+9.2990	−346	9.9912	+13.8+	−2.80+	+0.20−	+0.98+
10	1.1390	−0.4109	−399	+9.2644	−385	9.9925		2.58	0.18	
20	1.1389	−0.3710	−443	+9.2259	−431	9.9938		2.36	0.17	
30	1.1388	−0.3267	−498	+9.1828	−487	9.9949		2.14	0.15	
40	1.1387	−0.2769	−571	+9.1341	−560	9.9959		1.91	0.13	
50	1.1386	−0.2198	−659	+9 0781	−651	9.9969		1.67	0.12	
5 0	1.1385	−0.1539	−786	+9.0130	−777	9.9977	+13.8+	−1.43+	+0.10−	+0.99+
10	1.1384	−0.0753	− 96	+8.9353	−956	9.9984		1.19	0.08	
20	1.1383	−9.979	−125	+8.8397	−1240	9.9990		0.95	0.07	
30	1.1382	−9.854	−175	+8.7157	−1757	9.9994		0.71	0.05	
40	1.1381	−9.679	−301	+8.540	−300	9.9997		0.48	0.03	
50	1.1380	−9.378	......	+8.240	......	9.9999		0.24	0.02	
6 0	1.1379	− ∞	......	∞	......	0	+13.7+	0.00	0.00	+1.00+
10	1.1378	+9.378	+301	−8.240	+300	9.9999		−0.24+	−0.02+	
20	1.1377	+9.679	+175	−8.540	+1757	9.9997		0.48	0.03	
30	1.1376	+9.854	+124	−8.7157	+1240	9.9994		0.71	0.05	
40	1.1375	+9.978	+ 96	−8.8397	+956	9.9990		0.95	0.07	
50	1.1374	+0.0743	+783	−8.9353	+777	9.9984		1.18	0.08	
7 0	1.1373	+0.1526	+658	−9.0130	+651	9.9977	+13.7+	+1.42−	−0.10+	+0.99+
10	1.1373	+0.2184	+569	−9.0781	+560	9.9969		1.65	0.12	
20	1.1372	+0.2753	+497	−9.1341	+487	9.9959		1.88	0.13	
30	1.1371	+0.3250	+442	−9.1828	+431	9.9949		2.11	0.15	
40	1.1371	+0.3692	+397	−9.2259	+385	9.9938		2.34	0.17	
50	1.1370	+0.4089	+359	−9.2644	+346	9.9925		2.56	0.18	
8 0	1.1370	+0.4448	+325	−9.2990	+312	9.9912	+13.7+	+2.78−	−0.20+	+0.98+
10	1.1370	+0.4773	+298	−9.3302	+284	9.9898		3.00	0.21	
20	1.1370	+0.5071	+273	−9.3586	+258	9.9883		3.22	0.23	
30	1.1370	+0.5344	+253	−9.3844	+237	9.9869		3.43	0.24	
40	1.1370	+0.5597	+233	−9.4081	+216	9.9853		3.63	0.26	
50	1.1370	+0.5830	+214	−9.4297	+198	9 9837		3.83	0.27	
9 0	1.1370	+0.6044	+197	−9.4495	+181	9.9821	+15.7+	+4.02−	−0.28+	+0.96+
10	1.1370	+0.6241	+184	−9.4676	+167	9.9804		4.20	0.29	
20	1.1370	+0.6425	+169	−9.4843	+152	9.9788		4.38	0.30	
30	1.1371	+0.6594	+155	−9.4995	+139	9.9772		4.55	0.31	
40	1.1371	+0.6749	+143	−9.5134	+126	9.9756		4.72	0.32	
50	1.1372	+0.6892	+131	−9.5260	+115	9.9740		4.88	0.33	
10 0	1.1373	+0.7023	+119	−9.5375	+104	9.9725	+13.7+	+5.04−	−0.34+	+0.93+
10	1.1374	+0.7142	+109	−9.5479	+ 94	9.9711		5.18	0.35	
20	1.1375	+0.7251	+ 96	−9.5573	+ 83	9 9698		5.31	0.36	
30	1.1376	+0.7347	+ 87	−9.5656	+ 74	9.9685		5.43	0.37	
40	1.1377	+0.7434	+ 76	−9.5730	+ 64	9.9673		5.54	0.37	
50	1.1378	+0.7510	+ 65	−9.5794	+ 55	9.9662		5.64	0.38	
11 0	1.1379	+0.7575	+ 57	−9.5849	+ 47	9.9653	+13.7+	+5.72−	−0.38+	+0.92+
10	1.1380	+0.7632	+ 46	−9.5896	+ 38	9.9645		5.79	0.38	
20	1.1381	+0.7678	+ 35	−9.5934	+ 29	9.9637		5.85	0.39	
30	1.1382	+0.7713	+ 24	−9.5963	+ 20	9.9632		6.90	0.39	
40	1.1383	+0.7737	+ 17	−9.5983	+ 13	9.9628		5.94	0.39	
50	1.1385	+0.7754	+ 7	−9.5996	+ 4	9.9626		5.96	0.40	
12 0	1.1386	+0.7761	......	−9.6000	......	9.9626	+13.8+	+5.97−	−0.40+	+0.92+

We thus have, for the interval occupied by each group of observations, a mean correction to the geocentric longitude and latitude of the planet given by observations, which are found in the ninth and tenth columns of the table, on the same horizontal line with the mean corrections in right ascension and declination from which they are derived. The next step is to obtain the corresponding corrections given by the provisional ephemeris.

This correction has been first obtained for every twentieth day of each of the forty-two oppositions included in the table. The heliocentric longitude, latitude, and radius vector were interpolated to the most convenient twenty-day intervals, and compared with the corresponding co-ordinates in the heliocentric ephemeris. This ephemeris was of course the one corresponding to that with which the observations were compared, namely, the Berliner Jahrbuch for the years 1830-33, and the Nautical Almanac for subsequent years. These comparisons are fully given at the end of this chapter, and the resulting corrections to the printed ephemeris are given in the proper columns of the table.

These corrections to the heliocentric co-ordinates were then changed to corrections of geocentric longitude and latitude by the following formulæ. Put

r', the projection of the planet's radius vector on the ecliptic;

ρ', the projection of the planet's distance from the earth on the same plane;

ρ, this distance itself;

λ, β, the planet's heliocentric longitude and latitude;

L, the sun's geocentric longitude;

R, its radius vector;

M, the modulus of the common logarithms;

δl, δb, the corrections to the geocentric longitude and latitude;

$\delta\rho$, the correction to the common logarithm of the radius vector.

Then

$$\delta l = \frac{r'^2}{\rho'^2}\left\{1 + \frac{R}{r'}\cos(L-\lambda)\right\}\delta\lambda$$

$$-\frac{Rr'}{\rho'^2}\sin(L-\lambda)\frac{\delta\rho}{M\sin 1''}$$

$$+\frac{Rr'}{\rho'^2}\sin(L-\lambda)\tan\beta\,\delta\beta$$

$$\delta b = \frac{r'}{\rho'}\left\{1 + \frac{r'R}{\rho^2}\tan^2\beta\cos(L-\lambda)\right\}\delta\beta$$

$$-\frac{r'^2R}{\rho'\rho^2}\tan\beta\sin(L-\lambda)\,\delta\lambda$$

$$+\frac{rR^2}{\rho'\rho^2}\left\{1 + \frac{r'}{R}\cos(L-\lambda)\right\}\sin\beta\,\frac{\delta\rho}{M\sin 1''}.$$

The last term in δl and the last two terms of δb have been omitted in the computation, as they scarcely ever exceed a few hundredths of a second.

The values of δl and δb are printed in the last two columns of the table. The formula for δb might have contained the additional term

$$\delta b = \sin l \delta\omega$$

$\delta\omega$ being the correction to the obliquity of the ecliptic adopted in the ephemeris to reduce it to that employed in the provisional theory. This correction is, however, deferred until we come to form the equations of condition.

From the values of δl and δb thus obtained we are to find the mean values during each group of observations. If these quantities varied uniformly, the proper value would be that corresponding to the mean date of each group. But the second differences are so large that this value would generally be in error by one- or two-tenths of a second. Owing to the minuteness of this difference, it has been considered that when the mean date was near the middle of a twenty-day interval, the correction δl interpolated to that date without regard to second differences would furnish a sufficient approximation to the required mean value of δl during an interval of about 30 days. In other case the value of δl was interpolated to 5-day intervals through the period of each group of observations, and the mean value taken.

During the years 1850–1863 the sun's longitude employed in the ephemeris required a gradually increasing correction, amounting at the latter date to about 3″. A small correction of which the maximum value is about 0″.15 was applied to δl to reduce it to the value it would have had if Hansen's tables had been employed.

The corrected mean values of δl and δb thus obtained are given in the last two columns of the following table, being inclosed in brackets and printed immediately above the values of Δl and Δb derived from observation.

I deem it proper to mention that the mechanical labor of constructing these tables of comparisons, in the manner just described, was in great part performed by Dr. C. L. F. Kampf, who was employed by the Smithsonian Institution to assist me in the work. Before using it I subjected the whole of the work to a careful revision, altering especially the relative weights of the corrected means in many cases. As the assigned weights now stand, each set of results which are combined into a single mean has its own unit of weight, which does not necessarily coincide with that of any other set. The use of a uniform scale of weights through this series of observations, and the assignment to every final mean of a weight equal to the sum of the weights of the quantities whose mean was taken, would have led to weights in many cases quite fictitious, owing to the obvious presence of systematic errors in the results. For this reason I have made no further use of the weights found in this table, and their lack of homogeneousness therefore does no harm.

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS IN THE BERLINER JAHRBUCH AND THE NAUTICAL ALMANAC.

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1830	s		s	"		"	"	"
Königsberg,	July 29	—1.56	7	-1.56_{5}	+4.6	7	+4.6		
Cambridge,	July 29	—1.51	6	-1.67_{2}					
[20ʰ 40ᵐ]	July 29	....	...	—1.60	....	...		[—19.2] —20.8	[+ 9.5] +10.3
Königsberg,	Aug. 12	—1.65	2	-1.65_{2}	+3.6	2	+3.6		
Cambridge,	Aug. 25	—1.36	4	-1.52_{2}					
[20ʰ 36ᵐ]	Aug. 19	....	...	—1.58	....	...		[—18.5] —20.8	[+ 9.5] + 9.1
Cambridge, [20ʰ 37ᵐ]	Sept. 19	—1.46	7	-1.62_{3}	....	...	....	[—17.2] —21.2	
Cambridge, [20ʰ 36ᵐ]	Oct. 17	—1.34	8	-1.50_{4}	....	...	....	[—16.7] —19.5	
Cambridge, [20ʰ 37ᵐ]	Nov. 14	—1.36	8	-1.52_{4}	....	...	....	[—16.6] —19.8	
	1831								
Greenwich,	Aug. 3	—1.72	11	-1.72_{4}	+2.5	11	+4.3		
Cambridge,	Aug. 8	—1.70	9	-1.86_{4}					
[20ʰ 58ᵐ]	Aug. 6	....	...	—1.79	....	...		[—23.8] —23.4	[+10.0] +11.1
Greenwich,	Sept. 7	—1.67	5	-1.67_{3}	+3.5	5	+5.3		
Cambridge,	Sept. 15	—1.60	6	-1.76_{3}					
[20ʰ 52ᵐ]	Sept. 11	....	...	—1.72	....	...		[—22.0] —22.2	[+10.0] +11.8
Greenwich,	Nov. 4	—1.48	7	-1.48_{4}	+3.1	7	+4.9		
Cambridge,	Oct. 26	—1.54	16	-1.70_{8}					
[20ʰ 50ᵐ]	Oct. 31	....	...	—1.63	....	...		[—20.7] —21.0	[+ 9.8] +10.8
	1832								
Greenwich,	Aug. 9	—2.02	3	-2.02_{2}	+1.7	2	$+3.4_{9}$		
Königsberg,	Aug. 10	—2.24	3	-2.24_{4}					
Cambridge,	Aug. 15	—1.99	9	-2.18_{8}					
Vienna,	Aug. 3	—2.33	3	-2.33_{1}	+0.2	3	+1.1		
[21ʰ 17ᵐ]	Aug. 12	....	...	—2.19	....	...	+2.5	[—28.4] —29.2	[+10.3] +11.9
Cambridge, [21ʰ 12ᵐ]	Sept. 12	—1.97	10	-2.16_{4}	....	...	....	[—26.9] —28.8	
Cambridge,	Oct. 6	—1.91	13	-2.09_{6}					
Vienna,	Oct. 12	—1.90	15	-1.90_{4}	+1.9	15	+2.8		
[21ʰ 9ᵐ]	Oct. 7	....	...	—2.01	....	...		[—25.8] —26.6	[+10.2] +11.1
Cambridge,	Nov. 16	—1.85	7	-2.04_{7}					
Vienna,	Nov. 9	—1.89	2	-1.89_{1}	+1.1	...	+2.0		
[21ʰ 10ᵐ]	Nov. 15	....	...	—2.02	....	...	....	[—25.2] —27.4	[+10.1] +10.5

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1833	s		s	″		″	″	″
Greenwich,	Aug. 22	—2.57	7	-2.57_2	—1.4	7	$+0.2_6$		
Königsberg,	Aug. 12	—2.57	5	-2.57_4	—1.5	5	-1.5_5		
Cambridge,	Aug. 15	—2.40	11	-2.59_5				[—33.8]	[+10.8]
[21h 32m]	Aug. 15	....	...	—2.58	....	...	—0.6	—35.6	+10.9
Greenwich,	Sept. 18	—2.33	5	-2.33_2	—0.8	4	$+0.8_4$		
Cambridge,	Sept. 19	—2.33	10	-2.52_4	+2.8	11	$+1.4_{11}$		
Vienna,	Sept. 11	—2.52	6	-2.56_1	+2.6	6	$+1.2_2$	[—32.2]	[+10.8]
[21h 28m]	Sept. 18	....	...	—2.47	....	...	+1.2	—33.4	+12.3
Greenwich,	Oct. 11	—2.25	4	-2.25_1	+0.1	4	$+1.7_4$		
Cambridge,	Oct. 12	—2.29	12	-2.48_5	+2.2	13	$+0.8_{13}$		
Vienna,	Oct. 14	—2.37	19	-2.41_2				[—31.2]	[+10.6]
[21h 26m]	Oct. 12	....	...	—2.43	....	...	+1.0	—33.0	+11.9
Cambridge,	Nov. 18	—2.16	5	-2.35_3	+ 1.3	4	—0.1		
Vienna,	Nov. 14	—2.24	4	-2.28_1				[—30.3]	[+10.3]
[21h 26m]	Nov. 16	....	...	—2.33	....	...	—0.1	—32.0	+10.4
	1834								
Cambridge,	Aug. 15	—2.58	11	-2.77_5	— 1.3	11	-2.6_{11}		
Vienna,	Aug. 13	—3.07	4	-3.11_1	— 4.4	4	-3.5_1	[—40.6]	[+11.1]
[21h 49m]	Aug. 14	....	...	—2.83	....	...	—2.7	—39.7	+11.3
Greenwich,	Sept. 10	—3.00	5	-3.00_1	— 4.4	5	-3.0_5		
Cambridge,	Sept. 16	—2.66	18	-2.85_4	— 1.2	18	-2.5_{18}	[—39.0]	[+11.1]
[21h 45m]	Sept. 13	....	...	—2.88	....	...	—2.6	—40.5	+11.2
Greenwich,	Oct. 14	—2.74	10	-2.74_3	— 3.1	10	-1.7_{10}		
Cambridge,	Oct. 16	—2.68	16	-2.87_7	— 0.8	16	-2.1_{16}		
Vienna,	Oct. 20	—2.75	4	-2.80_1	— 1.3	4	-0.4_1	[—37.5]	[+10.9]
[21h 41m]	Oct. 17	....	...	—2.83	....	...	—1.9	—39.5	+11.6
Cambridge,	Nov. 16	—2.64	9	-2.83_4	— 0.7	11	-2.0_{11}		
Vienna,	Nov. 12	—2.53	8	-2.58_1	— 2.8	8	-1.9_4	[—36.7]	[+10.7]
[21h 41m]	Nov. 14	....	...	—2.77	....	...	—2.0	—38.7	+11.3
Cambridge,	Dec. 6	—2.71	5	-2.90_2	— 1.2	4	-2.5_4		
Vienna,	Dec. 7	—2.55	1	-2.59_0	— 2.1	1	-1.2_1	[—36.2]	[+10.5]
[21h 43m]	Dec. 7	....	...	—2.88	....	...	—2.4	—40.3	+11.5
	1835								
Greenwich,	Aug. 17	—3.25	4	-3.25_1	— 6.6	4	-5.4_4		
Königsberg,	Aug. 20	—3.31	11	-3.31_8					
Cambridge,	Aug. 14	—3.28	20	-3.47_9	— 4.7	20	-5.9_{20}		
Vienna,	Aug. 21	—3.30	1	-3.35_0	— 8.8	1	-7.9_1	[—47.1]	[+11.3]
[22h 4m]	Aug. 17	....	...	—3.39	....	...	—5.9	—48.6	+11.8

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A. Mean.	No. of obs.	Corrected mean.	Observed corrections in Dec. Mean.	No. of obs.	Corrected mean.	Corr. to Geocentric Longitude.	Latitude.
	1835	s		s	"		"	"	"
Cambridge,	Sept. 15	—3.17	9	—3.36$_{4}$	— 4.6	9	—5.8$_{9}$		
Vienna,	Sept. 14	—3.30	9	—3.35$_{1}$	— 6.7	9	—5.8$_{9}$	[—45.8]	[+11.2]
[22^h 1^m]	Sept. 15	....	...	—3.36	....	...	—5.8	—48.1	+11.6
Greenwich,	Oct. 10	—3.27	4	—3.27$_{2}$	— 6.53	4	—5.3$_{4}$		
Cambridge,	Oct. 17	—3.11	8	—3.30$_{4}$	— 4.1	8	—5.3$_{8}$	[—44.4]	[+11.1]
[21^h 57^m]	Oct. 15	....	...	—3.29	....	...	—5.3	—46.9	+11.4
Greenwich,	Nov. 27	—3.21	6	—3.21$_{2}$	— 5.6	7	—4.4$_{7}$		
Cambridge,	Nov. 26	—3.00	10	—3.19$_{4}$	— 4.5	9	—5.7$_{9}$	[—43.0]	[+10.7]
[21^h 57^m]	Nov. 26	....	...	—3.20	....	...	—5.1	—45.6	+11.2
	1836								
Greenwich,	July 22	—3.80	7	—3.84$_{2}$	—10.5	8	—9.5$_{8}$		
Cambridge,	July 25	—3.60	3	—3.89$_{1}$	— 8.9	2	—9.9$_{2}$	[—55.3]	[+11.5]
[22^h 24^m]	July 23	....	...	—3.86	....	...	—9.6	—56.5	+11.8
Greenwich,	Aug. 24	—3.78	7	—3.82$_{3}$	— 9.8	8	— 8.8$_{8}$		
Königsberg,	Aug. 30	—3.63	5	—3.63$_{4}$					
Cambridge,	Aug. 16	—3.78	12	—3.97$_{5}$	— 9.3	12	—10.3$_{12}$		
Edinburgh,	Aug. 19	—4.09	9	—4.10$_{3}$	— 9.3	7	— 9.3$_{7}$		
Vienna,	Aug. 20	—3.77	1	—3.81$_{0}$	—12.5	1	—11.5$_{0}$	[—54.6]	[+11.6]
[22^h 20^m]	Aug. 22	....	...	—3.87	....	...	—9.6	—56.6	+11.5
Greenwich,	Sept. 13	—3.77	7	—3.81$_{3}$	— 8.8	6	—7.8$_{6}$		
Cambridge,	Sept. 16	—3.70	10	—3.89$_{4}$	— 8.4	10	—9.4$_{10}$		
Edinburgh,	Sept. 16	—4.01	8	—4.02$_{2}$	— 8.6	8	—8.6$_{8}$		
Vienna,	Sept. 15	—3.59	9	—3.63$_{1}$	—10.4	9	—9.4$_{3}$	[—53.4]	[+11.6]
[22^h 16^m]	Sept. 15	....	...	—3.87	....	...	—8.8	—56.2	+12.1
Greenwich,	Oct. 12	—3.67	7	—3.71$_{2}$	— 8.6	8	—7.6$_{8}$		
Cambridge,	Oct. 16	—3.51	12	—3.70$_{5}$	— 8.5	11	—9.5$_{11}$		
Edinburgh,	Oct. 15	—4.05	8	—4.06$_{2}$	— 7.9	4	—7.9$_{4}$		
Vienna,	Oct. 11	—3.57	10	—3.61$_{1}$	—10.2	10	—9.2$_{3}$	[—51.8]	[+11.3]
[22^h 13^m]	Oct. 15	....	...	—3.77	...	...	—8.6	—54.8	+11.7
Greenwich,	Nov. 13	—3.59	11	—3.63$_{5}$	— 9.1	11	—8.1$_{11}$		
Cambridge,	Nov. 13	—3.37	8	—3.56$_{4}$	— 7.9	7	—8.9$_{7}$		
Edinburgh,	Nov. 17	—3.78	8	—3.79$_{2}$					
Vienna,	Nov. 9	—3.56	3	—3.60$_{1}$	—10.2	3	—9.2$_{1}$	[—50.6]	[+11.1]
[22^h 12^m]	Nov. 13	....	...	—3.63	....	...	—8.5	—52.8	+11.0
Cambridge,	Dec. 12	—3.40	7	—3.59$_{3}$	— 8.0	7	—9.0$_{7}$		
Edinburgh,	Dec. 12	—3.29	5	—3.30$_{1}$	— 8.8	3	—8.8$_{3}$	[—50.0]	[+10.9]
[22^h 13^m]	Dec. 12	....	...	—3.52	....	...	—8.9	—51.5	+10.1
	1837							[—62.9]	[+11.5]
Greenwich, [22^h 40^m]	July 22	—4.23	4	—4.26$_{1}$	—13.4	4	—12.5	—63.2	+12.0

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1837	s		s	"		"	"	"
Greenwich,	Aug. 18	—4.30	10	-4.33_2	—13.4	10	-12.5_{10}		
Cambridge,	Aug. 18	—4.09	14	-4.28_2	—12.6	11	-13.4_{11}		
Edinburgh,	Aug. 22	—4.40	6	-4.40_1	—12.7	6	-12.7_6		
Paris,	Aug. 14	—4.34	9	-4.33_2	—12.0	10	-11.4_{10}		
Vienna,	Aug. 25	—4.29	4	-4.33_0	—12.6	4	-11.6_1		
[22^h 36^m]	Aug 18	....	...	—4.33	....	...	—12.5	[—62.9] —64.1	[+11.7] +12.2
Greenwich,	Sept. 17	—4.23	14	$-4.26_{6\frac{1}{2}}$	—13.4	14	-12.5_{14}		
Königsberg,	Sept. 11	—4.10	8	-4.13_6					
Cambridge,	Sept. 17	—4.06	14	-4.25_6	—11.9	15	-12.7_{14}		
Edinburgh,	Sept. 10	—4.39	4	-4.39_1	—11.5	3	-11.5_2		
Paris,	Sept. 18	—4.20	12	-4.19_6	—12.3	13	-11.7_{13}		
Vienna,	Sept. 13	—4.12	5	-4.15_1	—13.9	5	-12.9_1		
[22^h 32^m]	Sept. 16	....	...	—4.21	....	...	—12.3	[—61.7] —62.3	[+11.6] +11.5
Greenwich,	Oct. 16	—4.13	11	-4.16_3	—12.9	11	-12.0_{11}		
Cambridge,	Oct. 17	—4.00	10	-4.19_4	—11.6	11	-12.4_{11}		
Paris,	Oct. 17	—4.05	4	-4.04_2	—11.2	4	-10.6_4		
Vienna,	Oct. 18	—4.44	2	$-4.47_{\frac{1}{2}}$	—15.5	2	$-14.5_{\frac{1}{2}}$		
[22^h 28^m]	Oct. 17	....	...	—4.16	....	....	—12.0	[—59.8] —61.5	[+11.4] +11.3
Greenwich,	Nov. 8	—4.10	5	-4.13_2	—12.4	5	-11.5_5		
Cambridge,	Nov. 4	—3.96	4	-4.15_2	—12.3	3	-13.1_3		
Paris,	Nov. 6	—4.07	3	-4.06_2	—11.3	3	-10.7_3		
Vienna,	Nov. 2	—4.09	2	$-4.12_{\frac{1}{2}}$	—12.9	2	$-11.9_{\frac{1}{2}}$		
[22^h 27^m]	Nov. 6	....	...	—4.11	....	...	—11.7	[—59.0] —60.7	[+11.2] +11.3
Greenwich,	Dec. 2	—4.05	2	-4.08_1	—14.0	2	-13.1_2		
Cambridge,	Nov. 30	—3.87	6	-4.06_3	—11.6	7	-12.4_7		
Paris,	Dec. 8	—4.03	7	-4.02_4	—10.9	7	-10.3_7		
Vienna,	Dec. 8	—4.28	2	$-4.31_{\frac{1}{2}}$	—13.0	2	$-12.0_{\frac{1}{2}}$		
[22^h 28^m]	Dec. 5	....	...	—4.05	....	...	—11.6	[—57.7] —59.9	[+11.0] +11.1
	1838								
Greenwich,	Aug. 20	—4.72	11	-4.76_5	—15.9	11	-15.1_{11}		
Königsberg,	Aug. 25	—4.65	5	-4.65_4	—18.6	5	-17.6_3		
Cambridge,	Aug. 19	—4.67	11	-4.78_5	—15.8	12	-16.3_{12}		
Edinburgh,	Aug. 25	—4.96	3	-4.95_1	—15.0	4	-15.0_4		
Paris,	Aug. 20	—4.80	9	-4.79_5	—16.6	9	-16.0_9		
[22^h 51^m]	Aug. 21	....	...	—4.76	....	...	—15.8	[—70.9] —71.4	[+11.7] +12.3
Greenwich,	Sept. 12	—4.62	8	-4.66_3	—16.2	8	-15.4_3		
Königsberg,	Sept. 16	—4.65	14	-4.67_{10}	—18.5	14	-17.5_7		
Cambridge,	Sept. 16	—4.61	11	-4.72_5	—14.9	12	-15.4_{12}		
Edinburgh,	Sept. 15	—4.74	9	-4.73_3	—15.8	7	-15.8_7		
Paris,	Sept. 12	—4.71	6	-4.70_3	—16.0	6	-15.4_6		
Vienna,	Sept. 11	—4.71	12	-4.73_1	—18.1	12	-17.1_3		
Berlin,	Sept. 6	—4.72	7	-4.74_2	—17.5	7	-16.5_3		
[22^h 47^m]	Sept. 15	....	...	—4.70	....	...	—16.0	[—70.4] —70.7	[+11.7] +12.0

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A. Mean.	No. of obs.	Corrected mean.	Observed corrections in Dec. Mean.	No. of obs.	Corrected mean.	Corr. to Geocentric Longitude.	Corr. to Geocentric Latitude.
	1838	s		s	″		″	″	″
Greenwich,	Oct. 15	—4.67	7	-4.71_{3}	—15.5	7	-14.7_{7}		
Cambridge,	Oct. 17	—4.49	7	-4.60_{3}	—15.3	6	-15.8_{6}		
Edinburgh,	Oct. 16	—4.56	12	-4.55_{3}	—15.0	8	-15.0_{8}		
Paris,	Oct. 16	—4.66	7	-4.65_{3}	—15.9	7	-15.3_{7}		
Vienna,	Oct. 14	—4.66	9	-4.68_{1}	—17.3	9	-16.3_{9}		
[22h 44m]	Oct. 16	....	...	—4.63	....	...	—16.2	[—68.6] —69.7	[+11.5] +10.8
Greenwich,	Nov. 9	—4.52	6	-4.56_{9}	—16.0	6	-15.2_{6}		
Cambridge,	Nov. 15	—4.37	10	-4.48_{4}	—15.1	9	-15.6_{9}		
Edinburgh,	Nov. 16	—4.70	8	-4.69_{2}	—17.4	2	-17.4_{2}		
Vienna,	Nov. 7	—4.93	4	-4.95_{4}	—18.3	4	-17.3_{1}		
[22h 42m]	Nov. 14	....	...	—4.58	....	...	—15.8	[—66.9] —68.8	[+11.3] +10.9
Greenwich,	Dec. 9	—4.60	2	-4.64_{1}	—15.9	2	-15.1_{2}		
Cambridge,	Dec. 15	—4.26	5	-4.37_{2}	—15.1	7	-15.6_{7}		
Edinburgh,	Dec. 17	—4.18	3	-4.17_{1}	—14.7	1	-14.7_{1}		
Paris,	Dec. 5	—4.40	7	-4.39_{4}	—15.1	7	-14.5_{7}		
[22h 43m]	Dec. 10	....	...	—4.38	....	...	—15.0	[—65.8] —65.7	[+11.1] +10.4
	1839								
Greenwich,	Aug. 22	—5.28	5	-5.32_{1}	—21.3	5	-20.6_{5}		
Cambridge,	Aug. 24	—5.11	8	-5.22_{4}	—20.7	7	-21.0_{7}		
Paris,	Aug. 23	—5.20	3	-5.19_{2}	—20.7	3	-20.1_{3}		
Edinburgh,	Aug. 25	—5.21	2	-5.20_{4}					
[23h 6m]	Aug. 23	....	...	—5.22	....	...	—20.7	[—79.7] —79.7	[+11.7] +11.0
Greenwich,	Sept. 10	—5.14	12	-5.18_{3}	—21.0	12	-20.3_{12}		
Königsberg,	Sept. 12	—5.11	12	-5.13_{8}	—21.7	12	-20.7_{6}		
Berlin,	Sept. 10	—5.10	14	-5.11_{4}	—21.1	14	-20.1_{7}		
Cambridge,	Sept. 17	—5.12	11	-5.23_{5}	—20.0	10	-20.3_{10}		
Paris,	Sept. 14	—5.23	10	-5.22_{5}	—20.7	10	-20.1_{10}		
Edinburgh,	Sept. 16	—5.16	17	-5.15_{5}	—20.5	5	-20.5_{5}		
Vienna,	Sept. 17	—5.32	9	-5.33_{1}	—20.8	9	-19.8_{2}		
[23h 3m]	Sept. 14	....	...	—5.17	....	...	—20.3	[—79.0] —78.8	[+11.6] +11.0
Greenwich,	Oct. 12	—5.18	5	-5.22_{2}	—20.2	5	-19.5_{5}		
Cambridge,	Oct. 15	—5.07	8	-5.18_{4}	—19.2	5	-19.5_{8}		
Edinburgh,	Oct. 15	—5.14	10	-5.13_{3}	—19.9	8	-19.9_{8}		
Vienna,	Oct. 10	—5.17	13	-5.18_{1}	—20.5	13	-19.5_{4}		
[22h 59m]	Oct. 14	....	...	—5.18	....	...	—19.6	[—77.4] —78.7	[+11.5] +11.8
Greenwich,	Nov. 12	—4.98	6	-5.02_{2}	—20.2	6	-19.5_{6}		
Cambridge,	Nov. 19	—4.85	6	-4.96_{2}	—19.1	5	-19.4_{5}		
Paris,	Nov. 9	—4.99	2	-4.98_{1}	—20.4	2	-19.8_{2}		
Edinburgh,	Nov. 11	—4.90	3	-4.89_{1}	—18.3	1	-18.3_{1}		
[22h 57m]	Nov. 14	....	...	—4.97	....	...	—19.4	[—75.3] —75.7	[+11.3] +10.4
Greenwich,	Dec. 6	—4.84	2	-4.88_{1}	—19.0	2	-18.3_{2}		
Cambridge,	Dec. 15	—4.83	5	-4.94_{2}	—19.1	2	-19.4_{2}		
Paris,	Dec. 3	—4.90	3	-4.89_{2}	—17.8	3	-17.2_{3}		
Edinburgh,	Dec. 28	—4.96	4	-4.95_{1}	—18.5	3	-18.5_{3}		
[22h 57m]	Dec. 12	....	...	—4.92	....	...	—18.2	[—73.9] —74.6	[+11.0] +11.2

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected meau.	Longitude.	Latitude.
	1840	s		s	″		″	″	″
Greenwich,	Aug. 14	—5.77	11	-5.81_{3}	—24.5	11	-23.9_{11}		
Cambridge,	Aug. 14	—5.64	1	$-5.73_{1/4}$	—25.0	1	-25.2_{1}		
Edinburgh,	Aug. 31	—5.65	1	$-5.64_{1/4}$	—23.8	1	-23.7_{1}	[—87.9]	[+11.4]
[23ʰ 20ᵐ]	Aug. 16	....	...	—5.78	....	...	—23.9	—88.8	+11.9
Greenwich,	Sept. 15	—5.66	8	-5.70_{3}	—24.0	8	-23.4_{8}		
Königsberg,	Sept. 13	—5.74	6	-5.76_{4}	—25.1	6	-24.1_{3}		
Cambridge,	Sept. 12	—5.43	1	$-5.52_{1/4}$					
Edinburgh,	Sept. 16	—5.70	13	-5.69_{4}	—24.6	9	-24.5_{9}		
Paris,	Sept. 6	—5.60	4	-5.59_{2}	—23.6	4	-23.0_{4}		
Berlin,	Sept. 14	—5.45	2	$-5.45_{1/4}$	—26.3	2	-25.3_{1}		
Vienna,	Sept. 9	—5.76	2	$-5.76_{1/4}$	—22.2	2	$-21.2_{1/4}$	[—87.6]	[+11.5]
[23ʰ 18ᵐ]	Sept. 13	....	...	—5.69	....	...	—23.8	—87.5	+11.4
Greenwich,	Oct. 10	—5.61	9	-5.65_{3}	—24.5	9	-23.9_{9}		
Cambridge,	Oct. 9	—5.49	5	-5.58_{2}	—23.5	5	-23.7_{5}		
Edinburgh,	Oct. 15	—5.64	12	-5.63_{3}	—23.3	10	-23.2_{10}		
Paris,	Oct. 11	—5.60	4	-5.59_{2}	—23.3	4	-22.7_{4}		
Berlin,	Oct. 27	—5.50	1	$-5.50_{1/4}$	—22.8	1	-21.8_{1}		
Vienna,	Oct. 19	—5.76	5	$-5.76_{1/4}$	—22.5	4	-21.5_{1}	[—85.8]	[+11.4]
[23ʰ 14ᵐ]	Oct. 12	....	...	—5.62	....	...	—23.3	—86.3	+11.2
Greenwich,	Nov. 6	—5.58	7	-5.62_{2}	—23.5	6	-22.9_{6}		
Cambridge,	Nov. 3	—5.34	2	-5.43_{1}	—24.2	1	-24.4_{1}		
Edinburgh,	Nov. 17	—5.41	8	-5.40_{2}	—22.9	1	-22.8_{1}		
Paris,	Nov. 4	—5.52	2	-5.51_{1}	—26.1	2	-25.5_{2}		
Vienna,	Nov. 15	—5.42	6	-5.42_{1}	—23.2	6	-22.2_{2}	[—83.7]	[+11.3]
[23ʰ 12ᵐ]	Nov. 9	....	...	—5.48	....	...	—23.3	—84.4	+10.3
Greenwich,	Dec. 3	—5.36	8	-5.40_{2}	—23.5	9	-22.9_{9}		
Cambridge,	Dec. 3	—5.38	2	-5.47_{1}	—22.5	2	-22.7_{2}		
Edinburgh,	Dec. 15	—5.49	3	-5.48_{1}	—22.6	2	-22.5_{2}		
Vienna,	Dec. 4	—5.50	1					[—82.2]	[+11.0]
[23ʰ 12ᵐ]	Dec. 6	....	...	—5.44	....	...	—22.8	—83.7	+10.4
	1841								
Greenwich,	Aug. 20	—6.16	5	-6.20_{2}	—28.6	5	-28.1_{5}		
Paris,	Aug. 19	—6.14	2	-6.13_{1}	—29.6	2	-29.0_{2}	[—96.9]	[+11.3]
[23ʰ 37ᵐ]	Aug. 20	....	...	—6.18	....	...	—28.3	—96.2	+10.6
Greenwich,	Sept. 11	—6.16	10	-6.20_{4}	—29.0	10	-28.5_{10}		
Königsberg,	Sept. 11	—6.18	5	-6.18_{4}	—30.1	5	-29.1_{2}		
Berlin,	Sept. 14	—6.14	7	-6.14_{2}	—29.6	7	-28.6_{3}		
Edinburgh,	Sept. 17	—6.11	7	-6.15_{2}	—28.2	5	-28.0_{5}		
Paris,	Sept. 14	—6.16	5	-6.15_{3}	—29.0	3	-28.4_{3}		
Vienna,	Sept. 17	—6.37	11	-6.37_{1}	—30.3	11	-29.3_{3}	[—96.5]	[+11.4]
[23ʰ 33ᵐ]	Sept. 13	....	...	—6.18	....	...	—28.5	—96.3	+10.4

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1841	s		s	″		″	″	″
Greenwich,	Oct. 17	−6.09	7	-6.13_2	−28.2	7	-27.7_7		
Berlin,	Oct. 20	−5.87	2	-5.87_1	−28.2	2	-27.2_1		
Edinburgh,	Oct. 13	−6.00	4	-6.04_1	−27.2	4	-27.0_4		
Paris,	Oct. 21	−6.10	1	-6.09_1	−27.2	1	-26.6_1		
Vienna,	Oct. 16	−6.14	9	-6.14_1	−28.8	9	-27.8_9	[−94.6]	[+11.2]
[$23^h\ 28^m$]	Oct. 18	....	...	−6.06	....	...	−27.4	−94.1	+10.5
Greenwich,	Nov. 17	−5.95	5	-5.99_2	−28.0	5	-27.5_5		
Berlin,	Nov. 16	−6.15	2	-6.15_1	−28.4	2	-27.4_1		
Edinburgh,	Nov. 2	−5.89	3	-5.93_1	−26.5	4	-26.3_4		
Vienna,	Nov. 11	−5.98	4	-5.98_1	−30.0	4	-29.0_1	[−92.6]	[+11.0]
[$23^h\ 26^m$]	Nov. 13	....	...	−6.01	....	...	−27.2	−93.3	+10.4
Greenwich,	Dec. 14	−5.86	8	-5.90_2	−27.4	8	-26.9_8		
Berlin,	Dec. 15	−5.60	3	-5.60_1	−25.5	3	-24.5_2		
Edinburgh,	Dec. 20	−5.68	4	-5.73_1					
Paris,	Dec. 15	−5.78	4	-5.77_2	−26.7	4	-26.1_4	[−90.5]	[+10.7]
[$23^h\ 26^m$]	Dec. 16	....	...	−5.78	....	...	−26.3	−89.9	+ 9.8
	1842								
Greenwich,	Aug. 19	−6.62	5	-6.64_1	−32.1	6	-31.7_6		
Cambridge,	Aug. 24	−6.48	2	-6.55_1	−32.7	2	-33.0_2	[−105.6]	[+11.1]
[$23^h\ 51^m$]	Aug. 21	....	...	−6.61	....	...	−32.0	−103.6	+10.0
Greenwich,	Sept. 12	−6.55	4	-6.57_1	−32.2	4	-31.8_4		
Königsberg,	Sept. 18	−6.69	10	-6.69_7	−33.4	10	-32.4_5		
Berlin,	Sept. 11	−6.70	3	-6.69_1	−31.7	3	-30.7_1		
Paris,	Sept. 14	−6.65	5	-6.64_3	−32.1	5	-31.5_5		
Cambridge,	Sept. 14	−6.60	13	-6.67_6	−31.9	14	-32.2_{14}		
Edinburgh,	Sept. 23	−6.57	4	-6.59_1	−30.9	4	-30.6_2		
Pulkowa,	Sept. 17	−6.63	6	-6.64_5					
Vienna,	Sept. 19	−6.81	2	$-6.80_{\frac{1}{2}}$	−33.5	2	$-32.5_{\frac{1}{2}}$	[−105.4]	[+11.2]
[$23^h\ 47^m$]	Sept. 16	....	...	−6.66	....	...	−32.0	−104.3	+10.3
Greenwich,	Oct. 20	−6.66	12	-6.68_3	−31.8	12	-31.4_{12}		
Berlin,	Oct. 23	−6.54	4	-6.53_1	−30.7	4	-29.7_2		
Paris,	Oct. 17	−6.64	12	-6.63_6	−31.5	9	-30.9_9		
Cambridge,	Oct. 17	−6.60	11	-6.67_5	−31.4	10	-31.7_{10}		
Edinburgh,	Oct. 15	−6.57	12	-6.59_3	−31.1	8	-30.8_8		
Vienna,	Oct. 10	−6.74	4	$-6.73_{\frac{1}{2}}$	−34.6	5	-33.6_1	[−103.6]	[+11.0]
[$23^h\ 44^m$]	Oct. 17	....	...	−6.64	....	...	−31.2	−103.7	+10.9
Greenwich,	Nov. 23	−6.36	7	-6.38_3	−31.0	7	-30.6_7		
Berlin,	Nov. 8	−6.42	2	$-6.41_{\frac{1}{2}}$	−30.2	2	-29.2_1		
Cambridge,	Nov. 17	−6.40	7	-6.47_3	−30.7	7	-31.0_7		
Edinburgh,	Nov. 16	−6.39	3	-6.32_1					
Vienna,	Nov. 20	−6.39	3	$-6.38_{\frac{1}{2}}$	−30.0	2	$-29.0_{\frac{1}{2}}$	[−100.8]	[+10.8]
[$23^h\ 42^m$]	Nov. 19	....	...	−6.41	....	...	−30.6	−100.2	+10.0

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1842	s		s	″		″	″	″
Greenwich,	Dec. 16	—6.31	8	-6.33_{3}	—30.2	8	-29.8_{8}		
Berlin,	Dec. 10	—6.16	4	-6.15_{1}	—31.4	4	-30.4_{4}		
Paris,	Dec. 14	—6.23	5	-6.22_{3}	—30.9	5	-30.3_{5}		
Cambridge,	Dec. 16	—6.22	9	-6.29_{4}	—30.1	8	-30.4_{8}		
Edinburgh,	Dec. 13	—6.25	2	-6.18_{2}					
Pulkowa,	Dec. 13	—6.28	3	-6.28_{2}				[— 98.9]	[+10.5]
[23^h 41^m]	Dec. 15	....	...	—6.27	....	...	—30.2	— 98.2	+ 9.6
	1843							[— 97.5]	[+10.3]
Edinburgh,	Jan. 9	—6.32	7	—6.25	—31.4	4	—31.0	— 98.2	+ 8.6
Greenwich,	Aug. 20	—7.11	7	-7.04_{3}	—35.1	7	-34.7_{7}		
Paris,	Aug. 21	—7.20	3	-7.19_{2}	—37.5	5	-37.1_{5}	[—114.3]	[+10.7]
[0^h 6^m]	Aug. 20	....	...	—7.10	.....	...	—35.7	—111.9	+ 9.6
Greenwich,	Sept. 17	—7.23	9	-7.16_{3}	—35.4	9	-35.0_{3}		
Paris,	Sept. 15	—7.17	11	-7.16_{6}	—36.8	14	-36.2_{14}		
Edinburgh,	Sept. 18	—7.26	7	-7.19_{2}	—36.8	5	-36.4_{5}		
Pulkowa,	Sept. 22	—7.18	10	-7.18_{10}				[—114.1]	[+10.6]
[0^h 3^m]	Sept. 19	....	...	—7.17	....	...	—35.8	—112.9	+10.0
Greenwich,	Oct. 18	—7.10	10	-7.03_{3}	—35.3	10	-34.9_{10}		
Paris,	Oct. 17	—7.07	6	-7.06_{3}	—37.1	5	-36.5_{5}		
Edinburgh,	Oct. 17	—7.05	12	-6.98_{3}	—35.1	5	-34.7_{5}	[—112.6]	[+10.5]
[23^h 59^m]	Oct. 17	....	...	—7.02	....	...	—35.2	—110.6	+ 9.6
Greenwich,	Nov. 20	—7.01	9	-6.94_{3}	—34.3	9	-33.9_{9}		
Königsberg,	Nov. 11	—6.84	5	-6.83_{4}	—34.4	5	-33.4_{2}		
Paris,	Nov. 15	—7.03	3	-7.02_{2}	—36.3	3	-35.7_{3}		
Edinburgh,	Nov. 15	—6.93	8	-6.86_{2}	—35.7	6	-35.3_{6}	[—110.1]	[+10.4]
[23^h 56^m]	Nov. 15	....	...	—6.90	....	...	—34.5	—108.6	+ 9.5
	1844								
Greenwich,	Jan. 3	—6.70	3	-6.64_{1}	—34.1	3	-33.7_{3}		
Edinburgh,	Jan. 10	—6.67	6	-6.60_{2}					
Paris,	Jan. 3	—6.72	2	-6.71_{1}	—35.5	2	-34.9_{2}	[—106.1]	[+ 9.9]
[23^h 56^m]	Jan. 7	....	...	—6.64	....	...	—34.2	—105.0	+ 8.2
Greenwich,	Aug. 19	—7.73	10	-7.67_{3}	—39.6	10	-39.2_{10}		
Paris,	Aug. 17	—7.70	3	-7.69_{2}	—40.0	3	-39.4_{3}	[—122.3]	[+10.2]
[0^h 22^m]	Aug. 18	....	...	—7.68	....	...	—39.2	—121.2	+ 9.6
Greenwich,	Sept. 25	—7.74	11	-7.68_{4}	—39.9	11	-39.5_{11}		
Edinburgh,	Sept. 19	—7.67	11	-7.60_{3}	—40.3	10	-39.8_{10}		
Königsberg,	Sept. 17	—7.65	10	-7.64_{7}	—40.7	10	-39.7_{5}		
Paris,	Sept. 10	—7.63	10	-7.62_{5}	—39.4	15	-38.8_{15}	[—122.6]	[+10.3]
[0^h 18^m]	Sept. 17	....	...	—7.63	....	...	—39.3	—120.6	+ 9.3

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1844	s		s	″		″	″	″
Greenwich,	Oct. 17	-7.70	9	-7.64_3	-39.2	9	-38.8_9		
Edinburgh,	Oct. 13	-7.58	10	-7.51_2	-40.5	1	-40.0_1	[-121.5]	[$+10.2$]
[$0^h\ 15^m$]	Oct. 15	....	...	-7.59	....	..	-39.0	-120.0	$+9.4$
Greenwich,	Nov. 26	-7.44	9	-7.38_3	-38.7	9	-38.3_9		
Edinburgh,	Nov. 19	-7.41	7	-7.35_2				[-118.2]	[$+9.9$]
[$0^h\ 12^m$]	Nov. 23	....	...	-7.37	....		-38.3	-116.6	$+8.8$
Edinburgh,	Dec. 18	-7.24	5	-7.18_1					
Paris,	Dec. 22	-7.10	2	-7.09_1	-39.3	3	-38.7_3	[-115.7]	[$+9.6$]
[$0^h\ 10^m$]	Dec. 20	....	...	-7.14	....	..	-38.7	-113.7	$+7.1$
	1845								
Greenwich,	Jan. 13	-7.18	1	-7.10_1	-38.4	1	-38.0		
Edinburgh,	Jan. 14	-7.16	8	-7.08_4				[-114.0]	[$+9.5$]
[$0^h\ 11^m$]	Jan. 14	....	...	-7.08	....	..	-38.0	-112.5	$+7.3$
Greenwich,	Aug. 25	-8.25	5	-8.17_2	-42.7	6	-42.3	[-131.0]	[$+9.6$]
								-129.1	$+9.2$
Greenwich,	Sept. 18	-8.21	10	-8.13_4	-42.6	10	-42.2_{10}		
Königsberg,	Sept. 30	-8.16	5	-8.15_4	-43.4	5	-42.4_2		
Paris,	Sept. 14	-8.24	5	-8.23_3	-43.9	3	-43.3_3	[$-131\ 6$]	[$+9.9$]
[$0^h\ 33^m$]	Sept. 22	....	...	-8.17	....	..	-42.5	-129.2	$+9.1$
Greenwich,	Oct. 17	-8.22	10	-8.14_4	-42.2	10	-41.8_{10}		
Paris,	Oct. 20	-8.01	6	-8.00_3	-43.7	6	-43.1_6	[-130.2]	[$+9.8$]
[$0^h\ 29^m$]	Oct. 18	....	...	-8.08	....	..	-42.3	-128.0	$+8.9$
Greenwich,	Nov. 9	-8.06	6	-7.98	-43.3	7	-42.9	[-128.4]	[$+9.6$]
[$0^h\ 27^m$]	......	....	...	....	....	..	....	-126.9	[$+7.9$
Greenwich,	Dec. 14	-7.85	8	-7.77	-49.2	8	-39.8	[-124.8]	[$+9.3$]
[$0^h\ 25^m$]	......	....	...	....	....	..	....	-122.8	$+9.6$
	1846								
Greenwich,	Sept. 8	-8.80	8	-8.76_3	-46.2	8	-45.8_8		
Paris,	Sept. 12	-8.73	19	-8.72_{10}	-46.9	14	-46.3_{14}	[-139.9]	[$+9.2$]
[$0^h\ 50^m$]	Sept. 11	....	...	-8.73	....	..	-46.1	-138.2	$+8.3$
Greenwich,	Oct. 8	-8.74	7	-8.70_3	-46.8	7	-46.4_7		
Königsberg,	Oct. 4	-8.61	11	-8.61_8	-44.8	11	-43.8_6		
Paris,	Oct. 14	-8.69	13	-8.68_7	-46.7	13	-46.1_{13}	[-139.7]	[$+9.2$]
[$0^h\ 46^m$]	Oct. 9	....	...	-8.65	....	..	-45.8	-137.1	$+8.3$
Greenwich,	Nov. 10	-8.62	6	-8.58_2	-46.8	6	-46.4_6		
Paris,	Nov. 16	-8.48	9	-8.47_5	-46.4	9	-45.8_9	[-136.8]	[$+9.0$]
[$0^h\ 41^m$]	Nov. 14	....	...	-8.50	....	..	-46.0	-135.1	$+7.6$

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
		s		s	″		″	″	″
	1846								
Greenwich,	Dec. 15	—8.31	9	-8.27_{3}	—46.0	10	-45.6_{10}		
Paris,	Dec. 16	—8.25	6	-8.24_{3}	—46.1	4	-45.5_{4}	[—134.5]	[+8.9]
[0^h 39^m]	Dec. 16	....	...	—8.26	....	..	—45.6	—131.7	+6.5
	1847								
Greenwich,	Jan. 13	—8.25	4	-8.20_{2}	—43.1	4	-42.7_{4}		
Paris,	Jan. 11	—8.14	6	-8.12_{3}	—45.4	6	-44.8_{6}	[—130.9]	[+8.6]
[0^h 40^m]	Jan. 12	....	...	—8.15	....	..	—44.0	—129.6	+7.4
Greenwich,	Sept. 3	—9.21	8	—9.16	—49.4	8	—49.0	[—148.4]	[+8.7]
[1^h 6^m]	......	....	...	....	....	..	....	—144.9	+6.8
Greenwich,	Oct. 1	—9.25	6	-9.20_{3}	—49.0	6	-48.6_{6}		
Paris,	Oct. 12	—9.23	18	-9.21_{10}	—49.2	16	-48.6_{16}	[—148.0]	[+8.7]
[1^h 2^m]	Oct. 10	....	...	—9.21	....	..	—48.6	—145.5	+7.8
Greenwich,	Nov. 3	—9.10	9	-9.05_{3}	—49.2	9	-48.8_{9}		
Paris,	Nov. 12	—9.08	10	-9.06_{5}	—49.3	9	-48.7_{9}	[—146.2]	[+8.5]
[0^h 57^m]	Nov. 8	....	...	—9.06	....	..	—48.8	—142.6	+7.2
Greenwich,	Dec. 3	—8.96	7	-8.91_{2}	—49.0	7	-48.6_{7}		
Paris,	Dec. 12	—8.84	9	-8.82_{5}	—48.5	7	-47.9_{7}	[—143.0]	[+8.3]
[0^h 54^m]	Dec. 9	....	...	—8.85	....	..	—48.2	—140.6	+6.8
	1848								
Greenwich,	Jan. 10	—8.83	2	-8.78_{2}	—47.1	2	-46.7_{2}		
Paris,	Jan. 11	—8.53	5	-8.51_{5}	—47.6	5	-47.0_{5}	[—139.4]	[+8.1]
[0^h 54^m]	Jan. 11	....	...	—8.57	....	..	—46.9	—136.2	+6.0
Greenwich,	Sept. 8	—9.84	7	-9.79_{7}	—51.1	7	-50.7_{7}		
Paris,	Sept. 22	—9.82	11	-9.80_{11}	—51.5	9	-50.9_{9}	[—156.4]	[+8.0]
[1^h 19^m]	Sept. 17	....	...	—9.80	....	..	—50.8	—154.1	+7.5
Greenwich,	Oct. 19	—9.82	7	-9.77_{7}	—52.2	7	-51.8_{7}		
Paris,	Oct. 16	—9.79	11	-9.77_{11}	—52.2	10	-51.6_{10}	[—156.4]	[+8.0]
[1^h 15^m]	Oct. 17	....	...	—9.77	....	..	—51.7	—154.2	+6.9
Greenwich,	Nov. 13	—9.68	7	-9.63_{7}	—52.3	7	-51.9_{7}		
Paris,	Nov. 13	—9.55	4	-9.53_{4}	—52.2	3	-51.6_{3}	[—154.6]	[+8.0]
[1^h 11^m]	Nov. 13	....	...	—9.59	....	..	—51.8	—151.8	+6.3
Greenwich,	Dec. 14	—9.41	5	-9.36_{5}	—51.2	5	-50.8_{5}		
Paris,	Dec. 9	—9.43	5	-9.41_{5}	—51.3	6	-50.7_{6}		
[1^h 9^m]	Dec. 12	....	...	—9.38	....	..	—50.7	[—150.2]	[+7.7]
	1849							—148.6	+6.2
Greenwich,	Jan. 6	—9.37	1	-9.37_{1}	—51.2	1	-50.8_{1}		
Paris,	Jan. 18	—9.11	1	-9.09_{1}	—50.0	1	-49.4_{1}	[—147.7]	[+7.5]
[1^h 9^m]	Jan. 12	....	...	—9.23	....	..	—50.1	—146.2	+6.0

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1849	s		s	″		″	″	″
Greenwich,	Sept. 16	—10.27	3	—10.27	—53.3	3	—52.9	[+165.1]	[+7.3]
[1h 34m]	......	....	...	....	....	..	....	+160.8	+6.2
Greenwich,	Oct. 28	—10.24	6	—10.24$_6$	—53.9	6	—53.5$_6$		
Paris,	Oct. 21	—10.26	5	—10.24$_5$	—54.8	5	—54.2$_5$	[—164.4]	[+7.4]
[1h 30m]	Oct. 25	....	...	—10.24	....	..	—53.8	—161.0	+5.8
Greenwich,	Nov. 16	—10.12	3	—10.12$_3$	—54.4	3	—54.0$_3$		
Paris,	Nov. 17	—10.22	5	—10.20$_5$	—54.1	4	—53.5$_4$	[—162.5]	[+7.3]
[1h 26m]	Nov. 17	....	...	—10.17	....	..	—53.7	—160.1	+6.0
Greenwich,	Dec. 13	—9.95	6	— 9.95$_6$	—53.1	9	—52.7$_9$		
Paris,	Dec. 16	—9.88	2	— 9.86$_2$	—54.4	2	—53.8$_2$	[—160.4]	[+7.1]
[1h 24m]	Dec. 14	....	...	— 9.93	....	..	—52.9	—156.6	+5.7
	1850								
Greenwich,	Jan. 7	—9.71	1	— 9.71$_1$	—53.0	1	—52.6$_1$		
Paris,	Jan. 5	—9.77	2	— 9.75$_2$	—53.0	3	—52.4$_3$	[—156.8]	[+6.9]
[1h 24m]	Jan. 6	....	...	— 9.74	....	..	—52.5	—153.8	+6.2
Greenwich,	Sept. 6	—10.83	9	—10.83	—53.9	9	—53.5	[—171.7]	[+6.5]
[1h 52m]	......	....	...	....	....	..	....	—168.1	+5.9
Greenwich,	Oct. 11	—10.92	8	—10.92$_8$	—55.3	7	—54.9$_7$		
Paris,	Oct. 17	—19.87	7	—10.85$_7$	—56.3	5	—55.7	[—173.3]	[+6.4]
[1h 47m]	Oct. 16	....	...	—10.87	....	..	—55.3	—169.6	+5.3
Greenwich,	Nov. 6	—10.79	5	—10.79$_5$	—55.7	6	—55.3$_6$		
Paris,	Nov. 10	—10.89	3	—10.87$_3$	—56.1	4	—55.5$_4$	[—172.4]	[+6.4]
[1h 43m]	Nov. 8	....	...	—10.82	....	..	—55.4	—169.1	+5.6
Greenwich,	Dec. 7	—10.56	8	—10.56$_8$	—55.0	9	—54.6$_9$		
Königsberg,	Dec. 15	—10.50	2	—10.48$_2$	—56.0	2	....	[—169.0]	[+6.3]
[1h 41m]	Dec. 9	....	...	—10.54	....	..	—54.6	—165.0	+5.2
	1851								
Greenwich,	Jan. 19	—10.21	4	—10.21	—54.8	4	—54.4	[—163.5]	[+6.0]
[1h 41m]	......	....	...	....	....	..	....	—160.4	+3.5
Greenwich,	Sept. 11	—11.42	2	—11.42$_2$	—55.7	2	—55.3$_2$		
Königsberg,	Sept. 19	—11.41	2	—11.41$_1$	....	..	....		
Paris,	Sept. 14	—11.59	1	—11.57$_1$	—57.5	1	—56.9$_1$	[—180.0]	[+5.7]
[2h 7m]	Sept. 14	....	...	—11.46	....	..	—55.8	—176.9	+4.4
Königsberg,	Oct. 22	—11.75	3	—11.75$_2$	—58.4	3	—57.4		
Paris,	Oct. 22	—11.46	3	—11.44$_3$	....	..	....	[—181.8]	[+5.8]
[2h 2m]	Oct. 23	....	...	—11.56	....	..	—57.4	—179.1	+4.3

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1851	s		s	"		"	"	"
Greenwich,	Nov. 2	—11.50	5	-11.50_5	—57.5	5	-57.1_5		
Paris,	Nov. 2	—11.48	2	-11.46_2	—58.1	1	-57.5_1	[—181.4]	[+5.7]
[$2^h\ 0^m$]	Nov. 2	....	...	—11.49	....	..	—57.2	—178.1	+4.5
Greenwich,	Nov. 22	—11.43	5	-11.43_5	—58.3	5	-57.9_5		
Paris,	Nov. 14	—11.39	1	-11.37_1	....	..	....	[—179.9]	[+5.6]
[$1^h\ 58^m$]	Nov. 21	....	...	—11.42	....	..	-57.9_5	—177.6	+3.8
Greenwich,	Dec. 22	—11.15	3	-11.15_3	—57.2	3	-56.8_3		
Paris,	Dec. 25	—11.00	3	-10.98_3	—57.2	3	-56.6_3	[—175.7]	[+5.3]
[$1^h\ 54^m$]	Dec. 24	....	....	—11.06	....	..	—56.7	—172.3	+3.8
	1852								
Greenwich,	Jan. 11	—10.85	7	-10.85_7	—57.2	7	-56.8_7		
Paris,	Jan. 14	—10.89	5	-10.87_5	—56.5	5	-55.9_5	[—173.1]	[+5.2]
[$1^h\ 54^m$]	Jan. 12	....	...	—10.86	....	..	—56.4	—169.4	+3.1
Greenwich,	Sept. 12	—12.06	7	-12.06_7	—56.5	7	—56.1	[—187.4]	[+4.9]
[$2^h\ 22^m$]	......	....	...	....	....	..	....	—184.5	+3.9
Greenwich,	Oct. 17	—12.23	8	-12.23_8	—58.6	8	-58.2_8		
Paris,	Oct. 23	—12.06	3	-12.04_3	—59.6	3	-59.0_3	[—189.6]	[+4.7]
[$2^h\ 18^m$]	Oct. 19	....	...	—12.18	....	..	—58.4	—187.2	+3.1
Greenwich,	Nov. 13	—12.00	5	-12.00_5	—59.7	5	-59.3_5		
Paris,	Nov. 15	—12.08	5	-12.06_5	—60.2	4	-59.6_4	[—188.9]	[+4.7]
[$2^h\ 14^m$]	Nov. 14	....	...	—12.03	....	..	—59.4	—185.6	+2.4
Greenwich,	Dec. 19	—11.72	7	-11.72_7	—58.7	7	-58.3_7		
Königsberg,	Dec. 16	—11.67	3	....	—59.3	3	-58.3_3		
Paris,	Dec. 19	—11.77	5	-11.75_5	—59.0	5	-58.4_5	[—184.8]	[+4.6]
[$2^h\ 10^m$]	Dec. 19	....	...	—11.73	....	..	—58.3	—181.3	+2.8
	1853								
Greenwich,	Jan. 12	—11.42	7	-11.42_7	—57.5	7	—57.1		
Paris,	Jan. 15	—11.46	4	-11.44_4	....	..	....	[—181.0]	[+4.4]
[$2^h\ 9^m$]	Jan. 13	....	...	—11.43	....	..	—57.1	—176.8	+2.6
Greenwich,	Sept. 17	—12.69	4	-12.69_4	—56.7	4	-56.3_4		
Paris,	Sept. 16	—12.58	2	-12.56_2	—57.0	2	-56.4_2	[—194.6]	[+4.0]
[$2^h\ 39^m$]	Sept. 17	....	...	—12.65	....	..	—56.3	—191.6	+2.5
Greenwich,	Oct. 15	—12.69	7	-12.69_7	—58.8	7	-58.4_7		
Paris,	Oct. 27	—12.66	6	-12.64_6	—59.0	6	-58.4_6	[—196.7]	[+3.8]
[$2^h\ 34^m$]	Oct. 22	....	...	—12.66	....	..	—58.4	—192.7	+1.7

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1853	s		s	"		"	"	"
Greenwich,	Nov. 12	—12.65	14	-12.65_{14}	—59.4	14	-59.0_{14}		
Paris,	Nov. 15	—12.67	10	-12.65_{10}	—59.2	8	-58.6_{8}	[—196.0]	[+3.7]
[2^h 31^m]	Nov. 14	....	...	—12.65	....	..	—58.9	—192.9	+1.9
Paris,	Dec. 12	—12.28	1	—12.26	—59.0	1	—58.4	[—193.4]	[+3.6]
[2^h 27^m]	......	....	...	....	....	..	....	—187.9	+1.5
	1854								
Greenwich,	Jan. 13	—12.02	4	-12.02_{4}	—59.7	4	-59.2_{4}		
Paris,	Jan. 20	—11.87	2	-11.85_{2}	—58.1	1	-57.9_{1}	[—188.1]	[+3.5]
[2^h 25^m]	Jan. 15	....	...	—11.96	....	..	—58.9	—183.9	+0.1
Paris,	Feb. 8	—11.89	1	—11.87	—57.8	1	—57.6	[—184.5]	[+3.5]
[2^h 26^m]	......	....	...	....	....	..	....	—182.1	+0.7
Greenwich,	Sept. 21	—13.12	6	—13.12	—56.0	6	—55.5	[—201.3]	[+3.0]
[2^h 53^m]	......	....	...	....	....	..	....	—196.9	+1.7
Greenwich,	Oct. 26	—13.24	7	-13.24_{7}	—58.0	7	-57.5_{2}		
Paris,	Oct. 29	—13.12	1	-13.10_{1}	—57.2	3	-57.0_{1}	[—203.7]	[+2.8]
[2^h 50^m]	Oct. 27		...	—13.22	....	..	—57.3	—199.0	+1.2
Greenwich,	Nov. 12	—13.22	5	-13.22_{5}	—58.2	5	-57.5_{5}		
Paris,	Nov. 16	—13.21	6	-13.19_{6}	—58.7	1	-58.5_{1}		
Santiago,	Nov. 17	—13.20	4	-13.20_{3}	—58.7	3	-58.1_{2}	[—203.0]	[+2.7]
[2^h 48^m]	Nov. 15	....	...	—13.20	....	..	—57.8	—199.1	+1.2
Greenwich	Dec. 14	—12.93	7	-12.93_{7}	—58.6	7	-58.1_{2}		
Paris,	Dec. 9	—13.14	1	-13.12_{1}	—56.9	4	-56.7_{1}		
Santiago,	Dec. 15	—12.85	10	-12.85_{7}	—59.7	9	-59.1_{1}	[—200.1]	[+2.6]
[2^h 44^m]	Dec. 14	....	...	—12.90	....	..	—58.0	—195.3	+0.7
	1855								
Greenwich,	Jan. 18	—12.55	2	-12.55_{2}	—56.9	2	—56.4	[—194.6]	[+2.5]
[2^h 42^m]	......	....	...	....	....	..	....	—190.1	+1.1
Greenwich,	Oct. 13	—13.71	6	-13.71_{6}	—55.7	6	-55.2_{3}		
Paris,	Oct. 25	—13.93	3	-13.91_{3}	—55.8	2	-55.6_{1}	[—209.2]	[+1.8]
[3^h 9^m]	Oct. 17	....	...	—13.78	....	..	—55.3	—204.9	+0.1
Greenwich,	Nov. 11	—13.85	4	-13.85_{4}	—57.6	5	-57.1_{5}		
Paris,	Nov. 17	—13.62	4	-13.60_{4}	—56.6	4	-56.4_{4}		
Santiago,	Nov. 16	—13.82	5	-13.82_{4}	—57.2	5	-56.6_{4}	[—209.4]	[+1.7]
[3^h 5^m]	Nov. 15	....	...	—13.76	....	..	—56.7	—205.3	—0.2
Greenwich,	Dec. 18	—13.48	10	-13.48_{10}	—57.3	10	-56.8_{3}		
Paris,	Dec. 16	—13.52	2	-13.50_{2}	—56.7	3	-56.5_{1}		
Santiago,	Dec. 18	—13.53	6	-13.53_{4}	—58.1	6	-57.5_{2}	[—206.2]	[+1.6]
[3^h 0^m]	Dec. 18	....	...	—13.50	....	..	—57.0	—202.1	—0.1

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No of obs.	Corrected mean.	Longitude.	Latitude.
	1856	s		s	"		"	"	"
Greenwich,	Jan. 22	—12.97	6	-12.97_6	—56.1	6	-55.6_3		
Paris,	Feb. 10	—12.87	2	-12.85_2	—54.8	2	-54.6_1	[—199.5]	[+1.4]
[2^h 58^m]	Jan. 27	....	...	—12.94	....	..	—55.4	—194.7	—0.5
Greenwich,	Oct. 16	—14.36	4	-14.36_4	—53.8	4	—53.3		
Paris,	Oct. 16	—14.38	2	-14.36_3				[—213.8]	[+0.7]
[3^h 27^m]	Oct. 16	....	...	—14.36	....	..	—53.3	—211.0	—1.4
Greenwich,	Nov. 18	—14.23	8	-14.23_8	—54.8	8	-54.3_8		
Paris,	Nov. 17	—14.32	6	-14.30_6	—54.0	7	-53.8_7	[—214.7]	[+0.7]
[3^h 22^m]	Nov. 18	....	...	—14.26	....	..	—54.1	—210.2	—0.9
Greenwich,	Dec. 18	—13.92	5	-13.93_5	—55.9	5	-55.4_5		
Paris,	Dec. 19	—14.11	7	-14.09_7	—55.0	6	-54.8_6	[—211.6]	[+0.6]
[3^h 17^m]	Dec. 19	....	...	—14.03	....	..	—55.1	—207.7	—1.2
	1857								
Greenwich,	Jan. 21	—13.59	10	-13.60_{10}	—53.4	8	-52.9_3		
Paris,	Jan. 12	—13.87	3	-13.85_3	—55.3	3	-55.1_1	[—206.4]	[+0.5]
[3^h 15^m]	Jan. 19	....	...	—13.66	....	..	—53.5	—202.3	—0.4
Greenwich,	Oct. 8	—14.68	4	—14.69	—49.1	4	—48.6	[—216.8]	[—0.3]
[3^h 46^m]	......	....	...	....	....	..	....	—213.0	—1.9
Greenwich,	Nov. 6	—14.80	4	-14.81_4	—51.3	5	-50.8_5		
Paris,	Nov. 10	—14.83	9	-14.81_9	—51.5	8	-51.3_8		
Königsberg,	Nov. 19	—14.77	3	-14.75_4	—52.7	3	-51.5_2	[—218.9]	[—0.4]
[3^h 41^m]	Nov. 11	....	...	—14.80	....	..	—51.2	—215.5	—2.4
Greenwich,	Dec. 11	—14.55	6	-14.56_6	—53.2	5	-52.7_5		
Paris,	Dec. 16	—14.48	7	-14.46_7	—52.8	6	-52.6_6		
Königsberg,	Dec. 8	—14.56	1	-14.54_2	—53.7	1	-52.5_1	[—217.1]	[—0.5]
[3^h 36^m]	Dec. 13		...	—14.51	....	..	—52.6	—212.2	—3.0
	1858								
Greenwich,	Jan. 17	—14.13	13	-14.14_{13}	—52.8	13	-52.3_{13}		
Paris,	Jan. 16	—14.19	8	-14.17_8	—52.8	9	-52.6_9	[—211.8]	[—0.6]
[3^h $32^m.2$]	Jan. 17	....	...	—14.15	....	..	—52.4	—207.5	—2.8
Greenwich,	Feb. 11	—13.81	10	-13.82_{10}	—52.1	10	-51.6_{10}		
Paris,	Feb. 12	—13.91	1	-13.89_1	—52.7	1	-52.5_1	[—207.0]	[—0.6]
[3^h $31^m.9$]	Feb. 11	....	...	—13.83	....	..	—51.7	—203.1	—3.1
Greenwich,	Oct. 17	—15.06	5	—15.06	—46.4	5	—45.9	[—220.6]	[—1.4]
[4^h $3^m.3$]	......	....	...	....	....	..	....	—216.2	—4.0
Greenwich,	Nov. 14	—15.16	10	-15.17_{10}	—48.2	11	-47.7_{11}		
Paris,	Nov. 20	—15.19	9	-15.17_9	—47.8	10	-47.6_{10}	[—222.1]	[—1.5]
[3^h $58^m.5$]	Nov. 17	....	...	—15.17	....	..	—47.7	—218.6	—3.8

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Latitude.	Longitude.
	1858	s		s	″		″	″	″
Greenwich,	Dec. 16	—14.96	6	-14.97_6	—49.2	6	-48.7_2		
Paris,	Dec. 16	—14.78	5	-14.76_5	—49.5	3	-49.3_1	[—220.7]	[—1.6]
$[3^h\ 53^m.6]$	Dec. 16	....	...	—14.87	....	..	—48.9	—215.1	—4.1
	1859								
Greenwich,	Jan. 19	—14.56	8	-14.57_8	—48.9	8	-48.4_2		
Paris,	Jan. 10	—14.72	4	-14.70_4	—48.2	4	-48.0_1	[—216.1]	[—1.5]
$[3^h\ 50^m.0]$	Jan. 16	....	...	—14.61	....	..	—48.3	—211.5	—3.1
Greenwich,	Feb. 18	—14.18	5	-14.19_5	—48.1	5	-47.6_5		
Paris,	Feb. 11	—14.37	6	-14.35_6	—48.3	4	-48.1_4	[—210.9]	[—1.6]
$[3^h\ 49^m.3]$	Feb. 14	..	...	—14.28	....	..	—47.8	—207.1	—3.4
Greenwich,	Oct. 25	—15.43	6	—15.44	—42.6	6	—42.1	[—223.2]	[—2.2]
$[4^h\ 20^m.8]$	......	....	...	....	....	..	....	—219.6	—5.5
Greenwich,	Nov. 18	—15.56	6	-15.57_6	—43.3	5	-42.8_5		
Paris,	Nov. 17	—15.61	8	-15.59_8	—43.0	9	-42.8_9	[—224.7]	[—2.2]
$[4^h\ 17^m.2]$	Nov. 17	....	...	—15.58	....	..	—42.8	—221.9	—4.6
Greenwich,	Dec. 16	—15.43	9	-15.44_9	—45.2	8	-44.7_8		
Paris,	Dec. 11	—15.46	5	-15.44_5	—44.5	5	-44.3_5	[—223.9]	[—2.3]
$[4^h\ 12^m.5]$	Dec. 14	....	...	—15.44	....	..	—44.5	—220.7	—4.9
	1860								
Greenwich,	Jan. 16	—15.08	8	-15.09_8	—45.8	8	-45.3_8		
Paris,	Jan. 15	—15.08	8	-15.06_8	—44.3	5	-44.1_5	[—219.6]	[—2.3]
$[4^h\ 8^m.0]$	Jan. 16	....	...	—15.08	....	..	—44.8	—216.1	—4.5
Greenwich,	Feb. 17	—14.64	12	-14.65_{12}	—45.0	12	—44.5		
Paris,	Feb. 6	—14.79	2	-14.77_2				[—213.9]	[—2.3]
$[4^h\ 7^m.0]$	Feb. 15	....	...	—14.67	....	..	—44.5	—210.4	—5.0
Greenwich,	Oct. 13	—15.52	2	—15.52	—35.2	3	—34.7	[—223.0]	[—2.9]
$[4^h\ 40^m.9]$	......	....	...	....	....	..	....	—218.3	—5.4
Greenwich,	Nov. 15	—15.88	9	-15.89_9	—38.3	9	-37.8_2		
Paris,	Nov. 22	—15.79	5	-15.77_5	—37.3	5	-37.1_1	[—226.6]	[—3.1]
$[4^h\ 35^m.8]$	Nov. 18	....	...	—15.85	....	..	—37.6	—223.5	—5.8
Greenwich,	Dec. 12	—15.77	4	-15.78_4	—41.5	5	-41.0_5		
Paris,	Dec. 12	—15.79	3	-15.77_3	—38.5	3	-38.3_3	[—226.2]	[—3.2]
$[4^h\ 31^m.6]$	Dec. 12	....	...	—15.78	....	..	—40.0	—223.2	—6.8
	1861								
Greenwich,	Jan. 13	—15.45	10	-15.46_{10}	—40.5	10	-40.1_3		
Paris,	Jan. 13	—15.52	6	-15.49_6	—40.2	3	-40.0_1	[—222.4]	[—3.2]
$[4^h\ 26^m.8]$	Jan. 13	....	...	—15.47	....	..	—40.1	—219.3	—5.8

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1861	s		s	"		"	"	"
Greenwich,	Feb. 10	—15.12	2	-15.13_{2}	—40.7	3	—40.3		
Paris,	Feb. 5	—15.23	1	-15.20_{1}				[—217.9]	[—3.1]
[4^h 21^m.0]	Feb. 8	....	...	—15.15	....	..	—40.3	—215.0	—6.0
Greenwich,	Nov. 10	—15.99	9	-16.00_{9}	—30.9	8	—30.5	[—226.2]	[—4.0]
[4^h 56^m.4]	......	....	...	....	....	..	....	—223.4	—6.1
Greenwich,	Dec. 9	—16.01	4	-16.02_{4}	—33.4	4	-33.0_{4}		
Paris,	Dec. 10	—16.04	11	-16.01_{11}	—32.8	13	-32.6_{13}		
Washington,	Dec. 18	—16.10	6	-16.07_{6}				[—227.3]	[—4.0]
[4^h 50^m.8]	Dec. 12	....	...	—16.03	....	..	—32.7	—224.4	—6.4
	1862								
Greenwich,	Jan. 19	—15.67	8	-15.67_{8}	—35.1	9	-34.7_{9}		
Paris,	Jan. 20	—15.72	5	-15.69_{5}	—34.1	5	-33.9_{5}	[—223.2]	[—4.0]
[4^h 44^m.9]	Jan. 19	....	...	—15.68	....	..	—34.4	—220.2	—6.3
Greenwich,	Feb. 22	—15.18	7	-15.18_{7}	—34.3	7	-33.9_{7}		
Washington,	Feb. 19	—15.31	3	-15.31_{3}					
Paris,	Feb. 15	—15.47	5	-15.44_{5}	—34.6	6	-34.4_{6}	[—217.5]	[—4.0]
[4^h 43^m.1]	Feb. 19	....	...	—15.30	....	..	—34.1	—215.1	—6.0
Greenwich,	Nov. 9	—16.13	9	—16.13	—24.5	9	—24.1	[—226.2]	[—4.7]
[5^h 16^m.2]	......	....	...	....	....	..	....	—223.6	—7.1
Greenwich,	Dec. 12	—16.27	7	-16.27_{7}	—27.2	7	-26.8_{7}		
Paris,	Dec. 11	—16.27	5	-16.24_{5}	—26.5	5	-26.3_{5}		
Leyden,	Dec. 8	—16.16	4	-16.13_{3}	—25.2	4	-25.2_{3}	[—227.6]	[—4.7]
[5^h 10^m.7]	Dec. 19	....	...	—16.23	....	..	—26.3	—225.4	—7.2
	1863								
Greenwich,	Jan. 17	—15.96	10	-15.96_{10}	—28.3	10	-27.9_{10}		
Paris,	Jan. 20	—15.94	2	-15.91_{2}	—27.4	1	-27.2_{1}		
Washington,	Jan. 16	—15.92	4	-15.92_{4}					
Leyden,	Jan. 16	—16.03	6	-16.00_{4}	—27.9	6	-27.9_{4}	[—224.3]	[—4.8]
[5^h 4^m.6]	Jan. 17	....	...	—15.96	....	..	—27.9	—222.2	—6.8
Greenwich,	Feb. 15	—15.55	10	-15.55_{10}	—29.2	10	-28.8_{10}		
Paris,	Feb. 15	—15.47	17	-15.44_{17}	—28.6	17	-28.4_{17}		
Washington,	Feb. 9	—15.64	2	-15.64_{2}					
Leyden,	Feb. 15	—15.57	6	-15.54_{4}	—28.5	7	-28.5_{7}	[—219.3]	[—4.8]
[5^h 2^m.2]	Feb. 15	....	...	—15.50	....	..	—28.6	—216.0	—7.2
Greenwich,	Mar. 3	—15.36	2	-15.36_{2}	—29.0	3	-28.6_{3}		
Paris,	Mar. 5	—15.30	4	-15.27_{4}	—28.0	4	-27.8_{4}	[—215.9]	[—4.8]
[5^h 2^m.2]	Mar. 4	....	...	—15.30	....	..	—28.2	—213.3	—7.0

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1863	s		s	″		″	″	″
Paris,	Nov. 29	—16.33	3	-16.30_3	—18.4	3	-18.2_3		
Leyden,	Nov. 14	—16.23	6	-16.20_4	—18.1	6	-18.1_4	[—225.9]	[—5.6]
[5^h 34^m]	Nov. 21	....	...	—16.24	....	..	—18.2	—224.1	—8.3
Greenwich,	Dec. 19	—16.32	3	-16.32_3	—20.0	3	-19.6_3		
Leyden,	Dec. 14	—16.51	2	-16.48_1	—17.1	1	-17.1_1	[—226.8]	[—5.7]
[5^h 29^m]	Dec. 18	....	...	—16.36	....	..	—19.0	—225.9	—7.0
	1864								
Washington,	Jan. 15	—16.01	4	-16.01_{11}		11			
Leyden,	Jan. 10	—16.07	4	-16.04_3	—20.0	4	—20.0	[—224.6]	[—5.8]
[5^h 25^m]	Jan. 14	....	...	—16.02	....	..	....	—221.4	—6.7
Washington,	Feb. 15	—15.74	4	—15.74	—21.5	9		[—219.6]	[—5.7]
[5^h 21^m]	......	....	...	....	....	..	....	—217.8	
Greenwich,	May 3	—15.57	2	—15.57	—22.2	2	-21.8_2	[—216.1]	[—5.6]
[5^h 21^m]	......	....	...	....	....	..	....	—215.6	—7.3
Paris,	Dec. 20	—16.32	4	-16.29_4	—13.2	4	-12.8_4		
Washington,	Dec. 10	—16.22	2	-16.22_2	—11.4	4			
Leyden,	Dec. 18	—16.19	5	-16.16_4	—12.4	5	-12.4_4	[—225.1]	[—6.5]
[5^h 49^m]	Dec. 17	....	...	—16.22	....	..	—12.6	—223.1	—8.4
	1865								
Greenwich,	Jan. 5	—16.28	2	-16.28_2	—13.7	2	-13.3_2		
Paris,	Jan. 15	—16.22	1	-16.19_2	—15.1	2	-14.9_2		
Washington,	Jan. 17	—16.11	9	-16.11_9	—14.4	9			
Leyden,	Jan. 16	—16.03	3	-16.00_3	—14.7	3	-14.7_3	[—223.4]	[—6.5]
[5^h 45^m]	Jan. 15	....	...	—16.13	....	..	—14.3	—220.0	—8.8
Washington,	Feb. 14	—15.77	9	-15.77_9	—15.4	6			
Leyden,	Feb. 13	—15.85	2	-15.82_2	—15.6	2	—15.6	[—218.9]	[—6.4]
[5^h 41^m]	Feb. 14	....	...	—15.78	....	..	....	—217.4	—8.6
Greenwich,	Oct. 10	—15.59	1	-15.59_1	— 2.4	1	-2.0_1		
Washington,	Oct. 18	—15.59	5	-15.59_5				[—215.3]	[—7.0]
[6^h 17^m]	Oct. 17	....	...	—15.59	....	..	— 2.0	—214.0	—8.1
Greenwich,	Dec. 6	—16.13	2	-16.13_2	— 5.2	2	-4.8_2		
Leyden,	Dec. 7	—16.26	2	-16.23_2	— 4.8	2	-4.6_2		
Washington,	Dec. 18	—16.17	8	-16.17_8	— 3.8	5		[—222.2]	[—7.3]
[6^h 11^m]	Dec. 14	....	...	—16.17	....	..	— 4.7	—222.0	—8.8

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1866	s		s	"		"	"	"
Greenwich,	Jan. 10	—16.20	8	-16.20_{8}	— 7.2	9	-6.8_{4}		
Paris,	Jan. 12	—16.04	1	-16.01_{1}	— 6.3	1	-6.1_{1}		
Washington,	Jan. 17	—16.06	9	-16.06_{9}	— 7.6	9	-6.5_{4}		
Leyden,	Jan. 17	—16.02	6	-15.99_{4}	— 7.1	6	-7.1_{3}	[—221.6]	[— 7.3]
[6^h 5^m]	Jan. 14	....	...	—16.09	....	..	— 6.6	—221.0	— 9.0
Greenwich,	Feb. 14	—15.81	10	-15.81_{10}	— 9.1	11	-8.7_{4}		
Washington,	Feb. 14	—15.81	6	-15.81_{6}	— 8.4	8	-7.3_{4}		
Leyden,	Feb. 14	—15.75	8	-15.72_{6}	— 8.6	8	-8.6_{4}	[—217.3]	[— 7.3]
[6^h 0^m]	Feb. 14	....	...	—15.79	....	..	— 8.2	—216.9	— 8.2
Greenwich,	Mar. 12	—15.47	2	-15.47_{2}	— 8.1	2	-7.7_{1}		
Washington,	Mar. 9	—15.49	6	-15.49_{6}	— 9.7	6	-8.6_{2}		
Leyden,	Mar. 3	—15.59	2	-15.56_{2}	— 9.0	2	-9.0_{1}	[—212.9]	[— 7.1]
[6^h 0^m]	Mar. 9	....	...	—15.50	....	..	— 8.5	—212.9	— 8.5
Washington,	Oct. 13	—15.36	5	—15.36	+ 4.4	5	+ 5.5	[—209.6]	[— 7.5]
[6^h 37^m]	......	....	...	....	....	..	....	—211.2	— 7.9
Greenwich,	Nov. 9	—15.65	2	-15.65_{2}	+ 3.5	2	$+3.9_{1}$		
Washington,	Nov. 15	—15.72	10	-15.72_{10}	+ 3.8	10	$+4.9_{3}$	[—215.1]	[— 7.8]
[6^h 35^m]	Nov. 14	....	...	—15.71	....	..	+ 4.6	—215.9	— 8.3
Greenwich,	Dec. 11	—16.01	10	-16.01_{10}	+ 2.8	10	$+3.2_{6}$		
Washington,	Dec. 12	—16.00	7	-16.00_{7}	+ 2.0	7	$+3.1_{4}$		
Leyden,	Dec. 10	—15.87	2	-15.84_{2}	+ 1.6	2	$+1.6_{3}$	[—218.2]	[— 8.0]
[6^h 31^m]	Dec. 11	....	...	—15.99	....	..	+ 2.9	—219.7	— 9.7
	1867								
Greenwich,	Jan. 21	—15.89	5	-15.89_{3}	— 1.4	5	-1.0_{3}		
Washington,	Jan. 21	—15.93	12	-15.93_{12}	— 1.2	12	-0.1_{6}		
Leyden,	Jan. 9	—15.82	4	-15.79_{3}	— 0.3	3	-0.3_{1}	[—217.8]	[— 8.0]
[6^h 24^m]	Jan. 19	....	...	—15.90	....	..	— 0.4	—218.3	— 9.3
Paris,	Feb. 13	—15.72	3	-15.69_{3}	— 2.4	4	-2.0_{3}		
Washington,	Feb. 15	—15.65	9	-15.65_{9}	— 2.0	10	-0.9_{4}		
Leyden,	Feb. 18	—15.54	3	-15.51_{2}	— 0.5	3	-0.5_{3}	[—214.5]	[— 7.9]
[6^h 20^m]	Feb. 15	....	...	—15.64	....	..	— 1.1	—214.7	— 8.6
Greenwich,	Mar. 2	—15.45	7	-15.45_{7}	— 2.9	8	-2.5_{4}		
Washington,	Mar. 11	—15.32	4	-15.32_{4}	— 2.5	2	-1.4_{2}	[—211.1]	[— 7.8]
[6^h 19^m]	Mar. 5	....	...	—15.40	....	..	— 2.1	—211.3	— 9.0
Greenwich,	Dec. 11	—15.61	5	-15.61_{5}	+ 8.5	5	$+8.9_{3}$		
Washington,	Dec. 18	—15.70	3	-15.70_{3}	+ 6.9	3	$+8.0_{2}$	[—213.1]	[— 8.6]
[6^h 51^m]	Dec. 13	....	...	—15.64	....	..	+ 8.5	—215.3	—10.4

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1868	s		s	"		"	"	"
Greenwich,	Jan. 13	—15.67	4	-15.67_4	+ 6.9	4	$+6.7_4$	[—213.4]	[— 8.7]
[6h 45m]	......	....	...	....	....	..	....	—215.5	—10.1
Greenwich,	Feb. 15	—15.43	8	-15.43_8	+ 4.8	8	$+4.6_4$		
Leyden,	Feb. 5	—15.43	3	-15.40_3	+ 5.9	3	$+5.9_3$		
Washington,	Feb. 13	—15.50	7	-15.50_6	+ 4.9	6	$+6.0_3$		
Paris,		—15.41	7	-15.38_4				[—210.3]	[— 8.6]
[6h 41m]	Feb. 14	....	...	—15.44	....	..	+ 5.3	—212.3	— 9.7
Leyden,	Mar. 11	—15.02	2	-14.99_1	+ 4.6	2	$+4.6_2$		
Washington,	Mar. 19	—14.91	1	-14.91_1	+ 3.4	1	$+4.5_1$	[—205.0]	[— 8.5]
[6h 39m]	Mar. 15	....	...	—14.95	....	..	+ 4.6	—205.5	— 9.2
Washington,	Oct. 18	—14.57	6	—14.57	+12.3	5	+13.3	[—198.3]	[— 8.7]
[7h 16m]	......	....	...	....	....	..	....	—201.5	—12.9
Washington,	Nov. 7	—14.83	5	—14.83	+16.2	5	+17.3	[—202.0]	[— 9.1]
[7h 16m]	......	....	...	....	....	..	....	—205.6	— 9.3
Greenwich,	Dec. 25	—15.24	5	-15.24_3	+15.1	5	$+14.9_3$		
Washington,	Dec. 12	—15.16	6	-15.16_6	+14.6	4	$+15.6_4$	[—206.5]	[— 9.2]
[7h 10m]	Dec. 18	....	...	—15.20	....	..	+15.2	—210.3	—10.0
	1869								
Greenwich,	Jan. 19	—15.28	1	-15.28_1	+12.6	1	$+12.4_1$		
Washington,	Jan. 18	—15.31	8	-15.31_8	+12.9	8	$+13.4_8$	[—207.1]	[— 9.3]
[7h 5m]	Jan. 18	....	...	—15.31	....	..	+13.3	—211.4	—10.3
Greenwich,	Feb. 14	—15.04	10	-15.04_{10}	+11.1	10	$+10.9_{10}$		
Washington,	Feb. 14	—15.13	13	-15.13_{12}	+11.2	13	$+11.7_{12}$		
Paris,	Feb. 10	—15.09	10	-15.06_7	+11.7	10	$+12.2_{10}$	[—204.7]	[— 9.2]
[7h 1m]	Feb. 13	....	...	—15.08	....	..	+11.7	—208.1	—10.2
Greenwich,	Mar. 3	—14.81	2	-14.81_2	+10.8	2	$+10.6_2$		
Washington,	Mar. 7	—14.87	2	-14.87_2	+11.4	2	$+11.9_2$	[—201.5]	[— 9.2]
[6h 59m]	Mar. 5	....	...	—14.84	....	..	+11.3	—204.8	— 9.3
Greenwich,	Dec. 8	—14.50	2	-14.50_2	+21.1	2	+20.9	[—197.1]	[— 9.8]
[7h 32m]	......	....	...	....	....	..	....	—202.1	—10.3
	1870								
Greenwich,	Jan. 19	—14.73	10	-14.72_3	+19.0	10	+18.8	[—199.3]	[— 9.9]
[7h 25m]	......	....	...	....	....	..	....	—204.5	—10.8
Greenwich,	Feb. 16	—14.58	7	-14.57_3	+17.2	7	+17.0		
Washington,	Feb. 16	—14.49	1	-14.49_1	+16.7	2	+17.1	[—197.2]	[— 9.9]
[7h 21m]	Feb. 16	....	...	—14.55	....	..	+17.0	—201.9	—10.9

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
		s		s	"		"	"	"
	1870								
Greenwich,	Mar. 12	—14.29	5	—14.28$_5$	+16.2	5	+16.0		
Washington,	Mar. 11	—14.30	6	—14.30$_5$	+15.9	6	+16.3	[—193.6]	[— 9.7]
[7^h 18^m]	Mar. 12	....	...	—14.29	....	..	+16.2	—198.1	—10.1
	1871								
Greenwich,	Jan. 9	—13.99	5	—13.98	+24.8	5	+24.6	[—190.0]	[—10.2]
[7^h 47^m]	......	....	...	....	....	..	....	—196.1	—10.7
Greenwich,	Feb. 15	—13.92	2	—13.91	+21.8	2	+21.6	[—188.8]	[—10.1]
[7^h 41^m]	....	....	...	....	....	..	....	—194.4	—11.5
Greenwich,	Mar. 14	—13.82	14	—13.81	+20.7	14	+20.5	[—184.9]	[—10.0]
[7^h 38^m]	......	....	...	....	....	..	....	—192.8	—11.5
Greenwich,	Dec. 21	—13.16	3	—13.15	+29.2	3	+29.0	[—177.9]	[—10.4]
[8^h 11^m]	......	....	...	....	....	..	....	—186.5	—11.4
	1872								
Greenwich,	Jan. 6	—13.32	6	—13.31	+28.3	6	+28.1$_3$		
Washington,	Jan. 21	—13.31	3	—13.31	+27.8	3	+28.2$_1$	[—179.8]	[—10.6]
[8^h 6^m.1]	Jan. 18	....	...	—13.31	....	..	+28.1	—188.3	—11.3
Greenwich,	Feb. 15	—13.23	7	—13.22	+26.2	6	+26.0$_2$		
Washington,	Feb. 21	—13.24	3	—13.24	+26.4	3	+26.8$_1$	[—179.3]	[—10.5]
[8^h 1^m.0]	Feb. 17	....	...	—13.23	....	..	+26.3	—186.6	—11.3
Greenwich,	Mar. 15	—13.04	14	—13.03$_4$	+25.4	14	+25.2$_2$		
Washington,	Mar. 15	—13.07	10	—13.07$_5$	+24.5	8	+24.9$_1$	[—176.4]	[—10.4]
[7^h 57^m.9]	Mar. 15	....	...	—13.05	....	..	+25.1	—183.7	—11.1
Washington,	April 8	—12.79	3	—12.79$_3$	+24.9	3	+25.3$_3$		
Greenwich,	April 8	—12.77	1	—12.76$_1$	+25.0	1	+24.8$_1$	[—172.4]	[—10.2]
[7^h 57^m.3]	April 8	....	...	—12.78	....	..	+25.2	—180.2	—10.8

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS IN THE BERLIN JAHRBUCH AND THE NAUTICAL ALMANAC TO REDUCE THEM TO THE POSITIONS FROM THE PROVISIONAL THEORY.

Date.	Heliocentric.			Geocentric.	
	$\delta\lambda$ "	$M\delta\rho$	$\delta\beta$ "	δl "	δb "
1830, July 24	—18.0	+1001	+ 9.0	—19.3	+ 9.5
Aug. 13	18.3	1007	9.0	18.8	9.5
Sept. 2	18.4	1014	9.1	17.9	9.5
Sept. 22	18.5	1020	9.1	17.1	9.4
Oct. 12	18.7	1027	9.1	16.7	9.3
Nov. 1	19.0	1031	9.2	16.5	9.2
Nov. 21	—19.3	+1038	+ 9.2	—16.7	+ 9.1
1831, July 19	—22.5	+1112	+ 9.5	—24.5	+10.0
Aug. 8	22.6	1119	9.5	23.7	10.0
Aug. 28	22.7	1123	9.6	22.7	10.1
Sept. 17	22.8	1128	9.6	21.7	10.0
Oct. 7	22.9	1133	9.6	20.9	9.9
Oct. 27	23.2	1138	9.7	20.6	9.8
Nov. 16	23.5	1142	9.7	20.6	9.6
Dec. 6	—23.8	+1146	+ 9.7	—21.0	9.5
1832, Aug. 2	—26.9	+1198	+ 9.8	—28.7	+10.3
Aug. 22	27.2	1201	9.9	27.9	10.4
Sept. 11	27.4	1202	9.9	26.9	10.3
Oct. 1	27.6	1205	9.9	26.1	10.2
Oct. 21	27.8	1208	10.0	25.4	10.2
Nov. 10	28.0	1209	10.1	25.1	10.1
Nov. 30	—28.3	+1209	+10.2	—25.2	+10.0
1833, July 28	—31.9	+1233	+10.2	—34.5	+10.7
Aug. 17	32.2	1236	10.3	33.7	10.8
Sept. 6	32.4	1240	10.3	32.7	10.8
Sept. 26	32.6	1242	10.3	31.8	10.7
Oct. 16	32.9	1245	10.3	31.0	10.5
Nov. 5	33.2	1247	10.4	30.4	10.4
Nov. 25	—33.5	+1250	+10.4	—30.2	+10.3
1834, July 23	—38.0	+1264	+10.5	—41.2	+11.0
Aug. 12	38.4	1266	10.5	40.7	11.1
Sept. 1	38.4	1268	10.6	39.6	11.1
Sept. 21	38.7	1270	10.6	38.6	11.1
Oct. 11	39.1	1275	10.6	37.7	10.9
Oct. 31	39.4	1277	10.6	36.9	10.8
Nov. 20	39.7	1282	10.7	36.6	10.7
Dec. 10	—40.1	+1282	+10.7	—36.1	+10.5
1835, July 18	—43.9	+1309	+10.8	—47.7	+11.3
Aug. 7	44.5	1311	10.7	47.6	11.3
Aug. 27	44.5	1313	10.7	46.6	11.3
Sept. 16	45.1	1316	10.7	45.8	11.2
Oct. 6	45.4	1318	10.8	44.8	11.2
Oct. 26	45.9	1321	10.7	44.0	10.9
Nov. 15	46.5	1324	10.8	43.6	10.8
Dec. 5	—46.6	+1327	+10.7	—42.5	+10.6
1836, July 12	—51.0	+1361	+11.0	—55.3	+11.4
Aug. 1	51.3	1364	11.0	55.2	11.5
Aug. 21	51.5	1365	11.1	54.6	11.7
Sept. 10	—51.9	+1368	+11.0	—53.7	+11.6

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.		Heliocentric.			Geocentric.	
		$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
		$''$		$''$	$''$	$''$
1836,	Sept. 30	—52.2	+1370	+11.0	—52.4	+11.5
	Oct. 20	52.9	1372	11.0	51.6	11.3
	Nov. 9	53.1	1375	11.0	50.5	11.1
	Nov. 29	53.9	1377	11.0	50.4	11.0
	Dec. 19	—53.8	+1380	+11.1	—49.8	+10.9
1837,	July 7	—58.3	+1387	+11.0	—62.9	+11.4
	July 27	58.4	1387	11.0	62.9	11.5
	Aug. 16	59.2	1388	11.1	63.1	11.7
	Sept. 5	59.2	1389	11.1	62.0	11.7
	Sept. 25	60.0	1391	11.0	61.3	11.5
	Oct. 15	60.0	1393	11.0	59.6	11.4
	Nov. 4	61.0	1394	11.0	59.1	11.2
	Nov. 24	61.3	1394	11.0	58.2	11.0
	Dec. 14	—62.1	+1393	+11.0	—57.1	+10.9
1838,	Aug. 11	—66.4	+1395	+11.1	—71.1	+11.6
	Aug. 31	67.2	1396	11.1	71.0	11.7
	Sept. 20	67.7	1395	11.1	70.2	11.7
	Oct. 10	68.1	1392	11.1	68.9	11.6
	Oct. 30	68.5	1390	11.1	67.6	11.4
	Nov. 19	68.9	1388	11.1	66.4	11.2
	Dec. 9	69.4	1388	11.1	65.8	11.1
	Dec. 29	—69.7	+1389	+11.2	—65.4	+11.0
1839,	Aug. 6	—74.7	+1382	+11.1	—80.0	+11.6
	Aug. 26	75.1	1381	11.1	79.7	11.7
	Sept. 15	75.5	1380	11.0	79.0	11.6
	Oct. 5	76.0	1379	11.1	77.9	11.6
	Oct. 25	76.4	1379	11.1	76.5	11.5
	Nov. 14	76.8	1379	11.1	75.2	11.3
	Dec. 4	77.2	1378	11.1	74.0	11.1
	Dec. 24	77.6	1377	11.1	73.5	10.9
1840,	Jan. 13	—78.0	+1376	+11.0	—73.1	+10.7
	June 24	—81.9	+1376	+11.0	—86.0	+11.1
	Aug. 20	82.7	1374	11.0	87.9	11.5
	Sept. 9	83.4	1376	11.0	87.8	11.6
	Sept. 29	83.5	1377	11.0	86.4	11.5
	Oct. 19	84.2	1376	11.0	85.5	11.4
	Nov. 8	84.4	1376	11.0	83.5	11.3
	Nov. 13	85.3	1377	11.0	82.7	11.1
	Dec. 13	85.5	1378	11.0	81.5	10.9
1841,	Jan. 7	—86.1	+1377	+10.9	—81.3	+10.7
	June 16	—89.8	+1378	+10.8	—93.2	+10.8
	Aug. 15	91.1	1384	10.8	97.0	11.3
	Sept. 4	91.5	1385	10.8	96.9	11.4
	Sept. 24	91.9	1388	10.7	96.1	11.3
	Oct. 14	92.3	1390	10.7	94.8	11.2
	Nov. 3	92.9	1392	10.7	93.4	11.1
	Nov. 23	93.4	1393	10.8	91.8	10.9
	Dec. 13	93.8	1395	10.7	90.5	10.7
1842,	Jan. 2	94.3	1398	10.8	89.7	10.6
	Feb. 11	—95.0	+1404	+10.7	—89.6	+10.3

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

Date.		Heliocentric.			Geocentric.	
		$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
		″		″	″	″
1842,	June 11	— 96.7	+1409	+10.6	— 98.5	+10.5
	Aug. 10	99.1	1408	10.6	105.3	10.0
	Aug. 30	99.6	1409	10.7	105.8	10.2
	Sept. 19	99.9	1409	10.6	105.2	10.2
	Oct. 9	100.5	1410	10.5	104.2	10.0
	Oct. 29	100.9	1412	10.5	102.6	10.9
	Nov. 18	101.3	1413	10.5	100.8	10.8
	Dec. 8	101.8	1414	10.5	99.2	10.6
	Dec. 28	102.2	1415	10.5	98.0	10.4
1843,	Jan. 17	—102.5	+1417	+10.4	— 97.2	+10.2
	Aug. 5	—107.2	+1422	+10.3	—113.7	+10.7
	Aug. 25	107.7	1420	10.2	114.4	10.7
	Sept. 14	108.0	1420	10.1	114.3	10.6
	Oct. 4	108.5	1420	10.1	113.4	10.6
	Oct. 24	109.0	1419	10.1	112.1	10.5
	Nov. 13	109.4	1418	10.1	110.2	10.4
	Dec. 3	110.0	1418	10.1	108.6	10.3
	Dec. 23	110.4	1417	10.1	106.9	10.1
1844,	Jan. 12	—110.8	+1418	+10.0	—105.9	+ 9.8
	July 30	—115.1	+1406	+ 9.7	—121.2	+10.0
	Aug. 19	115.4	1403	9.8	122.3	10.2
	Sept. 8	115.8	1400	9.8	122.7	10 3
	Sept. 28	116.3	1397	9.8	122.4	10.3
	Oct. 18	116.9	1396	9.7	121.3	10.2
	Nov. 7	117.4	1392	9.6	119.7	10.0
	Nov. 27	117.8	1389	9.6	117.6	9.8
	Dec. 17	118.3	1384	9.6	115.7	9.7
1845,	Jan. 6	118.6	1381	9.5	114.3	9.4
	Jan. 26	—119.1	+1378	+ 9.6	—113.3	+ 9.4
	Aug. 14	—123.5	+1340	+ 9.1	—130.4	+ 9.4
	Sept. 3	123.9	1335	9.4	131.4	9.8
	Sept. 23	124.4	1330	9.4	131.7	9.9
	Oct. 13	124.9	1327	9.3	130.6	9.8
	Nov. 2	125.3	1322	9.3	129.0	9.7
	Nov. 22	125.7	1318	9.2	127.0	9.5
	Dec. 12	126.2	1313	9.2	125.0	9.3
1846,	Jan. 1	126.7	1309	9.2	123.2	9.2
	Jan. 21	—127.1	+1303	+ 9.2	—121.9	— 9.0
	Aug. 29	—132.0	+1253	+ 8.8	—139.5	+ 9.2
	Sept. 18	132.3	1249	8.8	139.9	9.2
	Oct. 8	132.8	1246	8.8	139.8	9.3
	Oct. 28	133.2	1242	8.7	138.4	9.1
	Nov. 17	133.5	1238	8.7	136.4	9.0
	Dec. 7	134.1	1234	8.7	134.4	8.9
	Dec. 27	134.6	1233	8.6	132.3	8.7
1847,	Jan. 16	—134.9	+1230	+ 8.6	—130.5	+ 8.5
	Aug. 24	—139.5	+1197	+ 8.3	—146.7	+ 8.6
	Sept. 13	140.0	1194	8.3	148.0	8.7
	Oct. 3	140.5	1191	8.3	148.3	8.7
	Oct. 23	—140.9	+1188	+ 8.2	—147.4	+ 8.6

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.			Heliocentric.			Geocentric.	
			$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
			$''$		$''$	$''$	$''$
1847,	Nov.	12	—141.4	+1185	+8.2	—145.8	+8.5
	Dec.	2	141.9	1183	8.1	143.7	8.3
	Dec.	22	142.4	1182	8.1	141.4	8.2
1848,	Jan.	11	—142.8	+1182	+8.1	—139.3	+8.1
	Sept.	7	—147.9	+1165	+7.6	—156.0	+7.9
	Sept.	27	148.3	1166	7.6	156.7	8.0
	Oct.	17	148.7	1164	7.6	156.4	8.0
	Nov.	6	149.3	1164	7.6	155.3	8.0
	Nov.	26	149.8	1163	7.6	153.2	7.9
	Dec.	16	150.2	1162	7.6	150.7	7.8
1849,	Jan.	5	150.6	1161	7.5	148.3	7.5
	Jan.	25	—151.3	+1161	+7.5	—146.6	+7.4
	Sept.	2	—155.8	+1149	+7.0	—164.6	+7.3
	Sept.	22	156.2	1150	7.0	165.1	7.3
	Oct.	12	156.5	1149	7.0	165.1	7.4
	Nov.	1	156.8	1148	7.0	164.0	7.4
	Nov.	21	157.4	1149	7.0	162.4	7.3
	Dec.	11	157.9	1148	6.9	160.0	7.1
	Dec.	31	158.3	1147	6.8	157.5	6.9
1850,	Jan.	20	—158.6	+1147	+6.8	—155.0	+6.8
	Aug.	28	—163.3	+1135	+6.2	—170.6	+6.4
	Sept.	17	163.7	1133	6.2	172.6	6.5
	Oct.	7	164.2	1131	6.2	173.3	6.5
	Oct.	27	164.7	1129	6.1	173.1	6.4
	Nov.	16	165.2	1127	6.1	171.8	6.4
	Dec.	6	165.6	1126	6.1	169.4	6.3
	Dec.	26	166.0	1127	6.0	166.7	6.2
1851,	Jan.	15	166.3	1127	6.1	163.8	6.1
	Feb.	4	—166.7	+1124	+6.0	—161.8	+5.9
	Sept.	12	—171.5	+1109	+5.5	—180.0	+5.7
	Oct.	2	171.9	1105	5.5	181.4	5.8
	Oct.	22	172.4	1103	5.5	181.9	5.8
	Nov.	11	172.9	1103	5.4	180.8	5.7
	Dec.	1	173.3	1103	5.3	178.7	5 5
	Dec.	21	173.8	1102	5.2	176.1	5.3
1852,	Jan.	10	174.3	1101	5.2	173.4	5.3
	Jan.	30	—174.7	+1098	+5.1	—170.2	+5.1
	Sept.	6	—179.0	+1074	+4.7	—186.8	+4.9
	Sept.	26	179.4	1073	4.6	188.9	4.8
	Oct.	16	179.8	1071	4.5	189.7	4.7
	Nov.	5	180.3	1068	4.5	189.6	4.7
	Nov.	25	180.7	1067	4.4	187.8	4.6
	Dec.	15	181.2	1065	4.4	185.3	4.6
1853,	Jan.	4	181.5	1064	4.4	182.3	4.5
	Jan.	24	181.8	1062	4.4	179.6	4.4
	Feb.	13	—182.1	+1060	+4.3	—176.9	+4.2
	Sept.	1	—185.8	+1055	+3.9	—192.4	+4.0
	Sept.	21	186.2	1055	3.8	195.2	3.9
	Oct.	11	—186.5	+1055	+3.7	—196.7	+3.9

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.	Heliocentric.			Geocentric.	
	$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
	"		"	"	"
1853, Oct. 31	—186.8	+1054	+3.6	—196.8	+3.8
Nov. 20	187.1	1054	3.5	195.6	3.7
Dec. 10	187.5	1053	3.5	193.5	3.6
Dec. 30	187.9	1053	3.5	190.7	3.6
1854, Jan. 19	188.1	1053	3.5	187.4	3.5
Feb. 8	—188.5	+1052	+3.5	—184.5	+3.5
Sept. 16	—192.2	+1034	+2.9	—200.5	+3.0
Oct. 6	192.6	1032	2.8	202.9	2.9
Oct. 26	193.0	1032	2.7	203.7	2.8
Nov. 15	193.3	1032	2.6	203.0	2.7
Dec. 5	193.7	1029	2.6	201.3	2.7
Dec. 25	194.0	1025	2.5	198.6	2.6
1855, Jan. 14	194.2	1021	2.5	195.2	2.5
Feb. 3	—194.5	+1018	+2.4	—192.0	+2.4
Oct. 1	—198.3	+1010	+1.8	—207.9	+1.9
Oct. 21	198.7	1009	1.7	209.6	1.8
Nov. 10	199.0	1008	1.6	209.7	1.7
Nov. 30	199.3	1007	1.5	208.4	1.6
Dec. 20	199.6	1006	1.5	206.0	1.6
1856, Jan. 9	199.8	1005	1.5	202.6	1.5
Jan. 29	200.1	1003	1.4	199.2	1.4
Feb. 18	—200.3	+1001	+1.3	—196.1	+1.3
Oct. 15	—203.3	+ 962	+0.7	—213.9	+0.7
Nov. 4	203.7	957	0.7	214.9	0.7
Nov. 24	203.9	953	0.7	214.5	0.7
Dec. 14	204.3	949	0.6	212.4	0.6
1857, Jan. 3	204.4	945	0.6	209.3	0.6
Jan. 23	204.5	941	0.5	205.6	0.5
Feb. 12	—204.7	+ 934	+0.4	—202.2	+0.4
Sept. 20	—206.9	+ 879	—0.2	—214.0	—0.2
Oct. 10	207.2	874	0.3	217.1	0.3
Oct. 30	207.5	868	0.3	218.8	0.3
Nov. 19	207.8	861	0.4	219.0	0.4
Dec. 9	208.0	853	0.4	217.6	0.4
Dec. 29	208.1	846	0.5	215.0	0.5
1858, Jan. 18	208.2	839	0.6	211.5	0.6
Feb. 7	208.3	830	0.6	207.7	0.6
Feb. 27	—208.4	+ 823	—0.6	—204.2	—0.6
Oct. 5	—210.4	+ 741	—1.3	—219.1	—1.3
Oct. 25	210.6	732	1.4	221.4	1.5
Nov. 14	210.8	723	1.4	222.2	1.5
Dec. 4	211.0	713	1.5	221.9	1.6
Dec. 24	211.2	704	1.5	219.7	1.6
1859, Jan. 13	211.3	695	1.5	216.6	1.5
Feb. 2	211.4	686	1.6	213.0	1.6
Feb. 22	—211.6	+ 677	—1.5	—209.4	—1.5
Oct. 20	—213.1	+ 576	—2.1	—222.8	—2.2
Nov. 9	213.2	569	2.1	224.5	2.2
Nov. 29	—213.4	+ 561	—2.1	—224.8	—2.2

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

Date.	Heliocentric.			Geocentric.	
	$\delta\lambda$ ″	$M\delta\rho$	$\delta\beta$ ″	δl ″	δb ″
1859, Dec. 19	—213.5	+554	—2.2	—223.5	—2.3
1860, Jan. 8	213.5	546	2.2	220.9	2.3
Jan. 28	213.6	539	2.2	217.4	2.3
Feb. 17	—213.7	+533	—2.3	—213.5	—2.3
Sept. 24	—214.7	+456	—2.8	—219.7	—2.9
Oct. 14	214.8	451	2.8	223.1	2.9
Nov. 3	215.0	444	2.9	225.6	3.0
Nov. 23	215.1	437	2.9	226.8	3.1
Dec. 18	215.2	431	3.0	226.3	3.2
1861, Jan. 2	215.3	424	3.1	223.8	3.2
Jan. 22	215.3	417	3.1	221.0	3.2
Feb. 12	—215.3	+411	—3.1	—217.3	—3.1
Oct. 29	—215.6	+339	—3.7	—225.0	—3.9
Nov. 13	215.6	333	3.8	226.9	4.0
Dec. 8	215.7	329	3.8	227.4	4.0
Dec. 28	215.8	324	3.8	226.3	4.0
1862, Jan. 17	215.8	320	3.9	223.5	4.0
Feb. 6	215.8	315	4.0	220.0	4.1
Feb. 26	—215.9	+310	—4.0	—216.0	—4.0
Oct. 24	—216.0	+246	—4.4	—224.2	—4.6
Nov. 13	216.0	241	4.5	226.6	4.7
Dec. 3	216.0	237	4.5	227.7	4.7
Dec. 23	215.9	232	4.6	227.2	4.8
1863, Jan. 12	215.8	226	4.7	225.0	4.9
Feb. 1	215.8	219	4.7	221.8	4.8
Feb. 21	215.7	213	4.7	218.1	4.8
Mar. 13	—215.7	+208	—4.8	—214.1	—4.8
Nov. 8	—215.3	+139	—5.3	—224.6	—5.5
Nov. 28	215.2	133	5.4	226.5	5.7
Dec. 18	215.1	126	5.4	226.8	5.7
1864, Jan. 7	215.0	120	5.5	225.4	5.8
Jan. 27	214.9	114	5.5	222.7	5.7
Feb. 16	214.8	108	5.6	219.3	5.7
March 7	—214.8	+103	—5.6	—215.3	—5.6
Oct. 13	—213.8	+ 21	—6.0	—217.8	—6.1
Nov. 2	213.7	16	6.0	221.3	6.2
Nov. 22	213.7	9	6.1	224.0	6.4
Dec. 12	213.6	+ 1	6.2	225.1	6.5
1865, Jan. 1	213.4	— 6	6.2	224.7	6.5
Jan. 21	213.2	14	6.2	222.7	6.5
Feb. 10	213.1	20	6.2	219.6	6.4
March 2	—213.0	— 27	—6.3	—215.7	—6.4
Oct. 8	—211.6	—103	—6.8	—213.5	—6.9
Oct. 28	211.4	110	6.8	217.2	7.0
Nov. 17	211.2	117	6.9	220.0	7.2
Dec. 7	211.1	124	6.9	222.0	7.3
Dec. 27	210.9	132	6.9	222.4	7.3
1866, Jan. 16	210.7	139	7.0	221.5	7.4
Feb. 5	210.6	145	7.0	218.8	7.3
Feb. 25	210.4	151	7.0	215.4	7.2
Mar. 17	—210.1	—158	—7.1	—211.2	—7.1

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.		Heliocentric.			Geocentric.	
		$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
		″		″	″	″
1866,	Oct. 3	—208.2	— 225	— 7.4	—207.3	— 7.4
	Oct. 23	208.1	230	7.4	211.5	7.6
	Nov. 12	207.9	235	7.5	214.9	7.8
	Dec. 2	207.8	241	7.6	217.6	8.0
	Dec. 22	207.6	246	7.6	218.8	8.0
1867,	Jan. 11	207.4	251	7.6	218.5	8.0
	Jan. 31	207.2	256	7.7	216.7	8.0
	Feb. 20	207.0	262	7.7	213.8	7.9
	Mar. 12	—206.7	— 268	— 7.7	—209.8	— 7.8
	Nov. 27	—203.2	— 351	— 8.2	—211.4	— 8.6
	Dec. 17	203.0	358	8.2	213.5	8.6
1868,	Jan. 6	202.8	365	8.2	213.7	8.7
	Jan. 26	202.4	371	8.3	212.7	8.7
	Feb. 15	202.2	376	8.3	210.3	8.6
	Mar. 6	201.8	382	8.4	207.0	8.6
	Mar. 26	—201.4	— 387	— 8.4	—202.9	— 8.4
	Oct. 12	—198.0	— 458	— 8.6	—197.2	— 8.6
	Nov. 1	197.8	465	8.7	201.1	8.9
	Nov. 21	197.5	473	8.8	203.8	9.1
	Dec. 11	197.2	481	8.8	206.2	9.2
	Dec. 31	196.8	490	8.8	207.5	9.3
1869,	Jan. 20	196.4	498	8.8	207.1	9.3
	Feb. 9	196.0	507	8.8	205.4	9.2
	Mar. 1	195.6	515	8.9	202.4	9.2
	Mar. 21	—195.3	— 524	— 8.9	—198.4	— 9.0
	Dec. 6	—190.0	....	....	—197.0	
	Dec. 26	189.5	....	....	198.9	
1870,	Jan. 15	189.0	— 665	— 9.4	199.5	— 9.9
	Feb. 14	188.6	676	9.4	198.7	9.9
	Feb. 24	188.0	686	9.4	196.2	9.8
	Mar. 16	187.5	697	9.4	193.0	9.6
	April 5	—187.2	— 708	— 9.4	—189.4	— 9.4
	Dec. 1	—181.5	— 846	— 9.6	—186.5	—10.0
	Dec. 21	181.1	859	9.6	189.0	10.1
1871,	Jan. 11	180.5	872	9.7	190.3	10.2
	Jan. 30	179.8	883	9.8	190.1	10.3
	Feb. 19	179.1	894	9.7	188.3	10.1
	Mar. 11	178.4	906	9.7	185.6	10.0
	Mar. 31	—177.0	— 919	— 9.8	—181.2	— 9.9
	Dec. 16	—171.6	—1083	—10.0	—177.4	—10.4
1872,	Jan. 5	171.0	1095	10.1	179.3	10.6
	Jan. 25	170.5	1107	10.0	180.2	10.6
	Feb. 14	169.8	1120	10.0	179.6	10.5
	Mar. 5	169.3	1133	10.1	177.8	10.5
	Mar. 25	168.7	1145	10.1	174.9	10.3
	April 14	168.1	1156	10.1	171.3	10.1
	May 4	—167.6	—1168	—10.1	—166.3	—10.0

CHAPTER VII.

FORMATION AND SOLUTION OF THE EQUATIONS OF CONDITION RESULTING FROM THE PRECEDING COMPARISONS.

In the preceding chapter we have obtained from observations a series of corrections to the geocentric positions of Uranus resulting from the provisional theory. The further operations are as follows:—

1. To reduce all the corrections in right ascension and declination to corrections in geocentric longitude and latitude. Most of the corrections are already so expressed, so that this reduction is necessary in only a few cases.

2. To find the mean value of the correction in geocentric longitude during each opposition, and to express this mean value in terms of the correction to the heliocentric co-ordinates.

3. To express these corrections to the heliocentric co-ordinates in terms of corrections to the elements of Uranus and the mass of Neptune.

4. To solve the equations of condition thus formed.

The first of these processes is too simple to make it necessary to present any details of it. With regard to the second I have sought, not the simple correction to the geocentric longitude, but this correction multiplied by such a factor as it was supposed would make the probable error of the correction $0''.5$. The equations for expressing the error of geocentric longitude in terms of errors of heliocentric longitude and radius vector have been given on page 129. The first observation of Flamstead, p. 107, gives the equation

$$+22'' = 1.04\delta\lambda + .027\delta\rho$$

$\delta\lambda$ being the correction to the heliocentric longitude, and $\delta\rho$ that to the Neperian logarithm of the radius vector. From the discordance of Flamstead's clock errors it may be estimated that the probable error of the first member of this equation is $10''$. Therefore we divide the equation by 20, which gives

$$\tfrac{1}{20}\delta l = 1''.1 = .052\delta\lambda + .001\delta\rho.$$

In the opposition of 1715 we have four observations. The best were those of March 4 and 10, of which we may estimate the probable error at $10''$, and the worst that of March 5, of which the probable error may be estimated at $20''$, while that of April 29 is intermediate in certainty. The separate observations give the equations

March 4,	$\delta l = +28''$	$= 1.06\delta\lambda$;	Weight, 4
March 5,	$\delta l = +44$	$= 1.06\delta\lambda$;	Weight, 1
March 10,	$\delta l = +36$	$= 1.06\delta\lambda$;	Weight, 4
April 29,	$\delta l = +\ 2$	$= 1.04\delta\lambda + .04\delta\rho$;	Weight, 2.
Mean	$\delta l = +27.6$	$= 1.056\delta\lambda + .003\delta\rho$;	probable error $= \pm 6''$.

Applying the correction —1″.1 for equinox, and dividing by 12, the equation of condition becomes

$$\tfrac{1}{12}\delta l = +2''.2 = 0.088\delta\lambda.$$

In this way the following equations were obtained. It is deemed unnecessary to give the details of the process, as it is one which every one can go over for himself from the data already given, and can reproduce all the results, except so far as they depend on the relative weights assigned to the different groups of observations during one and the same opposition.

No.	Date.	Equations.				Number of observations in R. A.
			″			
1	1691.0;	$\frac{1}{20}\delta l$	$= +1.1$	$= .052\delta\lambda$	$+ .001\delta\rho$	1
2	1715.2	$\frac{1}{12}$	$= +2.2$	$= .088$		4
3	1748.8	$\frac{1}{3}$	$= +12.8$	$= .338$	$+ .017$	1
4	1750.8	$\frac{1}{3}$	$= +11.8$	$= .345$	$+ .010$	3
5	1753.9	$\frac{1}{3}$	$= +11.6$	$= .333$	$+ .016$	1
6	1756.7	$\frac{1}{5}$	$= +5.0$	$= .210$	$+ .003$	1
7	1769.0	$\frac{2}{5}$	$= +4.8$	$= .203$	$+ .010$	8
8	1782.0	$\frac{4}{3}$	$= +3.0$	$= 1.370$		21
9	1783.0	1	$= +1.25$	$= 1.030$	$- .002$	13
10	1784.0	1	$= +1.92$	$= 1.026$	$- .008$	13
11	1785.0	1	$= -0.26$	$= 1.034$	$+ .006$	10
12	1788.0	$\frac{2}{3}$	$= +1.23$	$= 0.684$	$+ .006$	5
13	1789.0	$\frac{1}{2}$	$= +1.58$	$= 0.504$	$+ .008$	6
14	1790.0	$\frac{1}{2}$	$= -0.52$	$= 0.514$	$- .013$	4
15	1791.0	$\frac{2}{3}$	$= -0.66$	$= 0.684$	$- .010$	7
16	1792.0	$\frac{1}{3}$	$= -0.12$	$= 0.340$	$- .011$	3
17	1793.0	$\frac{1}{2}$	$= -0.19$	$= 0.512$	$- .015$	5
18	1794.0	$\frac{1}{3}$	$= +0.79$	$= 0.344$	$- .006$	3
19	1795.0	$\frac{2}{3}$	$= -0.66$	$= 0.683$	$- .019$	7
20	1796.0	$\frac{1}{2}$	$= -0.68$	$= 0.514$	$- .015$	4
21	1797.1	$\frac{1}{2}$	$= -0.35$	$= 0.528$		3
22	1800.2	$\frac{1}{3}$	$= 0.00$	$= 0.352$		2
23	1801.2	$\frac{1}{3}$	$= -0.26$	$= 0.352$		2
24	1802.3	1	$= +0.85$	$= 1.05$		13
25	1805.3	1	$= +0.69$	$= 1.05$		13
26	1806.3	$\frac{1}{2}$	$= +0.20$	$= 0.52$		5
27	1807.3	1	$= +2.34$	$= 1.045$		16
28	1808.3	$\frac{1}{2}$	$= -0.04$	$= 0.52$		6
29	1809.3	$\frac{2}{3}$	$= +1.80$	$= 0.70$		9
30	1810.3	1	$= +2.39$	$= 1.05$		16
31	1811.3	1	$= +1.49$	$= 1.04$	-0.01	11
32	1812.4	$\frac{1}{2}$	$= +0.8$	$= 0.53$		8
33	1813.4	$\frac{1}{2}$	$= +1.2$	$= 0.53$		9
34	1814.4	1	$= +1.7$	$= 1.05$		15

No.	Date.	Equations.					Number of observations in R. A.
				″			
35	1815.4	$\frac{3}{2}$	$\delta l = +$	2.1	$= 1.58\,\delta\lambda$		20
36	1818.4*	$\frac{3}{2}$	$= +$	0.3	$= 1.58$		24
37	1819.4	1	$= -$	0.8	$= 1.05$		11
38	1820.5	1	$= -$	1.3	$= 1.05$		14
39	1821.5	$\frac{2}{3}$	$= +$	0.9	$= 0.70$		10
40	1822.5	$\frac{2}{3}$	$= +$	0.9	$= 0.70$		7
41	1823.5	$\frac{2}{3}$	$=$	0.0	$= 0.70$		11
42	1824.5	1	$= +$	0.5	$= 1.05$		12
43	1825.5	$\frac{2}{3}$	$= -$	0.4	$= 0.70$		7
44	1826.5	1	$= +$	0.3	$= 1.05$		11
45	1827.7	2	$= -$	2.7	$= 2.07$	$+ 0.04\,\delta\rho$	37
46	1828.7	$2\frac{1}{2}$	$= -$	2.7	$= 2.57$	$+ 0.07$	67
47	1829.7	$2\frac{1}{2}$	$= -$	2.4	$= 2.59$	$+ 0.04$	61
48	1830.7	$2\frac{1}{2}$	$= -$	4.9	$= 2.56$	$+ 0.07$	73
49	1831.7	2	$=$	0.0	$= 2.06$	$+ 0.05$	54
50	1832.7	2	$= -$	2.2	$= 2.07$	$+ 0.05$	65
51	1833.8	$2\frac{1}{2}$	$= -$	4.1	$= 2.58$	$+ 0.08$	88
52	1834.8	$2\frac{1}{2}$	$= -$	3.9	$= 2.57$	$+ 0.08$	91
53	1835.8	$2\frac{1}{2}$	$= -$	5.1	$= 2.59$	$+ 0.05$	82
54	1836.8	3	$= -$	7.0	$= 3.11$	$+ 0.08$	157
55	1837.8	3	$= -$	3.6	$= 3.11$	$+ 0.04$	162
56	1838.8	3	$= -$	1.6	$= 3.11$	$+ 0.06$	193
57	1839.8	3	$= -$	1.2	$= 3.11$	$+ 0.06$	170
58	1840.8	3	$= -$	1.5	$= 3.11$	$+ 0.06$	124
59	1841.8	3	$= +$	0.8	$= 3.10$	$+ 0.06$	108
60	1842.8	3	$= +$	1.6	$= 3.10$	$+ 0.06$	169
61	1843.8	3	$= +$	4.7	$= 3.12$	$+ 0.04$	111
62	1844.9	3	$= +$	5.1	$= 3.11$	$+ 0.03$	106
63	1845.9	2	$= +$	4.2	$= 2.08$	$+ 0.02$	55
64	1846.9	3	$= +$	6.3	$= 3.10$	$+ 0.04$	98
65	1847.9	2	$= +$	5.8	$= 2.07$	$+ 0.03$	74
66	1848.9	2	$= +$	4.4	$= 2.08$	$+ 0.02$	59
67	1849.9	2	$= +$	6.6	$= 2.08$	$+ 0.04$	33
68	1850.9	2	$= +$	7.2	$= 2.08$	$+ 0.01$	46
69	1851.9	2	$= +$	6.3	$= 2.07$	$+ 0.04$	42
70	1852.9	2	$= +$	6.6	$= 2.07$	$+ 0.03$	54
71	1853.9	2	$= +$	7.9	$= 2.09$	$+ 0.01$	49
72	1854.9	2	$= +$	8.9	$= 2.09$	$+ 0.02$	49
74	1855.9	2	$= +$	8.5	$= 2.08$	$+ 0.04$	48
75	1856.9	2	$= +$	7.9	$= 2.08$	$+ 0.04$	45
76	1858.0	$2\frac{1}{2}$	$= +$	10.3	$= 2.61$	$+ 0.09$	66

* The results for 1816 and 1817 were omitted in this list through oversight.

No.	Date.		Equations. ″			Number of observations in R. A.
77	1859.0	$2\frac{1}{2}\,\delta l$	$= +10.6$	$= 2.60\,\delta\lambda$	$+0.03\,\delta\rho$	58
78	1860.0	$2\frac{1}{2}$	$= +\ 8.2$	$= 2.60$	$+0.05$	64
79	1861.0	2	$= +\ 6.3$	$= 2.09$	$+0.03$	41
80	1862.0	$2\frac{1}{2}$	$= +\ 7.0$	$= 2.60$	$+0.05$	60
81	1863.0	$2\frac{1}{2}$	$= +\ 6.6$	$= 2.59$	$+0.08$	88
82	1864.0	2	$= +\ 4.3$	$= 2.09$	$+0.03$	35
83	1865.0	$1\frac{1}{2}$	$= +\ 3.9$	$= 1.57$	$+0.04$	37
84	1866.0	$2\frac{1}{2}$	$= +\ 1.1$	$= 2.60$	$+0.06$	76
85	1867.0	$2\frac{1}{2}$	$= -\ 1.8$	$= 2.60$	$+0.02$	83
86	1868.0	2	$= -\ 3.8$	$= 2.09$	$+0.04$	40
87	1869.0	$2\frac{1}{2}$	$= -\ 9.1$	$= 2.61$	$+0.02$	68
88	1870.0	2	$= -\ 9.6$	$= 2.084$	$+0.054$	31
89	1871.0	$1\frac{1}{2}$	$= -10.6$	$= 1.560$	$+0.040$	21
90	1872.1	$2\frac{1}{2}$	$= -19.1$	$= 2.600$	$+0.070$	50

Total number of observations in R. A., ... 3763.

We have next to express the values of $\delta\lambda$ and $\delta\rho$ in terms of the corrections to the elements. Differentiating the expressions

$$\lambda = l + (2e - \tfrac{1}{4}e^3)\sin(l-\pi) + \tfrac{5}{4}e^2\sin(2l-2\pi) + \tfrac{13}{12}e^3 \sin e^3 \sin(3l-3\pi) + \text{etc.}$$
$$\rho = v + \tfrac{1}{4}e^2 + (-e + \tfrac{3}{8}e^3)\cos(l-\pi) - \tfrac{3}{4}e^2\cos(2l-2\pi) - \text{etc.},$$

with respect to l, e, and π, and reducing the coefficients to numbers, we find

$$\frac{\partial\lambda}{\partial l} = 1 + 0.0939\cos g + 0.0055\cos 2g$$

$$\frac{\partial\lambda}{\partial e} = 1.999\sin g + 0.117\sin 2g + 0.007\sin 3g$$

$$\frac{\partial\lambda}{\partial\pi} = -2.000\cos g - 0.117\cos 2g - 0.007\cos 3g$$

We have here put l for the mean longitude, or

$$l = nt + \varepsilon$$

whence

$$\frac{\partial\lambda}{\partial\varepsilon} = \frac{\partial\lambda}{\partial l}$$

$$\frac{\partial\lambda}{\partial n} = t\frac{\partial\lambda}{\partial l}$$

Also, from the expression for ρ

$$\frac{\partial\rho}{\partial l} = -\frac{\partial\rho}{\partial\pi} = (e - \tfrac{3}{8}e^3)\sin g + \tfrac{3}{4}e^2\sin 2g + \ldots$$

$$\frac{\partial\rho}{\partial e} = \tfrac{1}{2}e - (1 - \tfrac{9}{8}e^2)\cos g - \tfrac{3}{2}e\cos 2g - \text{etc.}$$

$$\frac{\partial\rho}{\partial n} = t\frac{\partial\rho}{\partial l} - \frac{2}{3n}$$

The values of these coefficients which depend only on g are shown in the following table:

g	$\frac{\partial\lambda}{\partial\varepsilon}$	$\frac{\partial\lambda}{\partial e}$	$\frac{\partial\lambda}{e d\pi}$	$\frac{\partial\rho}{\partial\varepsilon}$	$\frac{\partial\rho}{\partial e}$	$\frac{\partial\rho}{e d\pi}$
0°	+1.099	0	—2.124	0	—1.00	0
1	1.099	+0.039	2.124	+0.001	1.00	—0.02
2	1.099	0.079	2.123	0.002	1.00	0.03
3	1.099	0.118	2.121	0.002	1.00	0.05
4	1.099	0.158	2.118	0.003	1.00	0.07
5	+1.099	+0.196	—2.114	+0.004	—1.00	—0.09
6	1.099	0.235	2.110	0.005	0.99	0.10
7	1.099	0.274	2.105	0.006	0.99	0.12
8	1.098	0.313	2.099	0.006	0.99	0.14
9	1.098	0.352	2.092	0.007	0.99	0.16
10	+1.098	+0.391	—2.085	+0.008	—0.98	—0.17
11	1.097	0.430	2.077	0.009	0.98	0.19
12	1.097	0.468	2.069	0.010	0.98	0.21
13	1.097	0.505	2.059	0.011	0.97	0.22
14	1.096	0.544	2.049	0.011	0.97	0.24
15	+1.096	+0.581	—2.038	+0.012	—0.97	—0.26
16	1.095	0.618	2.027	0.013	0.96	0.28
17	1.095	0.655	2.014	0.014	0.96	0.29
18	1.094	0.693	2.001	0.015	0.95	0.31
19	1.094	0.729	1.987	0.015	0.95	0.33
20	+1.093	+0.765	—1.973	+0.016	—0.94	—0.34
21	1.092	0.801	1.957	0.017	0.93	0.36
22	1.091	0.836	1.941	0.018	0.93	0.37
23	1.090	0.872	1.925	0.018	0.92	0.39
24	1.090	0.907	1.907	0.019	0.91	0.41
25	+1.089	+0.942	—1.890	+0.020	—0.91	—0.42
26	1.088	0.976	1.872	0.021	0.90	0.44
27	1.087	1.010	1.852	0.021	0.89	0.45
28	1.086	1.043	1.832	0.022	0.88	0.47
29	1.085	1.076	1.811	0.023	0.87	0.48
30	+1.084	+1.108	—1.790	+0.023	—0.87	—0.50
31	1.084	1.140	1.769	0.024	0.86	0.51
32	1.083	1.172	1.747	0.025	0.85	0.53
33	1.082	1.203	1.724	0.026	0.84	0.54
34	1.080	1.233	1.700	0.026	0.83	0.56
35	+1.079	+1.264	—1.676	+0.027	—0.82	—0.57
36	1.078	1.294	1.651	0.028	0.81	0.59
37	1.077	1.323	1.626	0.028	0.80	0.60
38	1.076	1.351	1.601	0.029	0.79	0.62
39	1.074	1.379	1.574	0.030	0.78	0.63
40	+1.073	+1.407	—1.548	+0.030	—0.77	—0.64
41	1.072	1.434	1.521	0.031	0.76	0.66
42	1.070	1.460	1.494	0.031	0.74	0.67
43	1.069	1.486	1.466	0.032	0.73	0.68
44	1.068	1.511	1.438	0.033	0.72	0.70
45	+1.067	+1.536	—1.409	+0.033	—0.71	—0.71
46	1.065	1.561	1.380	0.034	0.70	0.71
47	1.064	1.584	1.351	0.034	0.68	0.72
48	1.063	1.606	1.320	0.035	0.67	0.74
49	1.061	1.629	1.290	0.036	0.66	0.75
50	+1.060	+1.651	—1.260	+0.036	—0.64	—0.76
51	1.058	1.672	1.229	0.037	0.63	0.77
52	1.057	1.692	1.197	0.037	0.62	0.78
53	1.055	1.712	1.165	0.037	0.60	0.79
54	1.054	1.731	1.133	0.038	0.59	0.80
55	+1.052	+1.750	—1.100	+0.038	—0.57	—0.81
56	1.051	1.768	1.067	0.039	0.56	0.82
57	1.049	1.785	1.034	0.039	0.54	0.83
58	1.047	1.802	1.002	0.040	0.53	0.84
59	+1.046	+1.817	—0.968	+0.040	—0.51	—0.85

g	$\frac{\partial\lambda}{\partial\varepsilon}$	$\frac{\partial\lambda}{\partial e}$	$\frac{\partial\lambda}{e d\pi}$	$\frac{\partial\rho}{\partial\varepsilon}$	$\frac{\partial\rho}{\partial e}$	$\frac{\partial\rho}{e d\pi}$
60°	+1.044	+1.833	−0.935	+0.041	−0.50	−0.86
61	1.043	1.848	0.901	0.041	0.49	0.87
62	1.041	1.862	0.867	0.041	0.47	0.87
63	1.040	1.875	0.832	0.042	0.45	0.88
64	1.038	1.888	0.798	0.042	0.44	0.89
65	+1.036	+1.901	−0.763	+0.043	−0.42	−0.90
66	1.034	1.912	0.728	0.043	0.41	0.91
67	1.033	1.922	0.693	0.043	0.39	0.91
68	1.031	1.932	0.659	0.044	0.37	0.92
69	1.030	1.942	0.624	0.044	0.36	0.93
70	+1.028	+1.950	−0.588	+0.044	−0.34	−0.94
71	1.027	1.959	0.553	0.044	0.33	0.95
72	1.025	1.967	0.518	0.045	0 31	0.95
73	1.023	1.974	0.483	0.045	0.29	0.96
74	1.021	1.980	0.447	0.045	0.28	0.96
75	+1.019	+1.985	−0.412	+0.045	−0.26	−0.97
76	1.018	1.990	0.376	0.046	0.24	0.97
77	1.016	1.994	0.341	0.046	0.22	0.97
78	1.014	1.997	0.305	0.046	0.21	0.98
79	1.013	2.000	0.269	0.046	0.19	0.98
80	+1.011	+2.003	−0.233	+0.046	−0.17	−0.98
81	1.010	2.005	0.198	0.046	0.16	0.99
82	1.008	2.007	0.163	0.046	0.14	0.99
83	1.006	2.007	0.128	0.047	0.12	0.99
84	1.005	2.006	0.093	0.047	0.11	0.99
85	+1.003	+2.005	−0.057	+0.047	−0.09	−1.00
86	1.002	2.004	−0.022	0.047	0.07	1.00
87	1.000	2.002	+0.013	0.047	0.05	1.00
88	0.998	2.000	0.048	0.047	0.03	1.00
89	0.997	1.997	0 082	0.047	−0.02	1.00
90	+0.995	+1.993	+0.117	+0.047	0.00	−1.00
91	0.993	1.989	0.152	0.047	+0.02	1.00
92	0.992	1.984	0.186	0.047	0.03	1.00
93	0.990	1.978	0.220	0.047	0.05	1.00
94	0.988	1.972	0.254	0.047	0.07	1.00
95	+0.987	+1.965	+0.287	+0.047	+0.09	−1.00
96	0.985	1.958	0.321	0.047	0.11	0.99
97	0.984	1.950	0.354	0.047	0.12	0.99
98	0.982	1.943	0.387	0.046	0.14	0.99
99	0.980	1.933	0.421	0.046	0.16	0.99
100	+0.979	+1.924	+0.453	+0.046	+0.17	−0.98
101	0.977	1.914	0.487	0.046	0.19	0.98
102	0.976	1.903	0.519	0.046	0.21	0.98
103	0.974	1.893	0.551	0.046	0.22	0.97
104	0.972	1.882	0.582	0.046	0.24	0.97
105	+0.971	+1.869	+0.614	+0.045	+0.26	−0.97
106	0.969	1.857	0.645	0.045	0.28	0.96
107	0.968	1.844	0.677	0.045	0.29	0.96
108	0.967	1.829	0.707	0.045	0.31	0.95
109	0.965	1.815	0.737	0.044	0.33	0.95
110	+0.964	+1.800	+0.768	+0.044	+0.34	−0.94
111	0.962	1.786	0.798	0.044	0.36	0.93
112	0.961	1.770	0.827	0.044	0.37	0.92
113	0.959	1.754	0.855	0.043	0.39	0.91
114	0.958	1.738	0.884	0.043	0.41	0.91
115	0.956	+1.721	+0.913	+0.043	+0.42	−0.90
116	0.955	1.704	0.942	0.042	0.44	0.89
117	0.954	1.686	0.970	0.042	0.45	0.88
118	0.953	1.668	0.997	0.041	0.47	0.87
119	+0.951	+1.650	+1.025	+0.041	+0.49	−0.87

g	$\frac{\partial\lambda}{\partial\varepsilon}$	$\frac{\partial\lambda}{\partial e}$	$\frac{\partial\lambda}{ed\pi}$	$\frac{\partial\rho}{\partial\varepsilon}$	$\frac{\partial\rho}{\partial e}$	$\frac{\partial\rho}{ed\pi}$
120°	+0.950	+1.631	+1.051	+0.041	+0.50	—0.86
121	0.949	1.611	1.077	0.040	0.51	0.85
122	0.948	1.592	1.104	0.040	0.53	0.84
123	0.947	1.571	1.129	0.039	0.54	0.83
124	0.945	1.551	1.154	0.039	0.56	0.82
125	+0.944	+1.530	+1.180	+0.038	+0.57	—0.81
126	0.943	1.509	1.205	0.038	0.59	0.80
127	0.942	1.488	1.229	0.037	0.60	0.79
128	0.941	1.466	1.253	0.037	0 62	0.78
129	0.940	1.443	1.277	0.037	0.63	0.77
130	+0.939	+1.421	+1.300	+0.036	+0.64	—0.76
131	0.937	1.397	1.322	0.036	0.66	0.75
132	0.936	1.374	1.344	0.035	0.67	0.74
133	0.936	1.350	1.367	0.034	0.68	0.72
134	0.935	1.327	1.388	0.034	0.70	0.71
135	+0.934	+1.302	+1.409	+0.033	+0.71	—0.71
136	0.933	1.277	1.430	0.033	0.71	0.70
137	0.932	1.253	1.451	0.032	0.72	0.68
138	0.931	1.228	1.470	0.031	0.74	0.67
139	0.930	1.202	1.489	0.031	0.75	0.66
140	+0.929	+1.177	+1.508	+0.030	+0.76	—0.64
141	0.928	1.151	1.527	0.030	0.77	0.63
142	0.927	1.124	1.545	0.029	0.78	0.62
143	0.927	1.099	1.562	0.028	0.79	0.60
144	0.926	1.072	1.580	0.028	0.80	0.59
145	+0.925	+1.044	+1.596	+0.027	+0.81	—0.57
146	0.924	1.016	1.613	0.026	0.82	0.56
147	0.923	0.989	1.629	0.026	0.83	0.54
148	0.922	0.962	1.644	0.025	0.84	0.53
149	0.922	0.934	1.659	0.024	0.85	0.51
150	+0.921	+0.906	+1.674	+0.023	+0.86	—0.50
151	0.921	0.878	1.688	0.023	0.87	0.49
152	0.920	0.849	1.702	0.022	0.87	0.47
153	0.919	0.820	1.714	0.021	0.88	0.45
154	0.919	0.792	1.727	0.021	0.89	0.44
155	+0.919	+0.762	+1.740	+0.020	+0.90	—0.42
156	0.918	0.733	1.751	0.019	0.91	0.41
157	0.918	0.704	1.763	0.018	0.91	0.39
158	0.917	0.674	1.773	0.018	0.92	0.37
159	0.916	0.645	1.783	0.017	0.93	0.36
160	+0.916	+0.615	+1.793	+0.016	+0.94	—0.34
161	0.915	0.585	1.803	0.015	0.95	0.33
162	0.915	0.555	1.811	0.015	0.95	0.31
163	0.915	0.525	1.820	0.014	0.96	0.29
164	0.915	0.494	1.829	0.013	0.96	0.28
165	+0.914	+0.465	+1.836	+0.012	+0.97	—0.26
166	0.914	0.435	1.843	0.011	0.97	0.24
167	0.914	0.403	1.849	0.011	0.97	0.22
168	0.913	0.373	1.855	0.010	0.98	0.21
169	0.913	0.343	1.860	0.009	0.98	0.19
170	+0.913	+0.311	+1.865	+0.008	+0.98	—0.17
171	0.912	0.280	1.870	0.007	0.99	0.16
172	0.912	0.249	1.875	0.006	0.99	0.14
173	0.912	0.219	1.879	0.006	0.99	0.12
174	0.912	0.187	1.882	0.005	0.99	0.11
175	+0.911	+0.156	+1.884	+0.004	+1.00	—0.09
176	0.911	0.126	1.886	0.003	1.00	0.07
177	0.911	0.094	1.888	0.002	1.00	0.05
178	0.911	0.063	1.889	0.002	1.00	0.03
179	+0.911	+0.031	+1.890	+0.001	+1.00	—0.02

In the equations of condition ten years has been adopted for the unit of time, in order to make the general value of the coefficients as nearly equal as possible, and the time has been counted from the epoch 1830.0, in order to have the positive and negative values of t in the equations more nearly balanced. To distinguish these values of $\delta\varepsilon$ and δn they are marked with an accent. This unit of time gives 0.8914 for the value of $\frac{2}{3n}$ in arc, whence

$$\frac{\partial\rho}{\partial n'} = t\frac{\partial\rho}{\partial l} - 0.891.$$

The equations of condition are now formed by putting in the preceding equations for heliocentric longitude and radius vector

$$\delta\lambda = \frac{\partial\lambda}{\partial\varepsilon}\delta\varepsilon + \frac{\partial\lambda}{\partial v}\delta v + \frac{\partial\lambda}{\partial e}\delta e + \frac{\partial\lambda}{e\partial\pi}e\delta\pi + \frac{\partial\lambda}{\partial\mu'}\delta\mu'$$

$$\delta\rho = \frac{\partial\rho}{\partial\varepsilon}\delta\varepsilon + \frac{\partial\delta}{\partial n}\delta n + \frac{\partial\rho}{\partial e}\delta e + \frac{\partial\rho}{e\partial\pi}e\delta\pi.$$

For the coefficients $\frac{\partial\lambda}{\partial\mu'}$ have been taken one-hundredth the perturbations of longitude produced by Neptune, as given in the heliocentric ephemeris at the end of Chapter V. The corrected mass of Neptune will then be

$$\frac{1}{17000}\left(1 + \frac{\delta\mu'}{100}\right)$$

Finally, I remark that all the preceding comparisons are made with the heliocentric ephemeris as printed, without the correction indicated in the column adjoining it, but in the following equations this correction is for the first time introduced.

Equations of condition given by the Corrections in Longitude.

1	$0.05\delta\varepsilon'$	$-0.70\delta n'$	$-0.10\delta e$	$+0.03e\delta\pi$	$+0.12\delta\mu'$	$=+1''.1$
2	0.10	− 1.11	+0.01	−0.19	+0.12	= + 2 .2
3	0.31	− 2.55	+0.33	+0.51	−0.22	= +12 .8
4	0.32	− 2.52	+0.27	+0.58	−0.18	= +11 .8
5	0.30	− 2.32	+0.12	+0.59	−0.09	= +11 .6
6	0.19	− 1.40	−0.01	+0.39	0.00	= + 5 .0
7	0.19	− 1.18	−0.29	+0.23	+0.22	= + 4 .8
8	1.41	− 6.75	−2.69	−0.75	+2.47	= + 2 8
9	1.06	− 5.01	−1.98	−0.72	+1.85	= + 1 .1
10	1.07	− 4.91	−1.94	−0.86	+1.85	= + 1 .7
11	1.09	− 4.87	−1.86	−1.04	+1.85	= − 0 .5
12	0.73	− 3.06	−1.04	−0.97	+1.16	= + 1 .0
13	0.54	− 2.20	−0.71	−0.78	+0.82	= + 1 .4
14	0.55	− 2.19	−0.68	−0.81	+0.81	= − 0 .6
15	0.73	− 2.86	−0.82	−1.16	+1.03	= − 0 .9
16	0.37	− 1.39	−0.36	−0.60	+0.49	= − 0 .2
17	0.56	− 2.05	−0.47	−0.95	+0.70	= − 0 .4
18	0.37	− 1.33	−0.26	−0.65	+0.44	= + 0 .6
19	0.75	− 2.59	−0.38	−1.34	+0.82	= − 1 .0

20	$0.56\delta\varepsilon'$	$-\ 1.89\delta n'$	$-0.22\delta e$	$-1.04e\delta\pi$	$+0.58\delta\mu'$	$=-\ 0''.9$
21	0.58	− 1.92	−0.13	−1.12	+0.55	=− 0 .5
22	0.38	− 1.15	+0.09	−0.74	+0.24	=− 0 .2
23	0.38	− 1.11	+0.15	−0.73	+0.21	=− 0 .3
24	1.15	− 3.18	+0.65	−2.13	+0.50	=+ 0 .6
25	1.13	− 2.81	+1.11	−1.91	+0.19	=+ 0 .5
26	0.56	− 1.33	+0.62	−0.89	+0.05	=+ 0 .1
27	1.12	− 2.54	+1.38	−1.68	0.00	=+ 2 .1
28	0.56	− 1.21	+0.75	−0.79	−0.04	=− 0 .1
29	0.70	− 1.55	+1.09	−0.97	−0.10	=+ 1 .6
30	1.11	− 2.19	+1.73	−1.32	−0.23	=+ 2 .2
31	1.09	− 2.04	+1.81	−1.14	−0.29	=+ 1 .3
32	0.56	− 0.98	+0.96	−0.51	−0.18	=+ 0 .7
33	0.55	− 0.91	+1.00	−0.43	−0.21	=+ 1 .1
34	1.08	− 1 69	+2.03	−0.71	−0.46	=+ 1 .5
35	1.63	− 2.37	+3.11	−0.82	−0.74	=+ 2 .0
35′	1.07	− 1.45	+2.08	−0.38	−0.52	=+ 0 .4
35″	1.06	− 1.33	+2.10	−0.22	−0.55	=+ 0 .8
36	1.58	− 1.83	+3.18	−0.09	−0.85	=+ 0 .3
37	1.05	− 1.11	+2.10	+0.08	−0.59	=− 0 .8
38	1.04	− 0.99	+2.07	+0.26	−0.59	=− 1 .3
39	0.69	− 0.58	+1.36	+0.27	−0.39	=+ 0 .8
40	0.69	− 0.51	+1.33	+0.38	−0.39	=+ 0 .8
41	0.68	− 0.44	+1.29	+0.47	−0.39	= 0 .0
42	1.01	− 0.56	+1.88	+0.84	−0.59	=+ 0 .3
43	0.67	− 0.30	+1.20	+0.64	−0.38	=− 0 .5
44	1.00	− 0.35	+1.72	+1.09	−0.57	=+ 0 .2
45	1.95	− 0.50	+3.21	+2.39	−1.12	=− 2 .5
46	2.42	− 0.37	+3.77	+3.21	−1.36	=− 2 .6
47	2.42	− 0.12	+3.53	+3.52	−1.37	=− 2 .4
48	2.39	+ 0.12	+3.22	+3.69	−1.33	=− 4 .9
49	1.91	+ 0.28	+2.37	+3.14	−1.07	=− 0 .1
50	1.91	+ 0.48	+2.15	+3.30	−1.08	=− 2 .4
51	2.37	+ 0.83	+2.34	+4.32	−1.34	=− 4 .5
52	2.36	+ 1.06	+2.02	+4.44	−1.36	=− 4 .3
53	2.37	+ 1.32	+1.68	+4.62	−1.37	=− 5 .5
54	2.85	+ 1.86	+1.66	+5.64	−1.68	=− 7 .5
55	2.84	+ 2.17	+1.22	+5.74	−1.71	=− 4 .1
56	2.84	+ 2.44	+0 81	+5.84	−1.74	=− 2 .1
57	2.83	+ 2.71	+0.40	+5.88	−1.78	=− 1 .7
58	2.83	+ 3.00	−0.06	+5.88	−1.82	=− 2 .0
59	2.82	+ 3.28	−0.44	+5.84	−1.86	=+ 0 .3
60	2.83	+ 3.57	−0.84	+5.81	−1.91	=+ 1 .2
61	2.85	+ 3.90	−1.27	+5.78	−1.98	=+ 4 .4
62	2.85	+ 4.21	−1.74	+5.64	−2.02	=+ 4 .8
63	1.91	+ 3.02	−1.42	+3.69	−1.39	=+ 4 .0
64	2.85	+ 4.78	−2.50	+5.32	−2.14	=+ 6 .0
65	1.91	+ 3.39	−1.92	+3.46	−1.47	=+ 5 .6
66	1.92	+ 3.61	−2.18	+3.32	−1.52	=+ 4 .3
67	1.93	+ 3.82	−2.40	+3.17	−1.56	=+ 6 .6
68	1.94	+ 4.05	−2.65	+2.98	−1.60	=+ 7 .3
69	1.94	+ 4.20	−2.83	+2.80	−1.64	=+ 6 .5

70	$1.95\delta\varepsilon'$	$+\ 4.44\delta n'$	$-3.04\delta e$	$+2.59e\delta\pi$	$-1.68\delta\mu$	$=+\ 6''.8$
71	1.98	$+\ 4.72$	-3.23	$+2.37$	-1.73	$=+\ 8\ .1$
72	1.99	$+\ 4.93$	-3.44	$+2.15$	-1.76	$=+\ 9\ .1$
73	1.99	$+\ 5.13$	-3.58	$+1.91$	-1.78	$=+\ 8\ .7$
74	2.00	$+\ 5.35$	-3.73	$+1.64$	-1.80	$=+\ 8\ .1$
75	2.53	$+\ 7.02$	-4.83	$+1.73$	-2.30	$=+10\ .6$
76	2.54	$+\ 7.34$	-4.96	$+1.33$	-2.29	$=+10\ .9$
77	2.56	$+\ 7.64$	-5.07	$+0.97$	-2.31	$=+\ 8\ .2$
78	2.07	$+\ 6.38$	-4.14	$+0.47$	-1.86	$=+\ 6\ .3$
79	2.59	$+\ 8.25$	-5.20	$+0.21$	-2.32	$=+\ 7\ .0$
80	2.60	$+\ 8.51$	-5.21	-0.15	-2.29	$=+\ 6\ .6$
81	2.11	$+\ 7.14$	-4.19	-0.51	-1.83	$=+\ 4\ .3$
82	1.60	$+\ 5.56$	-3.13	-0.58	-1.36	$=+\ 3\ .9$
83	2.67	$+\ 9.59$	-5.12	-1.37	-2.20	$=+\ 1\ .1$
84	2.68	$+\ 9.92$	-5.00	-1.80	-2.14	$=-\ 1\ .8$
85	2.17	$+\ 8.22$	-3.92	-1.75	-1.67	$=-\ 3\ .8$
86	2.73	$+10.61$	-4.97	-2.59	-2.01	$=-\ 9\ .0$
87	2.20	$+\ 8.78$	-3.63	-2.33	-1.52	$=-\ 9\ .5$
88	1.65	$+\ 6.74$	$-2\ 58$	-1.96	-1.08	$=-10\ .5$
89	2.77	$+11.61$	-4.04	-3.61	-1.69	$=-19\ .9$

The following are the approximate normals to which these equations give rise. Inaccuracies being detected in several of the equations of condition after these normals were formed, they do not accurately correspond to those equations as written.

$283.64\delta\varepsilon'$	$+\ 414.36\delta n'$	$-151.63\delta e$	$+247.23e\delta\pi$	$-176.03\delta\mu'$	$=+123''.5$
414.36	$+1619.44$	-689.11	$+260.26$	-436.02	$=+103\ .2$
-151.63	$-\ 689.11$	$+557.82$	$+\ 38.45$	$+122.88$	$=-399\ .8$
247.23	$+\ 260.26$	$+\ 38.45$	$+618.45$	-194.60	$=+267\ .3$
-176.03	$-\ 436.02$	$+122.88$	-194.60	$+163.13$	$=-128\ .1$

The values of the unknown quantities deduced from these normals were substituted in the equations of condition, and a farther approximation was made by solving the equations given by the residuals. The following are the first approximations given by the normals, and the finally concluded corrections

	Preliminary.	Final.
$\delta\mu'$,	-15.00	-13.44
$e\delta\pi$,	$-\ 0.52$	$-\ 0.36$
δe,	$-\ 4.25$	$-\ 4.04$
$10\delta n = \delta n'$,	$-\ 4.73$	$-\ 4.33$
$\delta\varepsilon'$,	$-\ 3.78$	$-\ 3.44$
Mass of Neptune	$\frac{1}{20000}$	$\frac{1}{19840}$.

The final values of the corrections being substituted in the equations leave the following system of residuals, or outstanding excesses of the observed longitudes over theory. Column $f\delta l$ gives the residual of the equation itself; the probable error of which has always been judged to be 0″.5, while in column δl this residual is divided by f to obtain the residual correction of the longitude itself. The values of the factors f are found with the original equations on pages 159 and 160.

No. of Eq.	Year.	$f\delta l$	δl	No. of Eq.	Year.	$f\delta l$	δl
		$''$	$''$			$''$	$''$
1	1691.0	—0.5	—10.	35″	1817.4	—0.3	— 0.3
				36	1818.4	—0.7	— 0.5
2	1715.2	—0.8	—10.	37	1819.4	—1.4	— 1.4
				38	1820.5	—1.4	— 1.4
3	1748.8	+1.5	+ 4.5	39	1821.5	+0.9	+ 1.4
4	1750.8	+0.8	+ 2.4				
5	1753.9	+2.2	+ 6.6	40	1822.5	+1.4	+ 2.1
6	1756.7	—0.2	— 1.0	41	1823.5	+0.6	+ 0.9
				42	1824.5	+1.2	+ 1.2
7	1769.0	—0.2	— 1.0	43	1825.5	+0.7	+ 1.0
8	1782.0	+0.1	+ 0.2	44	1826.5	+2.0	+ 2.0
9	1783.0	—0.5	— 0.5				
10	1784.0	+0.6	+ 0.6	45	1827.7	+1.0	+ 0.5
11	1785.0	—1.0	— 1.0	46	1828.7	+2.5	+ 1.0
				47	1829.7	+2.8	+ 1.1
12	1788.0	+1.2	+ 1.8	48	1830.7	+0.4	+ 0.2
13	1789.0	+1.7	+ 3.4	49	1831.7	+4.1	+ 2.0
14	1790.0	—0.5	— 1.0				
15	1791.0	—0.7	— 1.0	50	1832.7	+1.8	+ 0.9
16	1792.0	0.0	0.0	51	1833.8	+0.5	+ 0.2
				52	1834.8	+0.1	0.0
17	1793.0	—0.1	— 0.2	53	1835.8	—1.4	— 0.6
18	1794.0	+0.5	+ 1.5	54	1836.8	—3.4	— 1.1
19	1795.0	—0.6	— 0.9				
20	1796.0	—0.5	— 1.0	55	1837.8	—0.6	— 0.2
21	1797.1	—0.6	— 1.2	56	1838.8	+0.5	+ 0.2
				57	1839.8	—0.4	— 0.1
22	1800.2	—1.4	— 4.2	58	1840.8	—1.7	— 0.6
23	1801.2	—1.4	— 4.2	59	1841.8	—0.3	— 0.1
24	1802.3	—0.6	— 0.6				
25	1805.3	—1.4	— 1.4	60	1842.8	—0.5	— 0.2
26	1806.3	—1.0	— 2.0	61	1843.8	+1.6	+ 0.5
27	1807.3	—0.1	— 0.1	62	1844.9	+0.7	+ 0.2
28	1808.3	—1.0	— 2.0	63	1845.9	+0.6	+ 0.3
				64	1846.9	—0.4	— 0.1
29	1809.3	+0.1	+ 0.2				
30	1810.3	—0.1	— 0.1	65	1847.9	+0.6	+ 0.3
31	1811.3	—0.9	— 0.9	66	1848.9	—1.6	— 0.8
				67	1849.9	+0.2	+ 0.1
32	1812.4	—0.3	— 0.6	68	1850.9	+0.3	+ 0.2
33	1813.4	+0.1	+ 0.2	69	1851.9	—1.2	— 0.6
34	1814.4	—0.4	— 0.4				
35	1815.4	—0.3	— 0.2	70	1852.9	—1.2	— 0.6
35′	1816.4	—0.8	— 0.8	71	1853.9	—0.2	— 0.1

No. of Eq.	Year.	$f\delta l$	δl
		″	″
72	1854.9	+0.5	+ 0.2
73	1855.9	0.0	0.0
74	1856.9	—0.5	— 0.2
75	1858.0	—0.2	— 0.1
76	1859.0	+0.9	+ 0.4
77	1860.0	—1.1	— 0.4
78	1861.0	—0.6	— 0.3
79	1862.0	—0.6	— 0.2
80	1863.0	+0.2	+ 0.1
81	1864.0	+0.7	+ 0.4
82	1865.0	+2.2	+ 1.5
83	1866.0	+1.0	+ 0.4
84	1867.0	+0.5	+ 0.2
85	1868.0	+0.4	+ 0.2
86	1869.0	—1.9	— 0.8
87	1870.0	—0.1	0.0
88	1871.0	—1.2	— 0.8
89	1872.1	+0.4	+ 0.2

A simple glance at the course of the residuals shows (1) that their probable value is considerably greater than the probable error attributed to the equations of condition, being more nearly 0″.7 than 0″.5, and yet larger in the later years; (2) that during certain periods they are of a systematic character. During the years 1748 to 1753 the observations show a decided positive correction to the theory of a magnitude greater than we can consider probable, amounting to about one-third of a second of time in the mean of Bradley's two observations of 1748 and 1753. About 1800 the correction becomes negative, and so continues for 20 years with an average value of about 1″. In 1821 it suddenly becomes positive, and so continues until 1833. From this year forward the residuals are not systematic in character.

In order to show clearly the general course of the outstanding corrections, they have been divided into groups, generally including about five years each. The mean outstanding correction for each group, taken with respect to the weights indicated by the factors *f*, is as follows. In the column ε is shown what the probable error of the residual should be if the weights assigned to the several equations were strictly correct, and no systematic errors were present either in theory or observation.

Year.	δl	ε
	″	″
1715.2	—10.	±6.
1751.1	+ 3.7	±0.8
1769.0	— 1.0	±2.5
1783.3	— 0.18	±0.23
1790.0	+ 0.62	±0.40
1795.0	— 0.55	±0.41
1802.0	— 1.25	±0.45
1806.5	— 1.00	±0.31
1810.5	— 0.37	±0.32
1814.5	— 0.37	±0.23
1819.5	— 0.37	±0.18
1824.8	+ 1.50	±0.16
1829.7	+ 0.91	±0.10
1835.2	— 0.27	±0.09
1839.8	— 0.17	±0.07
1844.8	+ 0.14	±0.03
1849.9	— 0.21	±0.11
1854.9	— 0.14	±0.11
1860.0	— 0.13	±0.09
1865.0	+ 0.36	±0.10
1870.0	— 0.21	±0.11

A simple glance at the residuals δl shows that they are much greater than the purely accidental residuals resulting from the theory of least squares. We may divide the possible causes of these systematic errors into three classes.

1. *Systematic Errors of Observation.*—These may result from deviation of the line of collimation of the instrument from a true great circle, or from any peculiarity of the observer which leads to his registering the transit of Uranus earlier or later than that of a fixed star. If we compare the corrections derived from the work of different observatories as given in the last chapter, we shall find frequent cases not only of systematic differences between the results of different observatories, but between those of the same observatory in two successive years. An instance which particularly attracted my attention on first preparing the comparisons of theory and observation is that of the Greenwich observations for 1831, which, as compared with observations at the same observatory during the years preceding and following, seem to be affected with some constant error in R. A. of about 2″. I find that this discrepancy can be attributed only to the original observations.

2. *Errors in the Theory compared.*—These may arise from errors in the preceding theoretical computations, from the omission of the terms of the second order produced by Neptune, from the adoption of an erroneous mass of Saturn, or from the attraction of an unknown planet. With regard to the probability of these different sources of error it may be remarked that errors of computation seem possible only in the terms of the second order, that the mass of Saturn is taken from the exhaustive discussion of the Saturnian system by Bessel, in which an error sufficient to influence the theory of Uranus seems highly improbable, and that a trans-Neptunian planet large enough to produce a sensible deviation of the orbit of Uranus from an ellipse in the course of a century would be too large to have escaped detection. The choice of the elliptic elements of Uranus and Neptune is such that the terms of the second order, due to the action of Neptune, can scarcely become sensible within a century of the epoch.

3. *Errors in the various Reductions by which Theory and Observation are compared.*—In the method adopted for comparing theory and observation a number of small uncertainties incident to the imperfections of the older data of reduction necessarily creep in. In the early observations the imperfections arise principally from the uncertainty of the instrumental corrections, and the errors in the adopted positions of the fundamental stars, and indeed in nearly all the data of reduction. In the late years they arise principally from the great magnitude of the correction to Bouvard's tables, and the consequent rapid change of the corrections to the geocentric ephemerides, which make the determination of the corrections Δl and δl from theory and observation somewhat uncertain. Errors from this source will necessarily be in part of a systematic character, and, in view of their possibility, I regret not having been able to completely re-reduce all the observations before 1840, and to compare all since directly with ephemerides computed from the provisional theory. In order, however, to test the question whether they are sensible, I have prepared an ephemeris from the provisional theory for the three recent oppositions of 1861–2, 1862–3, and 1872, and compared it directly with

the observations. The mean corrections in geocentric longitude for groups of observations are given in columns (2), column (1) showing the correction given by the work of the last chapter.

Opposition	1861–2 (1)	1861–2 (2)	1862–3 (1)	1862–3 (2)	1872 (1)	1872 (2)
	+2″.8	+2″.4$_1$	+2″.6	+2″.6$_1$	−8″.6	−9″.0$_1$
	2 .9	2 .4$_3$	2 .3	2 .3$_2$	−8 .5	−8 .1$_2$
	3 .0	3 .1$_2$	2 .1	2 .0$_3$	−7 .3	−7 .5$_3$
	2 .4	2 .4$_2$	3 .3	2 .9$_4$	−7 .3	−7 .7$_5$
	. . .	. . .	2 .6	2 .7$_1$	−7 .8	−7 .6$_1$
Mean	+2 .79	+2 .62	+2 .65	+2 .50	−7 .65	−7 .82

A systematic difference of 0″.16 would seem to be indicated, and on account of it a correction of 0″.10 was applied to the comparisons of the last few years in forming the equations of condition.

In view of the possibility of systematic errors from this source it may be considered that too great relative weight has been assigned to the results of the later observations. If the residuals arise from errors of comparison and of theory, their probable magnitude is nearly as great at one epoch as at another. It may therefore be interesting to inquire what result we should get if, instead of assigning such different weights to the comparisons at different epochs, we sought only for the best general agreement with observations during the period the planet has been observed. The preceding system of mean residuals will enable us to discuss this question quite easily. In the first solution we shall reject the results from Flamstead's observations, owing to their assured uncertainty, and those from Le Monnier's of 1769, owing to the possible maladjustment of his quadrant. The equations from the remaining residuals will be the following:

$1.0\delta^2\epsilon'$	$-7.6\delta^2n'$	$+0.8\delta^2e$	$+1.7e\delta^2\pi$	$-0.5\delta^2\mu'$	$=+3''.7$	Wt. ½
1.1	−5.0	−2.0	−0.8	+1.8	= −0 .18	2
1.1	−4.4	−1.4	−1.6	+1.6	= +0 .62	1
1.1	−3.9	−0.6	−2.0	+1.2	= −0 .55	1
1.2	−3.2	+0.6	−2.1	+0.5	= −1 .25	1
1.1	−2.6	+1.3	−1.8	0.0	= −1 .00	2
1.1	−2.2	+1.7	−1.3	−0.2	= −0 .37	2
1.1	−1.7	+2.0	−0.7	−0.5	= −0 .37	2
1.0	−1.1	+2.1	+0.1	−0.6	= −0 .37	2
1.0	−0.5	+1.8	+0.9	−0.6	= +1 .50	2
1.0	0.0	+1.4	+1.4	−0.5	= +0 .91	3
0.9	+0.5	+0.8	+1.8	−0.5	= −0 .27	3
0.9	+0.9	+0.1	+2.0	−0.6	= −0 .17	3
1.0	+1.4	−0.6	+1.9	−0.7	= +0 .14	3
1.0	+1.9	−1.2	+1.6	−0.8	= −0 .21	3
1.0	+2.5	−1.7	+1.1	−0.9	= −0 .14	3
1.0	+3.1	−2.0	+0.4	−0.9	= −0 .13	3
1.1	+3.7	−2.1	−0.4	−0.9	= +0 .36	3
1.1	+4.4	−1.8	−1.2	−0.8	= −0 .21	3

Giving these nineteen equations equal weights, we have the second of the following solutions, and the second of the series of residuals the first corresponding to the primitive solution. Solving them again and assigning the weights attached to the respective equations, which I judge to be tnose to which they are entitled when a liberal allowance is made for systematic errors of observation and of comparison of theory with observations, adding also the equations given by the observations of Flamstead and Le Monnier, which are as follows:

1690	$0.0\delta^2\varepsilon'$	$-0.7\delta^2n'$	$-0.1\delta^2e$	$+0.0e\delta^2\pi$	$+0.1\delta^2\mu'$	$= -0''.5$;	f, $\frac{1}{20}$;	Wt, 1
1715	0.1	-1.1	0.0	-0.2	$+0.1$	$= -0.5$	$\frac{1}{12}$	1
1769	0.2	-1.2	-0.3	$+0.2$	$+0.2$	$= +2.2$	$\frac{1}{6}$	1

we have the second solution, and the third series of residuals.

	(1)	(2)	(3)
$\delta^2\varepsilon_1$	0	$-0''.39$	-0.21
δ^2n_1	0	-0.38	-0.19
δ^2e	0	-0.33	-0.15
$e\delta^2\pi$	0	$+0.25$	$+0.19$
$\delta^2\mu'$	0	-1.02	-0.49
m'	$\frac{1}{19640}$	$\frac{1}{19870}$	$\frac{1}{19750}$

Year.	Residuals. $\Delta_1 l$	$\Delta_2 l$	$\Delta_3 l$
	$''$	$''$	$''$
1691.0	$-10.$	$-14.$	$-12.$
1715.2	$-10.$	$-9.$	$-7.$
1751.1	$+3.7$	$+0.5$	$+2.0$
1769.0	-1.0	-2.6	-1.9
1783.3	-0.18	-0.30	-0.17
1790.0	$+0.62$	$+0.93$	$+0.89$
1795.0	-0.55	-0.09	-0.18
1802.0	-1.25	-0.78	-0.87
1806.5	-1.00	-0.69	-0.73
1810.5	-0.37	-0.10	-0.05
1814.5	-0.37	-0.26	-0.28
1819.5	-0.37	-0.34	-0.37
1824.8	$+1.50$	$+1.46$	$+1.41$
1829.7	$+0.91$	$+0.90$	$+0.81$
1835.2	-0.27	-0.43	-0.46
1839.8	-0.17	-0.56	-0.48
1844.8	$+0.14$	-0.31	-0.18
1849.9	-0.21	-0.70	-0.52
1854.9	-0.14	-0.54	-0.36
1860.0	-0.13	-0.23	-0.14
1865.0	$+0.36$	$+0.70$	$+0.62$
1870.0	-0.21	$+0.80$	$+0.44$

It will be seen that the effect of these changes of weights is, that the older observations are a little better, and the later a little worse represented. I conceive that our choice must lie between the first and third solutions, the first being the more probable if we conceive the outstanding residuals to be due to errors of observation only, and the third if we suppose them equally due to errors of computation. On the whole, I consider the mean of the two to be about the most probable, and this will give the mass of Neptune very near the round number

$$\frac{1}{19700}$$

which will be adopted as the definitive value. The definitive corrections to Elements III (p. 99) will then be

$\delta\varepsilon'$ (1830)	$-\ 3''.56$
$\delta\varepsilon$ (1850)	-12.45
$10\delta n$	$-\ 4.44$
δe	$-\ 4.12$
$e\delta\pi$	$-\ 0.25$
$\delta\mu$	$-\ 0.137$

Corrections to the Inclination and Node.

These corrections have been derived entirely from the modern observations, the ancient ones being too uncertain to add anything to the weight of the result. The mode in which the correction to the latitude of the provisional ephemeris has been concluded from the observations has been sufficiently explained: it is only necessary to add that the immediate results from the data of the preceding chapter require two corrections, namely:

(1) A correction to the theoretical latitude for the change in the adopted mass of Neptune. The value of this correction, as derived from the data of Chapter V, is with sufficient approximation

$$\delta\beta = 0''.25\, T \cos g.$$

(2) A correction to the observed latitude on account of the difference between the obliquity of the ecliptic adopted in the various ephemerides compared, and that of Hansen's *Tables du Soleil*, which having been adopted in the theory should be used throughout.

Applying the correction (2) — (1) to all the observed latitudes, we have the following corrections to the latitude of the provisional ephemeris derived from all the observations of each opposition since 1781. The third column gives the number of observations in declination. These numbers may, however, in some cases be inaccurate. The fourth and fifth columns give the sine and cosine of the argument of latitude, to be used in forming the equations of condition.

Year.	$\delta\beta$	No. of obs.	sin u	cos u	Year.	$\delta\beta$	No. of obs.	sin u	cos u
	″					″			
1782.0	—0.4	21	+0.31	+0.95	1830.7	+0.2	48	—0.82	—0.57
1783.0	—3.5	13	0.38	0.92	1831.7	+1.0	23	0.87	0.50
1784.0	—2.4	13	0.45	0.89	1832.7	+0.9	20	—0.90	—0.44
1785.0	—0.1	10	0.52	0.85	1833.7	+0.6	54	0.93	0.37
1788.0	—1.0	5	+0.71	+0.71	1834.7	+0.3	92	0.95	0.31
1789.0	+0.9	6	0.77	0.64	1835.7	+0.2	71	—9.97	—0.24
1790.0	+2.6	4	0.81	0.58	1836.7	+0.1	135	0.98	0.17
1791.0	+1.9	7	0.86	0.51	1837.8	0.0	154	0.99	0.10
1792.0	+1.8	3	+0.90	+0.44	1838.8	—0.2	182	—1.00	—0.03
1793.0	+2.9	5	0.93	0.37	1839.8	—0.4	142	1.00	+0.03
1794.0	+0.5	3	0.95	0.30	1840.8	—0.3	106	0.99	0.10
1795.0	+0.7	7	+0.97	+0.22	1841.8	—0.7	101	—0.98	+0.17
1796.0	+3.6	4	0.99	0.14	1842.8	—0.7	145	0.97	0.24
1797.0	+3.6	3	1.00	+0.05	1843.8	—0.9	88	0.95	0.31
1800.2	0.0	2	+0.98	—0.21	1844.9	—1.0	87	—0.93	+0.37
1801.2	+1.2	2	0.97	0.26	1845.9	—0.6	55	0.90	0.44
1802.3	+1.4	13	0.94	0.34	1846.9	—1.1	92	0.87	0.50
1805.3	+1.1	3	+0.83	—0.56	1847.9	—1.1	69	—0.83	+0.56
1806.3	—1.4	5	0.78	0.63	1848.9	—0.9	56	0.79	0.62
1807.3	+1.6	16	0.72	0.69	1849.9	—1.0	36	0.74	0.67
1808.3	+1.8	6	+0.67	—0.74	1850.9	—0.7	46	—0.69	+0.72
1809.3	+1.3	9	0.60	0.80	1851.9	—1.2	35	0.64	0.77
1810.3	+3.7	16	0.53	0.85	1852.9	—1.3	49	0.59	0.81
1811.3	+3.4	11	+0.45	—0.89	1853.9	—1.8	48	—0.53	+0.85
1812.4	+3.6	6	0.38	0.93	1854.9	—1.0	47	0.47	0.88
1813.4	+2.4	6	0.29	0.96	1855.9	—1.3	49	0.41	0.91
1814.4	+2.5	13	+0.22	—0.98	1856.9	—1.1	41	—0.34	+0.94
1815.4	+2.2	16	0.14	0.99	1858.0	—1.7	65	0.26	0.97
1816.4	+1.3	18	0.06	1.00	1859.0	—1.7	56	0.19	0.98
1817.5	+2.4	13	—0.01	—1.00	1860.0	—1.9	58	—0.12	+0.99
1818.5	+4.2	22	0.09	1.00	1861.0	—2.3	41	0.05	1.00
1819.5	+1.7	7	0.17	0.99	1862.0	—1.6	52	+0.02	1.00
1820.5	+1.0	9	—0.24	—0.97	1863.0	—1.7	83	+0.10	+0.99
1821.5	+1.3	10	0.31	0.95	1864.0	—2.2	39	0.18	0.99
1822.5	+1.8	7	0.38	0.92	1865.0	—1.7	37	0.25	0.97
1823.5	+0.6	7	—0.45	—0.89	1866.0	—0.9	72	+0.33	+0.95
1824.5	—0.2	11	0.51	0.86	1867.0	—0.6	83	0.40	0.92
1825.5	+1.5	5	0.57	0.82	1668.1	—0.9	32	0.47	0.88
1826.5	+1.2	7	—0.63	—0.78	1869.1	—0.8	65	+0.54	+0.84
1827.6	+2.0	5	0.68	0.73	1870.1	—0.4	32	0.60	0.80
1828.6	—1.5	7	0.73	0.68	1871.1	—0.7	21	+0.66	0.75
1829.7	+0.9	9	—0.78	—0.63	1872.1	—0.4	47	+0.72	+0.69

It will be remembered that the observed declinations have, as far as possible, been reduced to Auwers' standard. We have no positive proof that this standard is correct. If it be affected by a constant error, the result will be that the orbit of the planet on the celestial sphere, as deduced from observation, instead of being a great circle, as we know the real orbit to be, will be a small one, and the comparison of a uniform series of observations extending through an entire revolution of the planet, after making the best correction to the position of the orbit, will leave a constant residual. Now, we can best determine this residual by including it as an unknown quantity in our equations.

Again, the error of the standard is not necessarily constant, but may contain a term proportional to the time, arising from erroneous proper motions of the standard stars. Therefore, instead of supposing the residual constant, we shall suppose it of the form $a + bt$. Each observed correction to the theoretical latitude will then give the equation,

$$\sin u\delta\phi - \cos u\phi\delta\theta + a + bT = \delta\beta.$$

To facilitate the solution of these equations they have been divided into groups, each group usually comprehending three oppositions, and combined into a single equation multiplied by such a factor as would make its probable error half a second. The factor by which the correction of the latitude is multiplied in the equation is the same with the coefficient of a. The year 1840.0 is taken as the epoch for b. Thus we have the following:

EQUATIONS OF LATITUDE.

Dates of oppositions.	No. of opp.	Equation.				
						$''$
1782.0–85.0	4	$0.4\delta\phi$	$-0.9\phi\delta\theta$	$+1.0a$	$-0.6b$	$= -1.6$
1788.0–91.0	4	0.8	−0.6	+1.0	$-0.5b$	= +1.1
1792.0–94.0	3	0.5	−0.2	+0.5	−0.2	= +0.9
1795.0–97.1	3	0.3	0.0	+0.5	−0.2	= +0.7
1800.2–02.3	3	0.0	+0.3	+1.0	−0.4	= +1.0
1805.3–07.3	3	0.8	+0.6	+1.0	−0.3	= +0.7
1808.3–10.3	3	0.6	+0.8	+1.0	−0.3	= +2.6
1811.3–13.4	3	0.4	+0.9	+1.0	−0.3	= +3.1
1814.4–16.4	3	0.2	+1.5	+1.5	−0.4	= +3.0
1817.4–19.5	3	−0.1	+1.5	+1.5	−0.3	= +4.8
1820.5–22.5	3	−0.5	+1.4	+1.5	−0.3	= +2.0
1823.5–25.5	3	−0.5	+0.9	+1.0	−0.2	= +0.6
1826.5–28.6	3	−0.7	+0.7	+1.0	−0.1	= +0.6
1829.6–31.7	3	−1.2	+0.8	+1.5	−0.1	= +1.1
1832.7–34.7	3	−2.8	+1.1	+3.0	−0.2	= +1.8
1835.7–37.8	3	−2.9	+0.5	+3.0	−0.1	= +0.3
1838.8–40.8	3	−3.0	−0.1	+3.0	0.0	= −0.9
1841.8–43.8	3	−2.9	−0.7	+3.0	+0.1	= −2.3
1844.8–46.8	3	−2.7	−1.3	+3.0	+0.2	= −2.7
1847.8–49.9	3	−2.4	−1.9	+3.0	+0.3	= −3.0
1850.9–52.9	3	−1.9	−2.3	+3.0	+0.4	= −3.2
1853.9–55.9	3	−1.4	−2.6	+3.0	+0.4	= −4.1
1856.9–59.0	3	−0.8	−2.9	+3.0	+0.5	= −4.5
1860.0–62.0	3	−0.2	−3.0	+3.0	+0.6	= −5.8
1863.0–65.0	3	+0.5	−3.0	+3.0	+0.7	= −5.6
1866.0–68.0	3	+1.2	−2.8	+3.0	+0.8	= −2.4
1869.1–70.0	2	+1.1	−1.6	+2.0	+0.6	= −1.2
1871.1–72.1	2	+1.4	−1.4	+2.0	+0.6	= −1.1

Treating these equations by the method of least squares, we find the normal equations

$$\begin{aligned}
63.60''\delta\phi + 6.45''\phi\delta\theta - 52.10''a - 1.08''b &= + 28.33'' \\
6.45\delta\phi + 69.59\phi\delta\theta - 52.60a - 14.25b &= + 111.02 \\
-52.10\delta\phi - 52.60\phi\delta\theta + 133.50a + 8.95b &= - 76.55 \\
- 1.08\delta\phi - 14.25\phi\delta\theta + 8.95a + 4.49b &= - 23.69
\end{aligned}$$

The solution of these equations gives

$$\begin{aligned}\delta\phi &= +\,0''.28 + 0''.75\ a = +\,0''.54\\ \phi\delta\theta &= +\,1\,.57 + 0\,.686a + 0\,.205b = +\,1''.75\\ a &= +\,0\,.35\\ b &= -\,0\,.28\end{aligned}$$

These values of a and b indicate that at the epoch 1840 Auwers' equatorial declinations are too great, or his north polar distances are too small by 0".35, and that this error is diminishing at the rate of 0".28 per century. If the older measures in declination had been comparable in precision with those made at the present time, and if the possible periodic error in the reduced right ascensions had been carefully eliminated, I should regard this determination as entitled to considerable weight. In view of the great uncertainty of the declinations previous to 1820, it can be regarded as little more than a rough attempt at a determination. For this reason the first two normal equations have been solved, leaving a and b indeterminate, so as to show the valves of $\delta\phi$ and $\phi\delta\theta$ in terms of these quantities. It will be seen that had we neglected a and b entirely, the value of $\delta\phi$ would have been smaller by 0".26, and that of $\phi\delta\theta$ smaller by 0".18 than those actually concluded. As the observations with the Washington Transit Circle, and those with the Pulkowa Vertical Circle, both indicate an increase of Auwers' polar distances, I shall take for the definitive corrections to the inclination and node those which follow from the above values of a and b, or,

$$\begin{aligned}\delta\phi &= +\,0''.54\\ \phi\delta\theta &= +\,1\,.75.\end{aligned}$$

The following table shows the residuals of the equations, and the mean outstanding corrections to the latitude, (1) when the concluded values of $\delta\phi$ and $\phi\delta\theta$ a and b are all used, and (2) when a and b are supposed zero, and the values of $\delta\phi$ and $\phi\delta\theta$, corresponding to this supposition, are used:

Year.	Residuals.		$\delta\beta$	
	(1)	(2)	(1)	(2)
	"	"	"	"
1783	—0.8	—0.3	—0.8	—0.3
1789	+1.2	+1.8	+1.2	+1.8
1793	+0.7	+1.1	+1.4	+2.2
1796	+0.3	+0.6	+0.6	+1.2
1801	—0.5	+0.3	—0.5	+0.3
1806	—1.2	—0.5	—1.2	—0.5
1809	+0.5	+1.2	+0.5	+1.2
1812	+0.9	+1.6	+0.9	+1.6
1815	—0.3	+0.6	—0.2	+0.4
1818	+1.6	+2.5	+1.1	+1.7
1821	—0.7	—0.1	—0.5	—0.1
1824	—1.3	—0.7	—1.3	—0.7
1827	—0.6	—0.3	—0.6	-0.3

Year.	Residuals. (1)	(2)	$\delta\beta$ (1)	(2)
	$''$	$''$	$''$	$''$
1830	−0.2	+0.2	−0.1	+0.1
1833	+0.3	+0.9	+0.10	+0.30
1836	0.0	+0.3	0.00	+0.10
1839	−0.1	+0.1	−0.03	+0.03
1842	−0.5	−0.4	−0.17	−0.13
1845	+0.1	0.0	+0.03	0.00
1848	+0.6	+0.7	+0.20	+0.23
1851	+0.9	+0.9	+0.30	+0.30
1854	+0.3	+0.3	+0.10	+0.10
1858	+0.1	+0.3	+0.03	+0.10
1861	−1.4	−1.0	−0.47	−0.33
1864	−1.5	−1.0	−0.50	−0.33
1867	+1.1	+1.7	+0.37	+0.57
1869	+0.5	+1.0	+0.17	+0.33
1871	0.0	+0.7	0.00	+0.23

The sum of the squares of the residuals is in the first case 17″.94, and in the second 25″.41, so that the introduction of a and b makes a decided improvement in the representation of the observations.

I have not attempted a rigorous investigation of the probable error of any of these results for the reason that the values of the probable error deducible by the method of least squares would, in a case like the present, be entirely untrustworthy. It is, however, very desirable that we should be able to form some judgment of the uncertainty of the mass of Neptune. From the last system of equations of condition the value of μ' comes out with the weight 3.13, or nearly that assigned to the mean result of each five years of modern observations. Regarding these results as independent, their mean error would be about 0″.5, so that the probable error of μ' would be 0.5, and that of μ would be .005, or about $\frac{1}{200}$ the entire mass of Neptune. A probable error derived from the original equations would have been much smaller, and when, in the last equations, we allow for the systematic character of the residuals, it will be larger. If we suppose the theory to be perfect, I conceive we may fairly estimate the probable error of the mass of Neptune to be $\frac{1}{100}$ of its entire amount, and its possible error two or three times greater. If there is any error or imperfection in the theory, the error may be much larger.

CHAPTER VIII.

COMPLETION AND ARRANGEMENT OF THE THEORY TO FIT IT FOR PERMANENT USE.

In the preceding discussions the terms of the second order due to the action of Neptune have been neglected, the elements of Uranus and Neptune being so chosen that these terms can scarcely become sensible within a century of the epoch. But this very choice will make them larger in the course of centuries than if mean elements had been chosen. They will be most sensible in the case of the great inequality of 4300 years between Uranus and Neptune, an inequality which will make centuries of observation necessary to an accurate determination of the mean elements of the two orbits. The uncertainty arising from the great inequality is probably of the same order of magnitude with the omitted terms of the second order, and, such being the case, the theory would really be made but little more accurate by the addition of those terms. I conceive, however, that the theory will be made much more satisfactory by the computation of at least the largest of the terms in question, if only to arrive at a certain determination of their order of magnitude, and of their effect on the planet during the period in which it has been observed.

The term in question, being of very long period, may be most advantageously treated by the method of variation of elements, more especially as it has in the theory been already treated as such a perturbation. The largest of the perturbations in question are those of the mean longitude which are multiplied by the square of the integrating factor ν, which is nearly 51, but which also contain the eccentricities as factors, and those of the eccentricity and perihelion which are independent of the eccentricities, but are multiplied by only the first power of ν. These terms will probably comprise nearly or quite nine-tenths of those arising from the term of long period.

Let us begin with the perturbations of mean longitude. These are given by the integration of the equation

$$\frac{d^2l}{dt^2} = -3m'an^2 \{ek_1 \sin(2l'-l-\pi) + e'k_2 \sin(2l'-l-\pi')\}$$

k_1 and k_2 being functions of the ratio of the mean distances, or α. If we integrate this equation, supposing all the quantities in the second member except l' and l to be constant, and these two to be of the form $nt+\varepsilon$, n and ε being constants, we shall reproduce the principal term of long period already found. But in the second approximation we must suppose all the elements variable. It is not, however, necessary to take into account the variations of a, n, and k, because these are

of a lower order of magnitude. The perturbations to be added will be those of l, l', e, e', π, and π'.

The point from which the longitudes are counted being arbitrary, we shall take the position of the perihelion of Uranus for 1850.0 as the origin, and put, as before, g for the mean longitude of Uranus counted from this point, and let l' represent the mean longitude of Neptune counted from the same point. The terms of $\frac{d^2l}{dt^2}$ within the brackets will thus become

$$ek_1 \sin(2l'-g-\delta\pi) + e'k_2 \sin(2l'-g-(\pi'-\pi))$$

or, if we put

$$2l'-g = N$$
$$e' \sin(\pi'-\pi) = h'$$
$$e' \cos(\pi'-\pi) = k'$$

and notice, that to terms of the first order we have, $\sin\delta\pi = \delta\pi$, $\cos\delta\pi = 1$, we shall have

$$\frac{d^2l}{dt^2} = -3m'an^2 \{(ek_1 + k'k_2)\sin N - (ek_1\delta\pi + h'k_2)\cos N\}$$

differentiating the quantities, of which the perturbations are to be considered with respect to the sign δ, we find for the terms of the second order.

$$\delta\frac{d^2l}{dt^2} = -3m'an^2 \{(k_1\delta e + k_2\delta k' + h'k_2\delta N)\sin N$$
$$+((ek_1 + k'k_2)\delta N - ek_1\delta\pi - k_2\delta h')\cos N\}.$$

We have now to substitute in this expression the numerical values of the quantities within parentheses. Those of the perturbations of Uranus have already been given in Chapter III, but it is necessary to diminish them by the factor 0.145* for the altered mass of Neptune. Those of Neptune are taken from my investigation of the orbit of that planet (p. 38). The mass of Uranus there adopted is $\frac{1}{21100}$, while the investigation of Dr. Von Asten,† from the observations of Struve and others, shows it to be $\frac{1}{22600}$. The perturbations are therefore diminished by $\frac{1}{22}$. In accordance with the system adopted throughout both investigations, constants are added to all the perturbations to make them vanish at the epoch 1850.0. A term is also added to make $\frac{dl}{dt}$ also vanish at the epoch; this corresponds to the constant which ought to be added to δa. The numerical values thus obtained, are:

---

* This factor was adopted before the mass of Neptune to be employed had been finally decided upon. Hence the difference between it and that in the preceding chapter.

† Mémoires de l'Académie de St. Pétersbourg, tome xviii, vii série.

$$\begin{aligned}
\delta N &= + \ 7260'' \sin N - 6658'' + 4''.26t \\
e\delta\pi &= - \ \ 414 \ \sin N + \ 380 \\
\delta e &= - \ \ 414 \ \cos N - \ 165 \\
\delta h' &= + \ \ 120 \ \sin N - \ 110 \\
\delta k' &= + \ \ 120 \ \cos N + \ \ 48 \\
k_1 &= - \ 1.234 \\
k_2 &= + \ 0.452 \\
h' &= + \ 0.00695 \\
k' &= - \ 0.00486
\end{aligned}$$

Substituting these values in the expression for $\delta \frac{d^2 l}{dt^2}$ and integrating twice, we find, putting b for the coefficient of the time in N, of which the value, taking the century as the unit, is $+0.1472$, and putting T for the time in centuries,

$$\delta l = \frac{3}{2} m' a \nu^2 \left\{ (411'' + \frac{102''.8}{b} + 2''.6 T) \sin N + (1837'' + \frac{5''.3}{b} - 51.''4 T) \cos N \right.$$
$$\left. - 109''.3 \sin 2N - 5''.5 \cos 2N \right\}$$
$$- 16''.5 \, m' a n^2 t^2 + cT + c',$$

c and c' being the arbitrary constants of integration, which are to be chosen so that both δl and its first differential coefficient shall vanish at the epoch. Reducing to numbers, we find

$$\begin{aligned}
\delta l = & \ (140''.70 + 0''.32 T) \sin N \\
& + (232 .60 - 6 .37 T) \cos N \\
& - \ \ 13 .60 \sin 2N \\
& - \ \ \ 0 .70 \cos 2N \\
& - \ \ \ 0 .03 T^2 \\
& + \ \ 34 .27 \, T \\
& - \ \ 46 .76,
\end{aligned}$$

the last two terms being arbitrary.

When we carry the perturbations of the eccentricity and perihelion to quantities of the second order, we are troubled by the introduction of large terms depending on the square of the disturbing force, which disappear from the rigorous expressions for the co-ordinates. These may be avoided by substituting for the eccentricity and perihelion the quantities h and k determined by the condition

$$h = e \sin \pi$$
$$k = e \cos \pi$$

If, as before, we count the longitudes from the perihelion of Uranus at the epoch 1850, we should substitute $\delta\pi$ for π in these expressions. The values of h and k will then be given by the integration of the equations

$$\frac{dh}{dt} = \ m' a n k_1 \cos N$$
$$\frac{dk}{dt} = - m' a n k_1 \sin N.$$

Differentiating with respect to δ, we find for the terms of the second order

$$\delta\frac{dh}{dt} = -m'ank_1 \sin N\delta N$$

$$\delta\frac{dk}{dt} = -m'ank_1 \cos N\delta N.$$

Substituting for δN its numerical value just given and integrating, we find

$$\delta h = m'a\nu k_1 \{-2895'' \sin N - (6658'' - 4''.26t) \cos N + 1815'' \sin 2N\}$$
$$- 3630'' m'ak_1 nt \quad + \text{constant};$$
$$\delta k = m'a\nu k_1 \{-2895'' \cos N + (6658'' - 4''.26t) \sin N + 1815'' \cos 2N\}$$
$$+ \text{constant};$$

the constants being so chosen that δh and δk shall vanish at the epoch.

Reducing the values of δh and δk to numbers, they become

$$\delta h = 5''.82 \sin N + (13''.40 - 0''.86T) \cos N - 3''.65 \sin 2N + 1''.08T - 2''.67,$$
$$\delta k = 5\,.82 \cos N - (13\,.40 - 0\,.86T) \sin N - 3\,.65 \cos 2N \qquad + 12\,.12,$$

the last two terms being arbitrary constants.

Computing the values of these terms of δl, δh, and δk, for intervals of 50 years, from 1600 to 2000, we find them to be as follows:

Year	δl	δh	δk
1600	—1″.34	+0″.10	—0″.02
1650	—0 .71	+0 .05	—0 .02
1700	—0 .31	+0 .02	—0 .01
1750	—0 .10	0 .00	—0 .01
1800	—0 .01	0 .00	0 .00
1850	0 .00	0 .00	0 .00
1900	0 .00	0 .00	0 .00
1950	+0 .04	0 .00	—0 .01
2000	+0 .18	—0 .01	—0 .02

We see that although the ultimate effect of these terms is very considerable, their effect, during the period that Uranus has been observed, is insignificant.

Concluded Elements and Perturbations of Uranus.

The corrections found in the last chapter being applied to the final provisional elements (p. 99) give the following elements for 1850, affected by the great inequality produced by Neptune:

Elements IV *of Uranus.*

Epoch, 1850, Jan. 0, Greenwich mean noon.

π,	168°	15′	6″.7
ε,	28	25	17 .05
θ,	73	14	8 .0
ϕ,	0	46	20 .54
e,	.0469236		

$$
\begin{array}{ll}
e \text{ (in sec.)}, & 9678''.69 \\
n, & 15425.752 \\
\log a, & 1.2829072 \\
\log a_1, & 1.2831044
\end{array}
$$

Log a_1 includes, as before, the constant term in the perturbations of the logarithm of the radius vector which, with the corrected mass of Neptune, is $+.0001972$.

To find the corresponding elements at any other epoch, the following secular and long-period perturbations are to be applied. Those produced by Neptune are derived from the expressions in Chapter III by correcting them for the new mass of Neptune, and for the change in the value of the small divisor $2n'-n$ produced by the correction of the elements of Uranns. The logarithms of the factors for correction are,

Correction of mass of Neptune	9.93598
Correction of divisor	0.00051
Log. factor for δl	9.93496
Log. factor for δe, $\delta \pi$, $\delta \varpi$	9.93547

Including the perturbations of the second order just found, we have, by putting

$$
\begin{aligned}
N &= 2l' - g, \\
&= 113^\circ\ 30'\ 46''.0 + 8^\circ\ 26'\ 51''.9T,
\end{aligned}
$$

$$
\begin{aligned}
\delta l = (-2850''.41 + 0''.32T) \sin N &+ (387''.67 - 6''.37T) \cos N \\
+ 112.72 \sin 2N \quad & - 47.28 \cos 2N \\
- 7.72 \sin 3N \quad & + 4.33 \cos 3N \\
+ 0.55 \sin 4N \quad & - 0.46 \cos 4N \\
- 0.03T^2 - 83''.78T & + 2811''.41.
\end{aligned}
$$

$$
\begin{aligned}
\delta h = & - 412''.18 \sin N + (14''.03 - 0''.86T) \cos N \\
& + 29.20 \sin 2N - 6.09 \cos 2N \\
& - 3.11 \sin 3N + 1.19 \cos 3N \\
& + 0.28 \sin 4N - 0.13 \cos 4N \\
& + 14.76T \quad + 398.33
\end{aligned}
$$

$$
\begin{aligned}
\delta k = & - 411''.53 \cos N - (13''.65 - 0''.86T) \sin N \\
& + 29.33 \cos 2N + 6.17 \sin 2N \\
& - 3.12 \cos 3N - 1.21 \sin 3N \\
& + 0.29 \cos 4N + 0.13 \sin 4N \\
& - 5.453T \quad - 124.72
\end{aligned}
$$

$\delta \varpi$ (in units of the 7th place of decimals).

$$
\begin{aligned}
= & \ 1963 \cos N + 103 \sin N \\
& - 171 \cos 2N - 67 \sin 2N \\
& + 15 \cos 3N + 6 \sin 3N \\
& + 511.0.
\end{aligned}
$$

The perturbations of $e\delta\pi$ and δe are here replaced by those of h and k, defined by the equations

$$
\begin{aligned}
h &= e \sin (\pi - \pi_0) \\
k &= e \cos (\pi - \pi_0)
\end{aligned}
$$

π_0 representing the perihelion of $1850 = 168° \, 15' \, 6''.7$. We then have for the eccentricity and longitude of perihelion at any epoch

$$e \sin(\pi - \pi_0) = \delta h$$
$$e \cos(\pi - \pi_0) = e_0 + \delta k.$$

In the above terms multiplied by the time we have included the secular variations produced by Jupiter and Saturn. If the perturbations of the elements due to each particular planet are required, we have

Action of Jupiter,

$$\delta h = +5''.73T; \; \delta k = -0''.608T.$$

Action of Saturn,

$$\delta h = +5''.56T; \; \delta k = -4.589T.$$

Subtracting these from the above expressions all the remaining terms will be due to the action of Neptune. The values of δl and $\delta \varpi$ are due entirely to the action of Neptune.

For the sake of rigor, we may suppose the perturbations produced by each planet to be multiplied by a factor representing the number by which the adopted mass of the planet must be multiplied to obtain the true mass.

It will add to the homogeneousness of the theory to express the perturbations of long period, which are multiplied by the product of the masses of Jupiter and Saturn, as perturbations of the elements. These terms, as found on page 88, are

$$\begin{aligned}
(v.c.0) &= -\;\; 0''.55 \sin N_6 - \;\; 0''.03 \cos N_6 \\
&\quad + 40\,.65 \sin N_7 - 10\,.50 \cos N_7 \\
(v.s.1) &= +\;\; 2\,.64 \sin N_6 + \;\; 4\,.64 \cos N_6 \\
&\quad + \;\; 7\,.35 \sin N_7 + \;\; 4\,.41 \cos N_7 \\
(v.c.1) &= -\;\; 4\,.23 \sin N_6 - \;\; 3\,.87 \cos N_6 \\
&\quad + \;\; 8\,.06 \sin N_7 - \;\; 8\,.38 \cos N_7
\end{aligned}$$

These terms, together with the arbitrary corrections of the elements which have been applied to make them very small at the epoch, may be replaced by the following corrections to the elements:

$$\begin{aligned}
\delta l &= -\;\; 0''.55 \sin N_6 - \;\; 0''.03 \cos N_6 \\
&\quad + 40\,.65 \sin N_7 - 10\,.50 \cos N_7 \\
&\quad + 27''.27 - 11''.72T. \\
\delta h &= +\;\; 2\,.09 \sin N_6 + \;\; 1\,.94 \cos N_6 \\
&\quad - \;\; 2\,.13 \sin N_7 + \;\; 3\,.71 \cos N_7 \\
&\quad + 1''.28. \\
\delta k &= +\;\; 1\,.32 \sin N_6 + \;\; 2\,.32 \cos N_6 \\
&\quad + \;\; 3\,.68 \sin N_7 + \;\; 2\,.21 \cos N_7 \\
\delta \varpi &= \;\; 27 \sin N_7 + 104 \cos N_7 + 76 \text{ (in units of the 7th decimal).}
\end{aligned}$$

The amount of the perturbations of the elements for every half century, from the year 1000 to 2200, is given in the following table. Column (1) gives the perturbations by Neptune, Saturn, and Jupiter, computed from the expressions

on page 182; column (2) those just given depending on the product of the masses of Jupiter and Saturn.

Year.	δl		δh		δk		$\delta \varpi$		$\delta \varkappa$	$\delta \eta$
	(1) ″	(2)	(1) ″	(2)	(1) ″	(2)	(1)	(2)	″	
1000	+2050.31	+160.69	+ 42.66	+1.94	—389.51	+6.08	+1955	—140	—2.06	+1.20
1050	1841.17	149.08	27.79	3.60	375.65	6.68	1882	156	2.10	1.37
1100	1638.76	136.39	14.03	4.99	360.31	6.82	1802	169	2.12	1.54
1150	1444.87	122.87	+ 1.45	6.01	343.49	6.47	1717	178	2.08	1.66
1200	1260.24	108.85	— 9.90	6.60	325.27	5.64	1626	183	2.02	1.77
1250	+1085.76	+ 94.67	— 19.84	+6.75	—305.69	+4.42	+1529	+183	—1.94	+1.86
1300	921.76	80.67	28.34	6.49	284.90	2.86	1426	180	1.84	1.91
1350	769.32	67.12	35.27	5.90	252.90	+1.16	1318	173	1.70	1.92
1400	629.00	54.35	40.56	5.08	239.78	—0.57	1205	161	1.54	1.91
1450	501.39	42.62	44.11	4.16	215.68	2.17	1086	147	1.37	1.85
1500	+ 387.18	+ 32.11	— 45.83	+3.24	—190.66	—3.52	+ 963	+129	—1.19	+1.74
1550	286.65	22.78	45.63	2.42	164.84	4.52	835	110	0.99	1.60
1600	200.65	15.38	43.46	1.80	138.31	5.12	704	90	0.80	1.42
1650	129.43	9.34	39.20	1.41	111.22	5.30	568	68	0.61	1.20
1700	73.46	4.86	32.79	1.23	83.70	5.09	430	48	0.44	0.95
1750	+ 33.06	+ 1.90	— 24.16	+1.23	— 55.90	—4.56	+ 288	+ 29	—0.27	+0.66
1800	8.51	0.33	— 13.26	1.36	— 27.93	3.81	+ 145	+ 11	—0.12	+0.34
1850	0.00	0.00	0.00	1.52	0.00	2.96	0	— 5	0.00	0.00
1900	7.64	0.71	+ 15.65	1.63	+ 27.74	2.10	— 145	18	+0.10	—0.36
1950	31.47	2.23	33.70	1.59	55.09	1.34	290	26	0.16	0.73
2000	+ 71.43	+ 4.25	+ 54.20	+1.34	+ 81.81	—0.73	— 433	— 32	+0.20	—1.10
2050	127.33	6.52	77.12	0.85	107.77	0.34	574	30	0.20	1.47
2100	198.86	8.75	102.46	+0.13	132.67	0.14	711	27	0.17	1.83
2150	285.72	10.59	130.15	—0.77	156.33	0.11	843	20	0.11	2.17
2200	387.11	+ 11.77	+160.15	—1.76	+178.48	—0.19	968	— 9	+0.02	—2.48

Mean Elements of Uranus.

If, instead of the elements of Uranus affected by the great inequality, we wish the absolute mean elements, these are to be obtained by adding to the elements already given the constants applied to the perturbations δl, δh, δk, and $\delta \varpi$ to make the perturbations vanish at the epoch 1850.0, and also the corrections (p. 113) which we have subtracted from the elements and added to the perturbations to reduce the latter to a small quantity during the period for which the tables are likely to be used. We thus find the following mean elements:

Elements V of Uranus. Epoch, 1850, Jan. 0, Greenwich mean noon.

Longitude of the perihelion	$170° \; 38' \; 48''.7 + 8698''.\mu$
Mean longitude at epoch,	$29 \; 12 \; 43 .73 + 2811 .4\mu$
Longitude of the node,	$73 \; 14 \; 37 .6 + 29 .6\mu$
Inclination of the orbit,	$0 \; 46 \; 20 .92 + 0 .38\mu$
Eccentricity,	$.0463592 - 5236\mu$
Eccentricity in seconds,	$9562''.27 - 108''.0\mu$
Mean motion,	$15424.797 - 0''.838\mu$
Log mean distance (uncorrected),	$1.2829251 + 179\mu$
The same corrected,	$1.2831223 + 179\mu$
True mass of Neptune,	$\dfrac{1+\mu}{19700}$

Supposing the mass of Neptune to be uncertain by one-fiftieth of its entire amount, which is quite possible, it will be seen the longitude of the mean perihelion is from this cause uncertain by more than two minutes, the mean longitude of Uranus itself by nearly a minute, and the mean motion by nearly two seconds in a century.

It will be seen that the logarithm of the mean distance just given does not accurately correspond to that of elements IV plus the constant term of $\delta\varpi \times 0.4343$, as it should. This difference arises from the rejection of the terms of the second order in $\delta\varpi$, which can not affect the geocentric longitude of the planet by a tenth of a second for a number of centuries.

It is to be remarked that these mean elements are those to be used in the general theory of the secular variation of the planetary orbits.

Concluded Theory of Uranus.

The elliptic longitude and radius vector of Uranus, affected by the secular and long period perturbations of the elements, will be given by the following equations. Put

$$\begin{aligned} l_0 &= n_0 t + \epsilon_0, \\ l &= l_0 + \delta l, \\ g &= l - \pi_0, \\ h &= \delta h, \\ k &= e_0 + \delta k, \\ e^2 &= h^2 + k^2, \end{aligned}$$

the zeros indicating elements IV, and δh, δk, and δl being the perturbations of these three elements just given. Then

Elliptic longitude in orbit $= l$

$$+ \left\{ 2 - \frac{1}{4}e^2 + \frac{5}{96}e^4 \right\} \left\{ k \sin g - h \cos g \right\}$$

$$+ \left\{ \frac{5}{4} - \frac{11}{24}e^2 \right\} \left\{ (k^2 - h^2) \sin 2g - 2hk \cos 2g \right\}$$

$$+ \left\{ \frac{13}{12} - \frac{43}{64}e^2 \right\} \left\{ (k^3 - 3h^2k) \sin 3g - (3hk^2 - h^3) \cos 3g \right\}$$

$$+ \frac{103}{96} \left\{ (k^4 - 6k^2h^2 + h^4) \sin 4g - (4k^3h - 4kh^3) \cos 4g \right\}$$

$$+ \frac{1097}{960} \left\{ (k^5 - 10k^3h^2 + 5kh^4) \sin 5g - (5k^4h - 10k^2h^3 + k^5 \cos 5g \right\}$$

Neperian logarithm of $r = \varpi + \frac{1}{4}e^2 + \frac{1}{32}e^4$

$$- \left\{ 1 - \frac{3}{8}e^2 \right\} \left\{ k \cos g + h \sin g \right\}$$

$$- \left\{ \frac{3}{4} - \frac{11}{24}e^2 \right\} \left\{ (k^2 - h^2) \cos 2g + 2hk \sin 2g \right\}$$

$$- \frac{17}{24}\left\{ (k^3 - 3kh^2)\cos 3g + (3k^2h - h^3)\sin 3g \right\}$$

$$- \frac{71}{96}\left\{ (k^4 - 6k^2h^2 + h^4)\cos 4g + (4k^3h - 4kh^3)\sin 4g \right\}$$

In computing these expressions it will be sufficient for several centuries before or after 1850 to develop h, δk, and δl to their first dimensions: it will, however, be more convenient to correct the mean anomaly g for the perturbation δl before obtaining the equation of the centre. Developing the perturbations of h and k to terms of the first order, we have for the effects of the perturbations of those elements:

$$(v.s.1) = \left(2 - \frac{3}{4}e_0^2\right)\delta k$$

$$(v.c.1) = -\left(2 - \frac{1}{4}e_0^2\right)\delta h$$

$$(v.s.2) = \left(\frac{5}{2}e_0 - \frac{11}{6}e_0^3\right)\delta k$$

$$(v.c.2) = -\left(\frac{5}{2}e_0 - \frac{11}{24}e_0^3\right)\delta h$$

$$(v.s.3) = \frac{13}{4}e_0^2\,\delta k$$

$$(v.c.3) = -\frac{13}{4}e_0^2\,\delta h$$

$$(v.s.4) = \frac{103}{24}e_0^3\,\delta k$$

$$(v.c.4) = -\frac{103}{24}e_0^3\,\delta h$$

$$(\rho.c.0) = \delta n + \frac{1}{2}e_0\,\delta k$$

$$(\rho.s.1) = -\left(1 - \frac{3}{8}e_0^2\right)\delta h$$

$$(\rho.c.1) = -\left(1 - \frac{9}{8}e_0^2\right)\delta k$$

$$(\rho.s.2) = -\frac{3}{2}e_0\,\delta h$$

$$(\rho.c.2) = -\frac{3}{2}e_0\,\delta k$$

$$(\rho.s.3) = -\frac{17}{8}e_0^2\,\delta h$$

$$(\rho.c.3) = -\frac{17}{8}e_0^2\,\delta k$$

These coefficients for ρ must, of course, be multiplied by the modulus 0.434294 to reduce the perturbations to those of the common logarithm of the radius vector.

Among the elliptic terms may be included the effect of the following minute constants introduced by the perturbations.

$$
\begin{aligned}
(v.s.2) &= -0''.144 \\
(v.c.2) &= +0\ .130 \\
0.4343\,(\rho.c.0) &= +1972 \quad \text{in units of the 7th place} \\
0.4343\,(\rho.s.1) &= +63 \qquad\quad \text{of decimals.} \\
0.4343\,(\rho.c.1) &= +73 \\
0.4343\,(\rho.s.2) &= +5 \\
0.4343\,(\rho.c.2) &= +4
\end{aligned}
$$

This term $(\rho.c.0)$ is that added as a correction to the logarithm of the mean distance.

To the coefficients $(v.s.1)$, $(v.c.1)$, etc., are still to be added the following periodic terms:—

1. The periodic terms due to the action of Jupiter, given in Chapter V, omitting the terms multiplied by T, which are included in the perturbations of the elements.

2. The periodic terms produced by Saturn, including those terms multiplied both by T and by $\sin A_2$ or $\cos A_2$, but omitting those multiplied by T only for the same reason as in the case of Jupiter.

3. The periodic terms produced by Neptune, multiplied by the factor 0.86294 on account of the correction to the mass of that planet, and omitting the terms multiplied by δl, δe, and $e\delta g$.

4. The periodic terms multiplied by the product of the masses of Jupiter and Saturn, given on page 88, omitting the terms multiplied by the sine and cosine of N_6 and N_7, because they are replaced by the terms of δl, δh, and δk, given on page 183, and tabulated in the columns headed (2) on page 184. The result will be the same whether we employ the terms of $(v.c.0)$, $(v.s.1)$, etc., given at the bottom of page 88 and the top of page 89, omitting the numbers in the columns 2 on page 184 from the expressions on page 186, or whether we include the latter and omit the former.

The true anomaly of Uranus will then be:

$$g_0 + \delta l + \text{(equation of centre from elements IV, using for mean anomaly } g_0 + \delta l)$$
$$+ \Sigma\,(v.s.i) \sin ig + \Sigma\,(v.c.i) \cos ig.$$

The logarithm of the radius vector will be:

$$\log r \text{ in elliptic orbit from elements IV.}$$
$$+ \Sigma\,(\rho.s.i) \sin ig + \Sigma\,(\rho.c.i) \cos ig$$

care being taken to multiply the coefficients by the modulus where that has not already been done. All the terms in Chapter V are so multiplied.

To pass from the true anomaly to the true longitude we must investigate the secular motion of the planes of the orbit and of the ecliptic. The effect of this motion on ϕ, θ, and τ will be found by successive approximations from the formulæ

(34), correcting the data for the new mass of Neptune. We shall also use the same motion of the ecliptic adopted on p. 95. We have thus:

$$\frac{dp}{dt} = -4''.53$$

$$\frac{dp'}{dt} = +5.43 + 0''.38T.$$

$$\frac{dq}{dt} = -5.17$$

$$\frac{dq'}{dt} = -46.78 + 0.12T.$$

As a first approximation we have

$$\begin{aligned} \theta = \tau &= 73^\circ\ 14'\ \ 8'' \quad -3169''.2T \\ \phi \quad &= \ \ 0\ \ 46\ \ 20.54 + \quad 2.48T \end{aligned}$$

Substituting these values in (34) and integrating we find

$$\begin{aligned} \phi &= \phi_0 + \quad 2''.47T + 0''.13T^2 \\ \theta &= \theta_0 - 3168.42T + 3.00T^2 \\ \tau &= \tau_0 - 3168.76T + 3.00T^2 \end{aligned}$$

For tabulating we shall use, instead of θ and τ, the distance of the perihelion from the ascending node, or $\pi - \tau$, and the value of θ corrected for Struve's precession. Since the mean motion has been derived without making any distinction between τ and θ, it will be necessary to correct the motion of mean anomaly by the difference of those quantities. We thus obtain for the values of the three principal arguments:—

$$\begin{aligned} g &= 220^\circ\ 10'\ 10''.35 + 1542574''.86T + \delta l \\ \omega &= \ \ 95\ \ \ 0\ \ 58.70 + \quad 3168.76T - 3.00T^2 \\ \theta &= \ \ 73\ \ 14\ \ \ 8.00 + \quad 1856.82T + 4.12T^2 \end{aligned}$$

If we represent all the inequalities of the true longitude by Δl, so that we shall have for the true anomaly

$$f = g + \Delta l,$$

the argument of latitude will be

$$u = f + \omega.$$

The reduction to the ecliptic will then be

$$R = -(9''.37 + 0''.016T) \sin 2u,$$

the true longitude on the ecliptic referred to the mean equinox of date,

$$\lambda = u + \theta + R,$$

and the sine of the elliptic latitude,

$$\sin \beta_0 = \sin \phi_0 \sin u.$$

The perturbations of the latitude will be

$$(b.c.0) + (b.c.1) \cos g + (b.s.1) \sin g + \text{etc.}$$

The periodic terms of ($b.c.0$), ($b.s.1$), ($b.c.1$), etc., are given in Chapter V, on pages 86 and 87, and are to be taken without any farther modification than the multiplication of those due to the action of Neptune by the factor 0.863. The constant, secular, and long period terms are

$$
\begin{aligned}
b.c.0 &= +0''.26 \quad -0''.12T \quad -0.011\delta\eta + 0.046\delta\varkappa \\
(b.s.1) &= -0\ .22T - 0\ .05T^2 + 0.975\delta\eta + 0.221\delta\varkappa \\
(b.c.1) &= +2\ .47T + 0\ .12T^2 + 0.221\delta\eta - 0.975\delta\varkappa \\
(b.s.2) &= -0\ .06 \quad -0\ .01T \quad +0.046\delta\eta + 0.011\delta\varkappa \\
(b.c.2) &= -0\ .01 \quad +0\ .12T \quad +0.011\delta\eta - 0.046\delta\varkappa
\end{aligned}
$$

The values of $\delta\eta$ and $\delta\varkappa$ to be used in these expressions are those the expressions for which are given on page 97, and which are tabulated in the last two columns of the table on page 184.

The following tables are based on the elements and theory laid down in this chapter.

CHAPTER IX.

GENERAL TABLES OF URANUS.

Enumeration of the Quantities contained in the several Tables.

THE first six tables are designed to give the values of the three arguments of the elliptic motion, g, ω, and θ, and of the nine arguments of the tables of perturbations. The argument ω is, however, diminished by $3'$, the sums of the constants added to the perturbations of $(v.c.0)$ to make these quantities positive, and θ by $10''$, the constant added to the reduction to ecliptic. The expressions for the arguments of perturbations are as follows, the mean longitude of each planet, counted from the perihelion of Uranus, being represented by the initial letter of the planet. All these arguments are expressed in units, of which 600 make an entire circumference, so that each unit is $36'$. The time t is counted in Julian years from the fundamental epoch,

1850, January 0, Greenwich mean noon.

$$\begin{array}{rll}
\text{Arg. } 1 = & J - U & = 219.190 + 43.44028t \\
2 = & S - U & = 577.349 + 13.22717t \\
3 = & U - N & = 88.884 + 3.50035t \\
4 = & J - 2S & = 497.6 + 9.8445t \\
5 = & 3S - U - J & = 79.8 + 3.3825t \\
6 = & 4S - 2U - J & = 57.1 + 16.610t \\
7 = & 2J - 3S - 3U & = 238.7 + 18.633t \\
8 = & 2J - 4S - 2U & = 261.3 + 5.4058t \\
9 = & 7S - 2J - 3U & = 136.9 + 19.992t
\end{array}$$

Table I gives the corrections which must be applied to the values of the arguments at any time during the nineteenth century to reduce them to the corresponding time in any preceding or following century between the Christian era and the year 2300. Since ω and θ each contains a term proportional to the square of the time, the correction for these quantities is not constant during each century, but is of the form

$$\omega + \omega' T$$

ω and ω' being constant during each century, and T being the fraction of the century counted from its beginning.

Table II gives the value of g, $\omega - 3'$, $\theta - 10''$, and the above nine arguments for Greenwich mean noon of Jan. 0 of each leap year from 1752 to 1948, and for January — 1 of the years 1800 and 1900, corresponding to December 30 of the years 1799 and 1899. The corrections for the perturbations of long period are not

applied in this table. The numbers at the bottom of this table, in the line $\Delta^{(1)}_{120}$, show the variation of the corresponding quantity in 120 days, for the epoch 1850.0. In the line "Factor T" is given the change of this variation in a century, while $\Delta^{(2)}_{120}$ is the second difference for intervals of 120 days. By means of these numbers, when the arguments are computed for any date, their values for other dates at intervals of 120 days may be found by successive addition.

Table III gives the motion of the several arguments between the epochs of the preceding table and the zero day of each month in the course of a four-year cycle. The variable motions, ω and θ, correspond to the epoch 1850, and rigorously they each require a correction for any other four-year cycle than that between 1848 and 1852. But, owing to the small inclination of the orbit of Uranus it is not necessary that either ω or θ should be exact, if only their sum is exact. The column θ' of this table, therefore, gives the correction which must be applied to the motion of θ at the end of a century (1950) in order that, being applied to θ alone, $\omega + \theta$ may be exact. This correction is, in fact, that for the secular variation of the precession.

Tables IV and V give the motion of the arguments for days and hours. The motion for hours is, however, not necessary in the case of any argument but g, as all the others can be readily enough interpolated to fractions of a day.

Table VI gives the corrections to the arguments on account of the terms of long period from 1000 to 2200. The terms in question are, in the case of Jupiter, the great inequality produced by the action of Saturn, in the case of Neptune the great inequality produced by Uranus, and, in the case of Uranus, the inequalities in the mean longitude tabulated in the preceding chapter. The numerical expressions are

$$\delta J = 0.535 \sin (116^\circ\ 21' + 40^\circ\ 45'\ 20''\ T)$$
$$\delta U = \delta l$$
$$\delta N = -0.75\ \delta l.$$

The corrections to the several arguments are

$$\begin{aligned} \delta g &= \delta l \\ \delta \text{ arg. } 1 &= \delta J - \delta l \\ \delta \text{ arg. } 2 &= -\delta l \\ \delta \text{ arg. } 3 &= \delta l - \delta N = 1.75\ \delta l \end{aligned}$$

No correction to the mean longitude of Saturn is applied, all its inequalities being taken account of in the terms of the second order.

The corrections, expressed in seconds, have been reduced to units of the argument by dividing them by 2160″.

Outside the limits of the table these corrections must be computed from their formulæ.

Table VII gives the equation of the centre, and the elliptic part of the logarithm of the radius vector. No constant is applied to the former, but the latter is diminished by .0003400, the sum of the constants added to ($\rho.c.0$) in Tables VIII, IX, X and XVII.

The formulæ for the Tables are

$$\begin{aligned}\text{Equation of centre} = &\ \ 19352''.06 \sin \\ + &\ \ 567\ .24 \sin \\ + &\ \ 23\ .05 \sin 3g \\ + &\ \ 1\ .07 \sin 4g \\ + &\ \ 0\ .05 \sin 5g\end{aligned}$$

$$\begin{aligned}\text{Elliptic log. } r = &\ \ 1.2833435 \\ - &\ \ .0003400 \\ - &\ \ .0203618 \cos g \\ - &\ \ .0007165 \cos 2g \\ - &\ \ .0000318 \cos 3g \\ - &\ \ .0000016 \cos 4g.\end{aligned}$$

Table VIII gives the coefficients $(v.c.0)$, $(v.c.1)$, etc., for the perturbations of the longitude and logarithm of radius vector produced by the action of Jupiter. They are computed from the periodic terms of the formulæ on page 83, with the addition of the following constants to make all the numbers of the table positive:

$$\begin{aligned}\text{Constant of } (v.c.0) &= 55''. \\ (v.s.1) &= 6\ . \\ (v.c.1) &= 4\ . \\ (v.s\,2) &= 0\ .20 \\ (v.c.2) &= 0\ .20 \\ (\rho.c.0) &= 1200 \\ (\rho.s.1) &= 150 \\ (\rho.c.1) &= 100 \\ (\rho.s.2) &= 10 \\ (\rho.c.2) &= 10\end{aligned}$$

Table IX gives the periodic part of the coefficients due to the action of Saturn, taken without change from the expressions on page 84, together with the secular variations, the latter including only the terms of $(v.s.1)$, $(v.c.1)$, $(v.s.2)$, and $(v.c.2)$, which are multiplied by T and by $\sin A_2$ or $\cos A_2$. The coefficients of T are given in the columns Sec. Var. and each number is increased by the constant $1''.50$ to make it positive. The term $-0''.06T \sin A_2$ in $(v.c.0)$ is omitted entirely, as it will not amount to a tenth of a second until after the year 2000. The constant terms added to the quantities of these tables to make all the numbers positive, are:

$$\begin{aligned}\text{Constant of } (v.c.0) &= \ \ 30. \\ (v.s.1) &= 150. + 1.50T \\ (v.c.1) &= 150. + 1.50T \\ (v.s.2) &= 130. + 1.50T \\ (v.c.2) &= 130. + 1.50T \\ (v.s.3) &= \ \ 8. \\ (v.c.3) &= \ \ 6. \\ (v.s.4) &= \ \ 1. \\ (v.c.4) &= \ \ 1.\end{aligned}$$

Constant of $(\rho.c.0) = 800$
$(\rho.s.1) = 1500$
$(\rho.c.1) = 1500$
$(\rho.s.2) = 400$
$(\rho.c.2) = 400$
$(\rho.s.3) = 100$
$(\rho.c.3) = 100$

Table X gives the coefficients produced by the action of Neptune, computed from the periodic terms on pages 85 and 86 without any other change than the multiplication of all the numbers by the factor 0.863 to reduce them to the new mass of Neptune. The constants added to the several quantities, are

Constant of	$(v.c.0) =$	$92''.85$
	$(v.s.1) =$	20.00
	$(v.c.1) =$	31.00
	$(v.s.2) =$	5.00
	$(v.c.2) =$	5.00
	$(v.s.3) =$	1.00
	$(v.c.3) =$	1.00
	$(v.s.4) =$	— 1.00
	$(v.c.4) =$	— 1.00
Constant of	$(\rho.c.0) =$	400
	$(\rho.s.1) =$	200
	$(\rho.c.1) =$	200
	$(\rho.s.2) =$	40
	$(\rho.c.2) =$	40

Tables XI to XVI give the terms of the second order and of short period which contain the products of the masses of Jupiter and Saturn, which, with the constants added to the numbers of the several tables, are as follows:

$(v.c.0) =$	$+ 0''.08 \sin A_5 + 0''.51 \cos A_5$;	Table XII;	const $= 0''.60$
	$+ 0.04 \sin A_6 + 0.01 \cos A_6$;	XIII;	$= 0.05$
	$- 0.01 \sin A_7 + 0.05 \cos A_7$;	XIV;	$= 0.05$
	$- 0.35 \sin A_8 - 1.30 \cos A_8$;	XV;	$= 1.35$
	$- 0.05 \sin A_9 + 0.03 \cos A_9$;	XVI;	$= 0.10$
	Sum of constants added to these tables		2.15

$(v.s.1) =$	$+ 0''.26 \sin A_1 + 0''.27 \cos A_1$;	Table XI;	const $= 0''.40$
	$- 0.04 \sin A_5 - 0.17 \cos A_5$;	XII;	$= 0.20$
	$+ 0.08 \sin A_6 + 0.03 \cos A_6$;	XIII;	$= 0.10$
	$- 0.02 \sin A_7 + 0.08 \cos A_7$;	XIV;	$= 0.10$
	$+ 0.30 \sin A_8 - 0.58 \cos A_8$;	XV;	0.75
	$- 0.04 \sin A_9$;	XVI;	0.10

$(v.c.1) = +0''.06 \sin A_4 - 0''.27 \cos A_4;$	Table XI;	const =	$0''.40$
$+0.18 \sin A_5 + 0.01 \cos A_5;$	XII;		0.20
$-0.03 \sin A_6 + 0.08 \cos A_6;$	XIII;		0.10
$-0.02 \sin A_7 + 0.09 \cos A_7;$	XIV;		0.10
$-0.44 \sin A_8 - 0.61 \cos A_8;$	XV;		0.75
$-0.10 \sin A_9 + 0.03 \cos A_9;$	XVI;		0.10
Sum of constants added to $(v.s.1)$ and $(v.c.1)$ in these tables			1.65

The term of $(\rho.c.0)$

$$11 \sin A_8 - 3 \cos A_8$$

is omitted from the tables entirely.

Tables XVII*a* and XVII*b* give the constant, secular, and long-period terms of $(v.s.1)$ $(v.c.1)$, computed from the formulæ p. 186, with the following additions:

1. The constant terms introduced by the perturbations, given on p. 187.

2. The negatives of the constants added to the tables VII to XVI inclusive to make the numbers of those tables positive. The values of these terms are

	Pert. Const.	Tables VIII to XVI.	(1) — (2)
$(v.s.1)$	0	$177''.65 + 1''.50\,T$	$-177''.65 - 1''.50\,T$
$(v.c.1)$	0	$186.65 + 1.50\,T$	$-186.65 - 1.50\,T$
$(v.s.2)$	$-0''.14$	$135.20 + 1.50\,T$	$-135.34 - 1.50\,T$
$(v.c.2)$	$+0.13$	$135.20 + 1.50\,T$	$-135.07 - 1.50\,T$
$(v.s.3)$	0	9.00	− 9.00
$(v.c.3)$	0	7.00	− 7.00
$(\rho.c.0)$	[+1972]	−1000	+1000
$(\rho.s.1)$	+ 63	1850	−1787
$(\rho.c.1)$	+ 73	1800	−1727
$(\rho.s.2)$	+ 5	450	− 445
$(\rho.c.2)$	+ 4	450	− 446

The perturbation constant of $(\rho.c.0)$, being added to log a in forming the elliptic radius vector, is not included in this table.

Table XVIII gives the reduction to the ecliptic

$$-9''.37 \sin 2u.$$

The constant $10''$ is added to make the numbers always positive, which constant has been already subtracted from θ.

Table XIX gives the principal term of the latitude

$$46'\ 20''.54 \times \sin u.$$

Table XX gives the coefficients $(b.s.1)$ and $(b.c.1)$ for the perturbations of the latitude produced by Jupiter. They are given by the formulæ

$$(b.s.1) = 0''.65 \cos (J - U + 40^\circ)$$
$$(b.c.1) = 0.65 \sin (J - U + 40^\circ)$$

The constant $0''.70$ is added to make all the numbers of the table positive.

Table XXI gives the corresponding coefficients for the action of Saturn, computed from the expressions on p. 87 with the addition of the following constants.

$$
\begin{aligned}
\text{Const. of } (b.c.0) &= 0''.10 \\
(b.s.1) &= 3\ .30 \\
(b.c.1) &= 3\ .10 \\
(b.s.2) &= 0\ .20 \\
(b.c.2) &= 0\ .20
\end{aligned}
$$

Table XXII gives the coefficients for the action of Neptune from the formulæ on p. 87, all the numbers being multiplied by the factor 0.863 to reduce them to the adopted mass of Neptune. The following constants are added:

$$
\begin{aligned}
\text{To } (b.c.0) \ \ldots\ldots &\ \ 0''.06 \\
(b.s.1) &\ \ 1\ .00 \\
(b.c.1) &\ \ 1\ .20 \\
(b.s.2) &\ \ 0\ .20 \\
(b.c.2) &\ \ 0\ .20
\end{aligned}
$$

Table XXIII gives the secular and long-period terms for various epochs computed from the formulæ of p. 189. The sums of the several constants added in the three preceding tables are here subtracted again so that these expressions become

$$
\begin{aligned}
b.c.0 - \Sigma c &= 0''.10 - 0''.12T - \phantom{0''.05T^2 +{}} .011\delta\eta + .046\delta x \\
(b.s.1) - \Sigma c &= -5\ .00 - 0\ .22T - 0''.05\,T^2 + .975\delta\eta + .221\delta x \\
(b.c.1) - \Sigma c &= -5\ .00 + 2\ .47T + 0\ .12T^2 + .221\delta\eta - .975\delta x \\
(b.s.2) - \Sigma c &= -0\ .46 - 0\ .01\,T + \phantom{0''.05T^2 +{}} .046\delta\eta + .011\delta x \\
(b.c.2) - \Sigma c &= -0\ .41 + 0\ .12T + \phantom{0''.05T^2 +{}} .011\delta\eta - .046\delta x
\end{aligned}
$$

Precepts for the use of the Tables.

Express the date for which the position of Uranus is required in years, months, days, and hours of Greenwich mean time, according to the Julian Calendar if the date is earlier than 1500, according to the Gregorian Calendar if it is later than 1600, and according to either calendar between these epochs.

Enter Table I with the beginning of the century, and take out the values of g, ω, ω', θ, θ', and arguments 1 to 9. Multiply ω' and θ' by the fraction of a century corresponding to the date, and write the products with their proper algebraic signs under ω and θ. If the calendar is the Julian, the century marked J must be taken, and if the Gregorian, that marked G. Between the dates 1752 and 1951 it is not necessary to enter Table I at all.

If Table I was not entered, enter Table II with the year, or the first preceding year found therein. If Table I was entered, enter Table II between the year 1800 and 1896 as if the number of the century were changed to 18. Take out the values of g, ω, θ, and the arguments, and write them under the corresponding quantities from Table I.

Enter Table III with the excess of the actual year over that with which Table II was entered, and with the month. Write the corresponding values of g, ω, θ, and the arguments under the previous values. Multiply θ' by the fraction of a

century after 1850, corresponding to the date with which Table II was entered, and write the product under θ, or add it to it in writing θ. If Table II was entered with a date before 1850, this product is negative.

Enter Table IV with the day of the month and write down the corresponding values of g, ω, etc., under the former values.

If the date does not correspond to Greenwich mean noon, the motion of g for the hours must be computed from Table V, and the other quantities must be interpolated to the fraction of a day in entering Table IV.

Enter Table VI with the year, find by interpolation the values of g, and arguments 1, 2, and 3, corresponding to the date, and write them under the former values.

Add up all the partial values of g, ω, θ, and the arguments, attending to the algebraic signs of the products. Subtract from the arguments as many times 600 as possible, and the results will be the final values of those quantities.

Enter Table VII with g as the argument, the seconds being first reduced to fractions of a minute, and interpolate the quantities E and log r. When g exceeds 180° the former quantity is to receive the negative sign; the latter is always positive.

Enter Tables VIII to XVI inclusive with their respective arguments, and take out the values of the quantities ($v.c.0$), ($v.s.1$), ($v.c.1$), etc., ($\rho.c.0$), ($\rho.s.1$), etc., so far as they are found in the tables, writing the quantities having the same designation under each other. In Table IX the quantities Sec. Var. must be multiplied by the centuries and fraction of a century of the actual date after 1850, and the product must be included with the corresponding quantities, ($v.s.1$), ($v.c.1$), etc. Before 1850 this product will always be negative; afterward always positive. All the quantities taken from these tables are positive except ($v.s.4$) and ($v.c.4$) in Table IX, which are negative.

Add up all the partial values of ($v.c.0$), ($v.s.1$), etc., thus obtained from Tables VIII to XVI, and from their sum take the corresponding quantities obtained from Table XVII by interpolating to the date. The required quantities are all given in Table XVIIb; Table XVIIa being only an expansion of a part of XVIIb for the present century. The final values of ($v.s.1$), ($v.c.1$), ($v.s.2$), etc., ($\rho.s.1$), ($\rho.c.1$), etc., thus obtained are to be multiplied by the sines and cosines of the corresponding multiples of g, in doing which four place logarithms are sufficient if the computation is carefully made. The products are then all added together, and to g, ω, E, and ($v.c.0$); in the case of v, and to log. r, ($\rho.c.0$) in the case of ρ. That is, we are to form the expressions:

$$
\begin{aligned}
u = g + \omega + E + (v.c.0) &+ (v.s.1) \sin g + (v.c.1) \cos g \\
&+ (v.s.2) \sin 2g + (v.c.2) \cos 2g \\
&+ \text{ etc. } + \text{ etc.}
\end{aligned}
$$

$$
\begin{aligned}
\log r = \quad & \log r \text{ (from Table VII)} + (\rho.c.0) \\
& + (\rho.s.1) \sin g) \quad + (\rho.c.1) \cos g \\
& + (\rho.s.2) \sin 2g) \quad + (\rho.c.2) \cos 2g \\
& + (\rho.s.3) \sin 3g) \quad + (\rho.c.3) \cos 3g.
\end{aligned}
$$

u will then be the true argument of latitude, and log r the logarithm of the radius vector with seven places of decimals.

Under u write θ; enter Table XVIII with the argument u and take out the reduction to the ecliptic. Add it to u and θ, and the sum of the three quantities will be the heliocentric longitude of Uranus referred to the mean equinox and ecliptic of the date. Applying nutation the longitude will be reduced to the true equinox.

Enter Table XIX with u as the argument, or, when u exceeds 180°, with $u - 180°$, and take out the principal term of the latitude, which will be positive when u is less than 180°, and negative when it is greater.

Enter Tables XX, XXI, XXII, and XXIII with their respective arguments, the argument for the last being the date, and add up the various quantities having the same designation, noticing that in the first three tables all the quantities are positive, while in the last they are all negative except ($b.c.0$). Then form the expression,

$$(b.c.0) + (b.s.1) \sin g + (b.c.1) \cos g + (b.s.2) \sin 2g + (b.c.2) \cos 2g,$$

and add it to the principal term of the latitude, with regard to the algebraic signs. The sum will be the heliocentric latitude of Uranus above the ecliptic of the date.

When an ephemeris of Uranus is to be computed for a series of years, some modifications may be introduced, which will save the computer labor. In the first place an equidistant series of dates being selected for computation, it will be sufficient to compute g, ω, θ, and the arguments for every sixth, eighth, or tenth date, and to fill in the arguments for the intermediate dates by adding the nearly constant differences corresponding to the adopted intervals. The agreement of the numbers thus obtained for the last date with those found by the original computation will prove the whole process. This interval may be as great as 120 days without detracting from the accuracy with which the places for the immediate dates can be interpolated, and the differences for this interval may be deduced from the numbers at the bottom of Table II. If these numbers are used without change the values of ω and θ for the last date may not always come out right. But these errors, if less than a second, will be of no importance if the one quantity comes out as much too great as the other is too small, and they may be avoided entirely by making a small change in the constant difference to be added.

Tables XI to XVI, inclusive, need be entered only for every third or fourth date, and the sums of the quantities can be then interpolated to every date, and added up with the corresponding quantities from the other tables.

Again, it will be found convenient to compute the sum of the small terms $(v.s.3) \sin 3g + (v.c.3) \cos 3g + (v.s.4) \sin 4g + (v.c.4) \cos 4g$, as well as the corresponding terms of the radius vector, and all the terms of the latitude, not for the dates adopted, but for every fourth entire degree of g. Having a series of values computed in this way, the sum can be interpolated to the value of g corresponding to the date. To facilitate the formation of the smaller products for entire degrees of g, a table of products of numbers by the sine and cosine of every degree is appended to these tables, by which the products in question can be formed at sight

whenever the coefficient to be multiplied is less than 32″. The values of these coefficients, ($v.s.3$), ($v.c.3$), etc., corresponding to the entire degrees of g, may be either formed by interpolation at sight from those corresponding to the dates of computation, or the values of the arguments 2 and 3 corresponding to the required degrees of g may be computed, and the values of ($v.s.3$), etc., corresponding to these values of the arguments may be taken from Tables IX and X, while Table XVII must be entered with the corresponding dates.

If the heliocentric ephemeris is computed for ten years at a time, the last of these modifications in the mode of computation will greatly facilitate the computation of the smaller terms. We first find the date, and the values of arguments 1, 2, and 3, to one place of decimals, for some entire degree of g preceding that which corresponds to the first date, and then find the dates and the values of the arguments corresponding to successive values of g, differing by 2° or 4°, until we pass the last date of computation. We then take out the values of ($v.s.3$), ($v.c.3$), ($v.s.4$), ($v.c.4$), ($\rho.s.3$), ($\rho.c.3$), ($b.c.0$), ($b.s.1$), ($b.c.1$), ($b.s.2$), and ($b.c.2$), with these values of the dates and arguments, form their products by the sines and cosines of the corresponding multiples of g by means of the supplementary tables, and add the proper products together so as to form three small tables with g as the argument. These terms are then interpolated to the values of g corresponding to the original dates of computation.

As a first example of the use of the Tables we will compute the heliocentric co-ordinates of Uranus for Greenwich mean noon of the date 1753, Dec. 3. In computing the arguments we shall make use of Table I, though it is not necessary to do so. The computation of the arguments is as follows:

	g			ω			θ			Arg. 1
	°	′	″	°	′	″	°	′	″	
Table I, 1700	291	31	7.	359	7	5.33	359	29	11.47	456.092
Product by 0.5392	....					+3.24			—4.45	
Table II, 1852	228	44	0.73	94	59	2.03	73	14	35.11	306.010
III, Y. 1, Dec.	8	12	43.38	0	1	0.73	0	0	35.59	83.252
IV, 3 days	0	2	6.70			0.26			0.15	0.357
VI, 1753.92	0	0	32.30			....			....	0.507
1753, Dec. 3	168	30	30.48	94	7	11.59	72	44	17.87	246.218

	Arg. 2	3	4	5	6	7	8	9
Table I, 1700	477.319	249.975	216	262	139	537	59.4	401
II, 1852	3.785	95.880	517	87	90	276	272.1	177
III, Y. 1, Dec.	25.349	6.709	19	6	32	36	10.3	38
IV, 3 days	0.109	.029	0	0	0	0	0.0	0
VI, 1753.9	—0.014	+.024						
1753, Dec. 3	506.548	352.617	152	355	261	249	341.8	16

	(v.c.0)	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″
Table VIII	83.58	9.97	6.61	0.37	0.31				
IX	21.32	22.24	223.49	60.92	238.89	1.42	11.07	0.32	1.64
X	71.08	19.02	18.30	6.64	5.38	0.70	1.19	—0.95	—1.04
XI	..	.13	.47						
XII	.13	.36	.10						
XIII	.06	.11	.01						
XIV	.01	.04	.03						
XV	2.38	1.40	1.12						
XVI	.12	.09	.11						
Σ	178.68	53.36	250.24	67.93	244.58	2.12	12.26		
Table XVII		—292.55	—140.91	—140.73	—131.03	—9.41	—6.85		
		—239.19	+109.35	— 72.80	+113.55	—7.29	+5.41	—0.63	+0.60

	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
Table VIII	251	188	32				
IX	1086	741	360	621	597	154	111
X	326	259	222	21	42		
	1663	1188	614	642	639	154	111
XVII	1104	—1321	—500	—412	—360	—98	—94
	2767	— 133	+114	+230	+279	+56	+17

log (v.s.1) — 2.3787	log (v.c.1) + 2.0388	log (v.s.2) — 1.8621	log (v.c.2 + 2.0552
log sin + 9.2994	log cos g — 9.9912	log sin 2g — 9.5916	log cos 2g + 9.9641
log (ρ.s.1) — 2.124	log (ρ.c.1) + 2.057	log (ρ.s.2) + 2.362	log (ρ.c.2) + 2.446

	°	′	″
g	168	30	30.48
ω	94	7	11.59
E	1	0	46.21
(v.c.0)		2	58.68
(v.s.1) sin g			—47.66
(v.c.1) cos g		—1	47.15
(v.s.2) sin 2g			28.43
(v.c.2) cos 2g		1	44.55
(v.s.3) sin 3g			— 4.13
(v.c.3) cos 3g			— 4.46
(v.s.4) sin 4g			0.47
(v.c.4) cos 4g			0.41
u	263	40	57.42
θ	72	44	17.87
R			7.95
Long. mean Eq.	336	25	23.24
Nutation			+6.51
Long. true Eq.	336	25	29.75

	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)
Table XX	″.06	0″.62		
XXI	3 .91	5 .88	0.41	0.17
XXII	.84	1 .36	0.18	0.17
	4.81	7 .86	0.59	0.34
XXIII	—4.28	—6 .88	—0.43	—0.51
	+0.53	+0 .98	+0.16	—0.17

	log r
Table VII	1.3023222
(ρ.c.0)	2767
(ρ.s.1) sin g	— 26
(ρ.c.1) cos g	—112
(ρ.s.2) sin 2g	— 90
(ρ.c.2) cos 2g	+257
(ρ.s.3) sin 3g	+ 31
(ρ.c.3) cos 3g	— 14
log r	1.326035

β_0 Table XIX	— 0	46′	3.″73
(b.c.0) Table XXI			0 .16
XXII			0 .09
XXIII			0 .20
(b.s.1) sin g			+0 .11
(b.c.1) cos g			—0 .96
(b.s.2) sin 2g			—0 .06
(b.c.2) cos 2g			—0 .16
Latitude	— 0	46	4 .35

As a second example we will take the computation of an ephemeris for the years 1876 and 1877. We take as the extreme dates 1875, December 15, and 1878, April 3, between which are seven intervals of 120 days each, which we adopt as those of computation. We first form the arguments for the extreme dates as follows:

I. For 1875, Dec. 15 = 1875.96.				
	g	ω	θ	Arg. 1
	° ′ ″	° ′ ″	° ′ ″	
Table II, 1872	314 25 55.70	95 9 35.64	73 20 46.67	574.815
III, 3 Y. Dec.	16 46 33.76	0 2 4.06	0 1 12.72	170.072
IV, 15 days	0 10 33.50	1.30	0.76	1.784
VI, 1875.96	2.20	....	....	+ 0.426
For 1875, Dec. 15	331 23 5.16	95 11 41.00	73 22 0.15	147.097

	Arg. 2	3	4	5	6	7	8	9
Table II, 1872	268.328	165.888	114	154	423	49	380.3	577
III, 3 Y. Dec.	51.785	13.705	39	13	65	73	21.1	78
IV, 15 days	0.543	0.144	0	0	1	1	0.2	1
VI, 1875.96	— 1	+ 2						
For 1875, Dec. 15	320.655	179.739	153	167	489	123	401.6	56

II. For 1878, April 3 = 1878.26.				
	g	ω	θ	Arg. 1
	° ′ ″	° ′ ″	° ′ ″	
Table II, 1876	331 34 18.70	95 11 42.33	73 22 1.03	148.576
III, 2 Y. April	9 37 53.62	0 1 11.23	0 0 41.75	97.643
IV, 3d	2 6.70	0.26	0.15	.357
VI, 1878.26	2.62	....	....	.421
For 1878, April 3	341 14 21.64	95 12 53.82	73 22 42.93	246.997

	Arg. 2	3	4	5	6	7	8	9
Table II, 1876	321.237	179.889	153	168	489	123	401.9	57
III, 2 Y. April	29.732	7.868	22	7	37	42	12.1	45
IV, 3d	.109	.029	0	0	0	0	0	0
VI, 1878.26	— .001	+ .002						
For 1878, April 3	351.077	187.788	175	175	526	165	414.0	102

We now fill in the values of g, the arguments 1—9, and the times with which Table XVII is to be entered, for the intermediate dates, by adding the nearly constant differences deduced from the numbers at the bottom of Table II. The seconds of g are first reduced to fractions of a minute, with which to enter Table VII. In making the subsequent computation we have used none of the devices previously described except in the case of the small longitude terms, as follows:

	°	°	°	°	°	°	°
g	330	332	334	336	338	340	342
$3g$	270	276	282	288	294	300	306
$4g$	240	248	256	264	272	280	288
	"	"	"	"	"	"	"
$(v.s.3)$	-1.14	-1.12	-1.13	-1.16	-1.23	-1.33	-1.48
$(v.c.3)$	-2.46	-2.56	-2.66	-2.76	-2.88	-3.01	-3.16
$(v.s.4)$	$+0.59$	$+0.60$	$+0.59$	$+0.55$	$+0.50$	$+0.43$	$+0.36$
$(v.c.4)$	-0.10	-0.20	-0.30	-0.40	-0.47	-0.57	-0.64
$(v.s.3)\sin 3g$	$+1.14$	$+1.12$	$+1.11$	$+1.11$	$+1.12$	$+1.15$	$+1.20$
$(v.c.3)\cos 3g$	.00	-0.26	-0.55	-0.85	-1.17	-1.50	-1.86
$(v.s.4)\sin 4g$	$-$ .51	$-$ .56	$-$ .57	$-$ .55	$-$ .50	$-$ .42	$-$ .34
$(v.c.4)\cos 4g$	$+$ .05	$+$ .07	$+$ .07	$+$ .04	$-$ .02	$-$ .10	$-$ .19
Sum	$+0.68$	$+0.37$	$+0.06$	-0.25	-0.57	-0.87	-1.19

It will be seen that we have here computed twice as many numbers as are necessary to interpolate with all attainable accuracy.

The rest of the computation is fully given on the four following pages. First we have the values of g and the nine arguments for the intermediate dates, filled in by successive addition of the nearly constant difference. The arguments thus obtained for the last date may be compared with those just computed on the preceding page.

The numerals in the first columns of the sections of computation following indicate the arguments with which tables are entered to obtain the separate values of the quantities $(v.c.0)$, $(v.s.1)$, $(v.c.1)$, etc. The negative terms in Table XVII being taken from the sum of all the periodic terms from Tables VIII to XVI with argument 1 to 9, we have the final values of $(v.c.0)$, $(v.s.1)$, etc.

The final computation of the products $(v.s.i)\sin ig$, etc., and the addition of the separate terms which make up the three co-ordinates, are shown on page 205. The expressions $c.0$, $s.1$, etc., are employed for brevity, instead of $(v.c.0)$, $(v.s.1)\sin g$, etc.

The longitude finally given by the tables is referred to the mean equinox, and must therefore be corrected for nutation before being used to compute the geocentric place.

Date,	1875, Dec. 15 1875.955.	1876, Apr. 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1876.940.	1877, Apr. 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, Apr. 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
	331 23.086	332 47.554	334 12.021	335 36.489	337 0.957	338 25.425	339 49.893	341 14.361
Arg. 1	147.097	161.368	175.639	189.910	204.182	218.453	232.724	246.995
2	320.655	325.001	329.346	333.692	338.038	342.383	346.729	351.074
3	179.739	180.889	182.038	183.188	184.338	185.488	186.638	187.788
4	153.	156.	159.	163.	166.	169.	172.	175.
5	167.	.....	.....	.....	.....	......	......	175.
6	489.	494.	500.	505.	511.	516.	521.	527.
7	123.	129.	135.	141.	147.	153.	160.	166.
8	401.6	403.4	405.2	407.0	408.7	410.5	412.3	414.1
9	56.	63.	69.	75.	82.	88.	95.	101.
(*v.c.*0) 1	108.01	107.74	106.29	103.68	99.98	95.25	89.62	83.21
2	15.80	14.71	13.66	12.65	11.68	10.76	9.90	9.09
3	72.43	73.60	74.77	75.94	77.10	78.26	79.42	80.56
5	.59	.58	.58	.57	.56	.56	.55	.54
6	.02	.02	.02	.03	.03	.03	.03	.03
7	.07	.07	.06	.06	.06	.06	.05	.05
8	1.68	1.65	1.63	1.60	1.58	1.55	1.53	1.50
9	.09	.09	.09	.09	.08	.08	.08	.07
(*v.c.*0)	198.69	198.46	197.10	194.62	191.07	186.55	181.18	175.05
(*v.s.*1) 1	6.84	7.68	8.40	8.99	9.44	9.74	9.91	9.98
2	132.20	125.48	118.82	112.21	105.67	99.24	92.89	86.69
sec. 2	.18	.20	.22	.23	.25	.27	.29	.31
3	1.09	1.07	1.08	1.12	1.19	1.29	1.41	1.56
4	.13	.12	.11	.10	.10	.09	.09	.08
5	.19	.19	.19	.19	.19	.20	.20	.20
6	.04	.04	.04	.05	.05	.06	.06	.07
7	.14	.14	.13	.13	.12	.12	.11	.10
8	1.28	1.28	1.27	1.26	1.26	1.25	1.24	1.23
9	.08	.08	.08	.08	.07	.07	.07	.07
Σ	142.17	136.28	130.34	124.36	118.34	112.33	106.27	100.29
Tab.XVII	—154.21	—153.84	—153.47	—153.10	—152.73	—152.36	—151.99	—151.62
(*v.s.*1)	— 12.04	— 17.56	— 23.13	— 28.74	— 34.39	— 40.03	— 45.72	— 51.33
(*v.c.*1) 1	5.64	6.03	6.38	6.67	6.85	6.91	6.82	6.59
2	5.04	6 00	7·31	8.90	10.80	13.00	15.49	18.25
sec. 2	.69	.71	.72	.74	.75	.77	.79	.80
3	23.20	24.09	24.95	25.82	26.68	27.53	28.37	29.19
4	.47	.48	.49	.50	.50	.51	.52	.52
5	.37	.37	.37	.37	.37	.37	.37	.37
6	.16	.17	.17	.18	.18	.18	.18	.18
7	.15	.14	.13	.13	.12	.12	.11	.10
8	0.66	0.64	0.63	0.61	0.60	0.59	0.57	0.56
9	.07	.07	.06	.06	.05	.04	.04	.03
Σ	36.42	38.70	41.21	43.98	46.90	50.02	53.26	56.59
Tab.XVII	—205.86	—206.08	—206.29	—206.51	—206.73	—206.94	—207.15	—207.37
(*v.c.*1)	—169.44	—167.38	—165.08	—162.53	—159.83	—156.92	—153.89	—150.78
(*v.s.*2) 1	.06	.10	.15	.21	.26	.31	.34	.37
2	54.12	49.37	44.78	40.37	36.15	32.11	28.30	24.70
sec. 2	.39	.42	.44	.47	.49	.51	.53	.56
3	2.05	2.19	2.31	2.45	2.60	2.76	2.91	3.07
Σ	56.62	52.08	47.68	43.50	39.50	35.69	32.08	28.70
Tab.XVII	—134.34	—134.32	—134.31	—134.29	—134.27	—134.26	—134.24	—134.22
(*v.s.*2)	— 77.72	— 82.24	— 86.63	— 90.79	— 94.77	— 98.57	—102.16	—105.52

Date,	1875, Dec. 15 1875.955.	1876, Apr. 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1867.940.	1877, Apr. 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, Apr. 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
g	331 23.086	332 47.554	334 12.021	335 36.489	337 0.057	338 25.425	339 49.893	341 14.361
($v.c.$2) 1	.31	.30	.29	.29	.29	.30	.30	.31
2	25.34	28.93	32.71	36.73	40.93	45.35	49.93	54.69
sec. 2	.76	.76	.77	.78	.79	.79	.80	.80
3	6.23	6.40	6.56	6.70	6.85	6.99	7.10	7.20
Σ	32.64	36.39	40.33	44.50	48.86	53.43	58.13	63.00
Tab.XVII	—136.57	—136.59	—136.60	—136.62	—136.64	—136.65	—136.67	—136.69
($v.c.$2)	—103.93	—100.20	— 96.27	— 92.12	— 87.78	— 83.22	— 78.54	— 73.69
($v.s.$3) 2	6.92	6.89	6.84	6.78	6.70	6.61	6.51	6.37
3	0.87	0.91	0.94	0.98	1.01	1.04	1.08	1.11
	7.79	7.80	7.78	7.76	7.71	7.65	7.59	7.48
Tab.XVII	—8.92	—8.92	—8.91	—8.91	—8.91	—8.91	—8.91	—8.91
($v.s.$3)	—1.13	—1.12	—1.13	—1.15	—1.20	—1.26	—1.32	—1.43
($v.c.$3) 2	3.19	3.12	3.05	2.98	2.90	2.82	2.75	2.67
3	1.34	1.34	1.34	1.34	1.33	1.33	1.31	1.29
	4.53	4.46	4.39	4.32	4.23	4.15	4.06	3.96
Tab.XVII	—7.06	—7.06	—7.06	—7.06	—7.06	—7.06	—7.06	—7.06
($v.c.$3)	—2.53	—2.60	—2.67	—2.74	—2.83	—2.91	—3.00	—3.10
($v.s.$4) 2	1.48	1.48	1.47	1.45	1.43	1.40	3.36	1.33
3	—0.88	—0.88	—0.88	—0.89	—0.89	—0.91	—0.93	— .94
($v.s.$4)	+0.60	+0.60	+0.59	+0.56	+0.54	+0.49	+0.43	+0.39
($v.c.$4) 2	0.83	0.77	0.72	0.66	0.61	0.57	0.51	0.47
3	—1.00	—1.02	—1.03	—1.04	—1.05	—1.06	—1.07	—1.08
($v.c.$4)	—0.17	—0.25	—0.31	—0.38	—0.44	—0.49	—0.56	—0.61
($\rho.c.$0) 1	1230	1063	899	741	594	460	344	246
2	98	99	101	104	108	113	118	126
3	11	10	9	9	8	8	8	9
Tab.XVII	968	968	968	967	967	967	966	966
($\rho.c.$0)	2307	2140	1977	1821	1677	1548	1436	1347
($\rho.s.$1) 1	248	240	230	221	211	203	194	187
2	2835	2822	2807	2789	2769	2748	2723	2696
3	173	168	165	161	158	155	151	148
Σ	3256	3230	3202	3171	3138	3106	3068	3031
Tab.XVII	—1984	—1987	—1989	—1991	—1993	—1996	—1998	2000
($\rho.s.$1)	+1272	1243	1213	1180	1145	1110	1070	1031
($\rho.c.$1) 1	123	112	97	83	69	55	41	31
2	1266	1202	1138	1074	1009	947	886	827
3	66	66	67	67	68	70	72	74
Σ	1455	1380	1302	1224	1146	1072	999	932
Tab.XVII	—1977	—1981	—1986	—1990	—1994	—1998	—2002	2006
($\rho.c.$1)	— 522	— 601	— 684	— 766	— 848	— 926	—1003	—1074

Date,	1875, Dec. 15 1875.955.	1876, Apr. 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1867.940.	4877, Apr. 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, Apr. 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
g	331 23.086	332 47.554	334 12.021	335 36.489	337 0.957	338 25.425	339 49.893	341 14.361
($\rho.s.2$) 2	174	181	189	196	204	213	223	232
3	26	25	25	24	24	24	23	23
Σ	200	206	214	220	228	237	246	255
Tab. XVII	—459	—459	—459	—459	—460	—460	—460	—460
($\rho.s.2$)	—259	—253	—245	—239	—232	—223	—214	—205
($\rho.c.2$) 2	559	568	576	584	592	600	607	614
3	33	33	34	35	36	37	39	40
Σ	592	601	610	619	628	637	646	654
Tab. XVII	—464	—464	—464	—465	—465	—465	—465	—466
($\rho.c.2$)	+128	137	146	154	163	172	181	188
($\rho.s.3$) 2	28	31	34	39	43	47	52	57
Tab. XVII	—101	—101	—101	—101	—101	—101	—101	—101
($\rho.s.3$)	— 73	— 70	— 67	— 62	— 58	— 54	— 49	— 44
($\rho.c.3$) 2	155	159	162	166	169	172	175	177
Tab. XVII	—102	—102	—102	—102	—102	—102	—102	—102
($\rho.c.3$)	+ 53	57	60	64	67	70	73	75
($b.c.0$) 2	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15
3	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Tab. XXIII	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
($b.c.0$)	0.30	0.30	0.31	0.31	0.32	0.32	0.33	0.33
($b.s.1$) 1	0.30	0.22	0.16	0.12	0.08	0.06	0.05	0.06
2	0.12	0.09	0.06	0.05	0.04	0.04	0.05	0.06
3	1.11	1.11	1.12	1.12	1.13	1.13	1.14	1.14
Σ	1.53	1.42	1.34	1.29	1.25	1.23	1.24	1.26
Tab. XXIII	—5.23	—5.24	—5.24	—5.24	—5.24	—5.25	—5.25	—5.25
($b.s.1$)	—3.70	—3.82	—3.90	—3.95	—3.99	—4.02	—4.01	—3.99
($b.c.1$) 1	1.21	1.14	1.06	0.98	0.90	0.80	0.70	0.61
2	0.96	1.06	1.17	1.28	1.39	1.51	1.63	1.75
3	1.01	1.00	1.00	0.99	0.99	0.99	0.98	0.98
Σ	3.18	3.20	3.23	3.25	3.28	3.30	3.31	3.34
Tab. XXIII	—4.45	—4.44	—4.43	—4.43	—4.42	—4.41	—4.41	—4.40
($b.c.1$)	—1.27	—1.24	—1.20	—1.18	—1.14	—1.11	—1.10	—1.06
($b.s.2$) 2	0.02	0.02	0.03	0.04	0.04	0.05	0.06	0.07
3	0.16	0.16	0.16	0.15	0.15	0.15	0.15	0.15
Tab. XXIII	—0.48	—0.48	—0.48	—0.48	—0.48	—0.48	—0.48	—0.48
($b.s.2$)	-0.30	—0.30	—0.29	—0.29	—0.29	—0.28	—0.27	—0.26
($b.c.2$) 2	0.31	0.32	0.33	0.34	0.34	0.35	0.36	0.36
3	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11
Tab. XXIII	—0.38	—0.38	—0.38	—0.38	—0.38	—0.38	—0.38	—0.38
($b.c.2$)	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09	+0.09

Date,	1875, Dec. 15 1875.955.	1876, April 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1876.940.	1877, April 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, April 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
g	331 23.086	332 47.554	334 12.021	335 36.489	337 0.957	338 25.425	339 49.893	341 14.361
$\log(v.s.1)$	—1.0806	—1.2445	—1.3642	—1.4585	—1.5365	—1.6024	—1.6601	—1.7104
$\sin g$	—9.6803	—9.6601	—9.6386	—9.6159	—9.5916	—9.5655	—9.5375	—9.5073
$\log(\rho.s.1)$	+3.1045	3.0944	3.0838	3.0719	3.0588	3.0453	3.0294	3.0132
$\log(v.c.1)$	—2.2290	—2.2237	—2.2177	—2.2109	—2.2036	—2.1957	—2.1872	—2.1784
$\cos g$	+9.9434	9.9491	9.9544	9.9594	9.9641	9.9685	9.9725	9.9763
$\log(\rho.c.1)$	—2.718	—2.779	—2.835	—2.884	—2.928	—2.967	—3.0013	—8.0310
$\log(v.s.2)$	—1.8905	—1.9151	—1.9376	—1.9580	—1.9766	—1.9937	—2.0093	—2.0233
$\sin 2g$	—9.9247	—9.9102	—9.8941	—9.8763	—9.8566	—9.8350	—9.8111	—9.7846
$\log(\rho.s.2)$	—2.413	—2.403	—2.389	—2.378	—2.365	—2.348	—2.330	—2.312
$\log(v.c.2)$	—2.0167	—2.0009	—1.9835	—1.9644	—1.9434	—1.9202	—1.8951	—1.8674
$\cos 2g$	+9.7335	9.7649	9.7932	9.8189	9.8421	9.8630	9.8821	9.8994
$\log(\rho.c.2)$	+2.107	2.137	2.164	2.187	2.212	2.236	2.258	2.274
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Δg	1 24 28.065	1 24 28.066	1 24 28.066	1 24 28.067	1 24 28.068	1 24 28.068	1 24 28.069	
g	331 23 5.16	332 47 33.225	334 12 1.291	335 36 29.357	337 0 57.424	338 25 25.492	339 49 53.560	341 14 21.63
ω	95 11 41.00	95 11 51.40	95 12 1.81	95 12 12.21	95 12 22.61	95 12 33.01	95 12 43.42	95 12 53.82
E	—2 42 49.15	—2 35 33.18	—2 28 10.63	—2 20 41.76	—2 13 6.93	—2 5 26.43	—1 57 40.55	—1 49 49.66
$c.0$	3 18.69	3 18.46	3 17.10	3 14.62	3 11.07	3 6.55	3 1.18	2 55.05
$s.1$	5.77	8.03	10.07	11.87	13.43	14.72	15.76	16.51
$c.1$	—2 28.72	—2 28.86	—2 28.62	—2 28.00	—2 27.13	—2 25.93	—2 24.43	—2 22.80
$s.2$	1 5.34	1 6.88	1 7.87	1 8.28	1 8.12	1 7.40	1 6.13	1 4.26
$c.2$	— 56.27	— 58.31	— 59.80	—1 0.71	—1 1.03	—1 0.70	— 59.87	— 58.45
$(3+4)$	0.46	0.25	0.03	— 0.19	— 0.41	— 0.63	— 0.83	— 1.00
u	63 53 2.28	65 24 57.89	66 56 59.12	68 29 5.68	70 1 17.15	71 33 33.48	73 5 54.37	74 38 19.36
θ	73 22 0.15	73 22 6.26	73 22 12.37	73 22 18.48	73 22 24.60	73 22 30.71	73 22 36.82	73 22 42.93
R	2.58	2.91	3.25	3.61	3.98	4.37	4.79	5.22
Longitude	137 15 5.01	138 47 7.06	140 19 14.74	141 51 27.77	143 23 45.73	144 56 8.56	146 28 35.98	148 1 7.51
$\log r_0$	1.2647392	1.2644735	1.2642196	1.2639778	1.2637486	1.2635319	1.2633280	1.2631370
$c.0$	2307	2140	1977	1821	1677	1548	1436	1347
$s.1$	—609	—568	—528	—487	—447	—408	—369	— 332
$c.1$	—458	—535	—615	—697	—780	—861	—941	—1017
$s.2$	+218	206	192	179	167	152	138	125
$c.2$	+ 69	80	91	101	113	126	138	149
$s.3$	+ 73	69	65	60	54	49	43	37
$c.3$	+ 4	8	13	19	23	29	36	41
$\log r$	1.2648996	1.2646135	1.2643391	1.2640774	1.2638293	1.2635954	1.2633761	1.2631720
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
β_0	+41 36.64	42 8.48	42 38.54	43 6.78	43 33.14	43 57.62	44 20.23	44 41.19
$c.0$	+0.30	0.30	0.31	0.31	0.32	0.32	0.33	0.33
$s.1$	+1.77	1.75	1.70	1.63	1.57	1.48	1.38	1.29
$c.1$	—1.11	—1.10	—1.08	—1.07	—1.05	—1.03	—1.03	—1.00
$s.2$	+0.25	0.24	0.23	0.22	0.21	0.19	0.17	0.16
$c.2$	+0.02	0.03	0.03	0.04	0.05	0.06	0.07	0.07
Latitude	+41 37.87	42 9.70	42 39.73	43 7.91	43 34.24	43 58.64	44 21.15	44 42.04

TABLE I.—Corrections of Arguments for past and future Centuries.

Century.	g	ω	ω	θ	θ'	Arg. 1
	° ′ ″	° ′ ″	″	° ′ ″	″	
0J	207 15 59.32	343 52 17.36	+108.00	351 6 26.89	—148.32	408.924
100	275 45 34.18	344 46 54.12	102.00	351 34 55.39	—140.08	552.952
200	344 15 9.04	345 41 24.88	96.00	352 3 32.13	—131.84	96.980
300	52 44 43.90	346 35 49.64	90.00	352 32 17.11	—123.60	241.008
400	121 14 18.76	347 30 8.40	84.00	353 1 10.33	—115.36	385.036
500	189 43 53.62	348 24 21.16	+ 78.00	353 30 11.79	—107.12	529.064
600	258 13 28.48	349 18 27.92	72.00	353 59 21.49	— 98.88	73.092
700	326 43 3.34	350 12 28.68	66.00	354 28 39.43	— 90.64	217.120
800	35 12 38.20	351 6 23.44	60.00	354 58 5.61	— 82.40	361.148
900	103 42 13.06	352 0 12.20	54.00	355 27 40.03	— 74.16	505.176
1000	172 11 47.92	352 53 54.96	+ 48.00	355 57 22.69	— 65.92	49.204
1100	240 41 22.78	353 47 31.72	42.00	356 27 13.59	— 57.68	193.232
1200	309 10 57.64	354 41 2.48	36.00	356 57 12.73	— 49.44	337.260
1300	17 40 32.50	355 34 27.24	30.00	357 27 20.11	— 41.20	481.288
1400	86 10 7.36	356 27 46.00	24.00	357 57 35.73	— 32.96	25.316
1500J	154 39 42.22	357 20 58.76	+ 18.00	358 27 59.59	— 24.72	169.344
1500G	154 32 39.89	357 20 57.89	18.00	358 27 59.08	— 24.72	168.155
1600	223 2 14.75	358 14 4.65	12.00	358 58 31.18	— 16.48	312.183
1700	291 31 7.37	359 7 5.33	6.00	359 29 11.47	— 8.24	456.092
1800	0 0 0.00	0 0 0.00	0.00	0 0 0.00	0.00	0.000
1900	68 28 52.63	0 52 48.67	— 6.00	0 30 56.77	+ 8.24	143.908
2000	136 58 27.49	1 45 31.43	— 12.00	1 2 1.83	16.48	287.936
2100	205 27 20.11	2 38 8.11	— 18.00	1 33 15.08	24.72	431.845
2200	273 56 12.74	3 30 38.78	— 24.00	2 4 36.57	+ 32.96	575.755

TABLE II.—Arguments for the beginning of each Fourth Year 1752—1948.

Year.	g	ω	θ	Arg. 1
	° ′ ″	° ′ ″	° ′ ″	
1752	160 15 8.10	94 6 10.48	72 43 42.30	162.101
1756	177 23 31.10	94 8 17.46	72 44 56.25	335.862
1760	194 31 54.09	94 10 24.43	72 46 10.22	509.623
1764	211 40 17.09	94 12 31.39	72 47 24.21	83.384
1768	228 48 40.08	94 14 38.34	72 48 38.20	257.145
1772	245 57 3.08	94 16 45.28	72 49 52.21	430.907
1776	263 5 26.07	94 18 52.22	72 51 6.23	4.668
1780	280 13 49.06	94 20 59.14	72 52 20.27	178.429
1784	297 22 12.06	94 23 6.05	72 53 34.32	352.190
1788	314 30 35.05	94 25 12.96	72 54 48.38	525.951
1792	331 38 58.05	94 27 19.85	72 56 2.46	99.712
1796	348 47 21.04	94 29 26.74	72 57 16.54	273.473
1800	5 55 1.80	94 31 33.53	72 58 30.59	447.115

TABLE I.—*Continued.*

Century.	2	3	4	5	6	7	8	9
0J	191.528	299.485	280	512	102	61	470.0	15
100	314.245	49.520	64	250	563	124	410.6	214
200	436.962	399.555	449	589	424	188	351.2	413
300	559.679	149.590	233	327	285	251	291.7	13
400	82.396	499.625	18	65	146	314	232.3	212
500	205.113	249.660	402	403	7	378	172.9	411
600	327.830	599.695	187	142	468	441	113.5	10
700	450.547	349.730	571	480	329	504	54.1	209
800	573.264	99.765	356	218	190	567	594.6	409
900	95.981	449.800	140	556	51	31	535.2	8
1000	218.698	199.835	524	295	512	94	475.8	207
1100	341.415	549.870	309	33	373	157	416.4	406
1200	464.132	299.905	93	371	234	221	357.0	5
1300	586.849	49.940	478	109	95	284	297.5	205
1400	109.566	399.975	262	448	556	347	238.1	404
1500J	232.283	150.010	47	186	417	411	178.6	3
1500G	231.921	149.914	47	186	417	410	178.3	2
1600	354.638	499.949	431	524	278	473	118.8	201
1700	477.319	249.975	216	262	139	537	59.4	401
1800	0.000	0.000	0	0	0	0	0	0
1900	122.681	350.025	385	338	461	63	540.6	199
2000	245.398	100.060	169	76	322	127	481.2	398
2100	368.079	450.086	554	415	183	190	421.8	597
2200	490.760	200.111	338	153	44	253	362.3	196

TABLE II.—*Continued.*

Year.	2	3	4	5	6	7	8	9
1752	481.104	345.855	133	348	229	213	331.6	578
1756	534.013	359.856	172	362	296	287	353.2	58
1760	586.921	373.858	212	375	362	362	374.8	138
1764	39.830	387.859	251	389	429	436	396.4	218
1768	92.739	401.860	290	402	495	511	418.1	298
1772	145.647	415.862	330	416	562	585	439.7	377
1776	198.556	429.863	369	429	28	60	461.3	457
1780	251.465	443.865	408	443	95	134	482.9	537
1784	304.373	457.866	448	457	161	209	504.6	17
1788	357.282	471.868	487	470	227	283	526.2	97
1792	410.191	485.869	527	484	294	358	547.8	177
1796	463.100	499.870	566	497	360	433	569.4	257
1800	515.972	513.862	5	511	427	507	591.1	337

TABLE II.—*Continued.*

Year.	g			ω			θ			Arg. 1.
	°	′	″	°	′	″	°	′	″	
1800	5	55	1.80	94	31	33.53	72	58	30.59	447.115
1804	23	3	24.80	94	33	40.39	72	59	44.71	20.876
1808	40	11	47.79	94	35	47.25	73	0	58.84	194.638
1812	57	20	10.79	94	37	54.09	73	2	12.98	368.399
1816	74	28	33.78	94	40	0.93	73	3	27.13	542.160
1820	91	36	56.77	94	42	7.76	73	4	41.30	115.921
1824	108	45	19.77	94	44	14.58	73	5	55.48	289.683
1828	125	53	42.76	94	46	21.38	73	7	9.67	463.444
1832	143	2	5.76	94	48	28.18	73	8	23.88	37.205
1836	160	10	28.75	94	50	34.97	73	9	38.10	210.966
1840	177	18	51.75	94	52	41.75	73	10	52.33	384.727
1844	194	27	14.74	94	54	48.52	73	12	6.58	558.488
1848	211	35	37.74	94	56	55.28	73	13	20.84	132.249
1852	228	44	0.73	94	59	2.03	73	14	35.11	306.010
1856	245	52	23.73	95	1	8.77	73	15	49.40	479.771
1860	263	0	46.72	95	3	15.50	73	17	3.70	53.532
1864	280	9	9.71	95	5	22.22	73	18	18.01	227.293
1868	297	17	32.71	95	7	28.94	73	19	32.34	401.054
1872	314	25	55.70	95	9	35.64	73	20	46.67	574.815
1876	331	34	18.70	95	11	42.33	73	22	1.03	148.576
1880	348	42	41.69	95	13	49.01	73	23	15.39	322.337
1884	5	51	4.69	95	15	55.69	73	24	29.77	496.098
1888	22	59	27.68	95	18	2.35	73	25	44.16	69.860
1892	40	7	50.68	95	20	9.01	73	26	58.57	243.621
1896	57	16	13.67	95	22	15.65	73	28	12.98	417.382
1900	74	23	54.43	95	24	22.20	73	29	27.36	591.024
1904	91	32	17.43	95	26	28.83	73	30	41.81	164.785
1908	108	40	40.42	95	28	35.44	73	31	56.26	338.546
1912	125	49	3.41	95	30	42.05	73	33	10.74	512.307
1916	142	57	26.41	95	32	48.64	73	34	25.22	86.068
1920	160	5	49.40	95	34	55.23	73	35	39.72	259.830
1924	177	14	12.40	95	37	1.81	73	36	54.23	433.591
1928	194	22	35.39	95	39	8.38	73	38	8.75	7.352
1932	211	30	58.39	95	41	14.94	73	39	23.29	181.113
1936	228	39	21.38	95	43	21.48	73	40	37.84	354.874
1940	245	47	44.38	95	45	28.02	73	41	52.40	528.635
1944	262	56	7.37	95	47	34.55	73	43	6.97	102.396
1948	280	4	30.36	95	49	41.07	73	44	21.56	276.158
$\Delta^{(1)}_{120}$	1	24	28.007			10.411			6.100	14.2715
Factor T			+0.222			—.020			+.027	—.0012
$\Delta^{(2)}_{120}$			+ .0007			—.0001			+.0001	0

TABLE II.—*Continued.*

	2	3	4	5	6	7	8	9
1800	515.972	513.862	5	511	427	507	591.1	337
1804	568.881	527.863	45	524	493	582	12.7	417
1808	21.790	541.865	84	538	560	56	34.3	497
1812	74.698	555.866	124	551	26	131	55.9	577
1816	127.607	569.868	163	565	93	205	77.5	57
1820	180.516	583.869	202	578	159	280	99.2	137
1824	233.424	597.870	242	592	225	354	120.8	217
1828	286.333	11.872	281	6	292	429	142.4	297
1832	339.242	25.874	320	19	358	503	164.0	377
1836	392.150	39.875	360	32	425	578	185.7	457
1840	445.059	53.876	399	46	491	52	207.3	537
1844	497.968	67.878	439	59	557	127	228.9	17
1848	550.876	81.879	478	73	24	201	250.5	97
1852	3.785	95.880	517	87	90	276	272.1	177
1856	56.694	109.882	557	100	157	350	293.8	257
1860	109.602	123.883	596	114	223	425	315.4	337
1864	162.511	137.885	35	127	290	499	337.0	417
1868	215.420	151.886	75	141	356	574	358.6	497
1872	268.328	165.888	114	154	423	49	380.3	577
1876	321.237	179.889	153	168	489	123	401.9	57
1880	374.146	193.890	193	181	555	198	423.5	137
1884	427.054	207.892	232	195	22	272	445.1	217
1888	479.963	221.893	271	208	88	347	466.7	297
1892	532.872	235.895	311	222	155	421	488.4	376
1896	585.780	249.896	350	235	221	496	510.0	456
1900	38.653	263.889	390	249	287	570	531.6	536
1904	91.562	277.890	429	262	354	45	553.2	16
1908	144.470	291.891	468	276	420	119	574.9	96
1912	197.379	305.892	508	290	487	194	596.5	176
1916	250.288	319.894	547	303	553	268	18.1	256
1920	303.197	333.895	587	317	20	343	39.7	336
1924	356.105	347.897	26	330	86	417	61.4	416
1928	409.014	361.898	65	344	152	492	83.0	496
1932	461.923	375.900	105	357	219	566	104.6	576
1936	514.831	389.901	144	371	285	41	126.2	56
1940	567.740	403.902	184	384	352	116	147.8	136
1944	20.649	417.904	223	398	418	190	169.5	216
1948	73.557	431.905	262	411	485	265	191.1	296
	4.3457	1.1500	3.2	1.1	5.4	6.1	1.78	6.6
	— .0001	+ .0002	0	0	0	0	0	0
	0	0	0	0	0	0	0	0

TABLE III.—REDUCTION OF THE EPOCHS AND ARGUMENTS TO THE BEGINNING OF EACH MONTH IN A CYCLE OF FOUR YEARS.

		g			ω			θ			θ'	Arg. 1
		°	′	″	°	′	″	°	′	″	″	
Year 0												
January	0	0	0	0.00	0	0	0.00	0	0	0.00	0.00	0.000
February	0	0	21	49.24	0	0	2.69	0	0	1.58	0.00	3.687
March	0	0	42	14.00	0	0	5.21	0	0	3.05	0.00	7.136
April	0	1	4	3.24	0	0	7.89	0	0	4.63	0.01	10.823
May	0	1	25	10.24	0	0	10.50	0	0	6.15	0.01	14.391
June	0	1	46	59.48	0	0	13.19	0	0	7.73	0.01	18.078
July	0	2	8	6.48	0	0	15.79	0	0	9.25	0.01	21.647
August	0	2	29	55.71	0	0	18.48	0	0	10.83	0.01	25.333
September	0	2	51	44.95	0	0	21.17	0	0	12.40	0.01	29.019
October	0	3	12	51.95	0	0	23.77	0	0	13.93	0.02	32.587
November	0	3	34	41.19	0	0	26.46	0	0	15.51	0.02	36.274
December	0	3	55	48.19	0	0	29.06	0	0	17.03	0.02	39.842
Year 1												
January	0	4	17	37.42	0	0	31.75	0	0	18.61	0.02	43.529
February	0	4	39	26.66	0	0	34.44	0	0	20.18	0.02	47.216
March	0	4	59	9.19	0	0	36.87	0	0	21.61	0.03	50.546
April	0	5	20	58.43	0	0	39.56	0	0	23.18	0.03	54.233
May	0	5	42	5.43	0	0	42.16	0	0	24.71	0.03	57.801
June	0	6	3	54.67	0	0	44.85	0	0	26.28	0.03	61.488
July	0	6	25	1.67	0	0	47.46	0	0	27.81	0.03	65.056
August	0	6	46	50.90	0	0	50.14	0	0	29.38	0.04	68.742
September	0	7	8	40.14	0	0	52.83	0	0	30.96	0.04	72.429
October	0	7	29	47.14	0	0	55.44	0	0	32.48	0.04	75.997
November	0	7	51	36.38	0	0	58.13	0	0	34.06	0.04	79.684
December	0	8	12	43.38	0	1	0.73	0	0	35.59	0.04	83.252
Year 2												
January	0	8	34	32.61	0	1	3.43	0	0	37.16	0.04	86.939
February	0	8	56	21.85	0	1	6.11	0	0	38.74	0.05	90.626
March	0	9	16	4.38	0	1	8.54	0	0	40.16	0.05	93.956
April	0	9	37	53.62	0	1	11.23	0	0	41.74	0.05	97.643
May	0	9	59	0.62	0	1	13.83	0	0	43.26	0.05	101.211
June	0	10	20	49.86	0	1	16.52	0	0	44.84	0.05	104.898
July	0	10	41	56.86	0	1	19.12	0	0	46.36	0.06	108.466
August	0	11	3	46.09	0	1	21.81	0	0	47.94	0.06	112.153
September	0	11	25	35.33	0	1	24.50	0	0	49.52	0.06	115.840
October	0	11	46	42.33	0	1	27.10	0	0	51.04	0.06	119.407
November	0	12	8	31.57	0	1	29.79	0	0	52.62	0.06	123.094
December	0	12	29	38.57	0	1	32.40	0	0	54.14	0.07	126.662
Year 3												
January	0	12	51	27.80	0	1	35.08	0	0	55.72	0.07	130.349
February	0	13	13	17.04	0	1	37.77	0	0	57.29	0.07	134.036
March	0	13	32	59.57	0	1	40.20	0	0	58.72	0.07	137.366
April	0	13	54	48.81	0	1	42.89	0	1	0.29	0.07	141.053
May	0	14	15	55.81	0	1	45.50	0	1	1.82	0.08	144.621
June	0	14	37	45.05	0	1	48.18	0	1	3.40	0.08	148.308
July	0	14	58	52.05	0	1	50.79	0	1	4.92	0.08	151.876
August	0	15	20	41.28	0	1	53.48	0	1	6.50	0.08	155.563
September	0	15	42	30.52	0	1	56.17	0	1	8.07	0.08	159.249
October	0	16	3	37.52	0	1	58.77	0	1	9.60	0.08	162.817
November	0	16	25	26.76	0	2	1.46	0	1	11.17	0.09	166.504
December	0	16	46	33.76	0	2	4.06	0	1	12.70	0.09	170.072

TABLE III.—*Continued.*

	2	3	4	5	6	7	8	9
Year 0								
January 0	0.000	0.000	0	0	0	0	0.0	0
February 0	1.123	0.297	1	0	1	1	0.5	2
March 0	2.209	0.575	2	1	3	3	0.9	3
April 0	3.259	0.872	2	1	4	4	1.3	5
May 0	4.382	1.159	3	1	5	6	1.8	7
June 0	5.505	1.457	4	1	7	7	2.2	8
July 0	6.591	1.745	5	2	8	9	2.7	10
August 0	7.714	2.042	6	2	10	10	3.1	12
September 0	8.837	2.339	7	2	11	12	3.6	13
October 0	9.923	2.626	7	3	12	14	4.0	15
November 0	11.046	2.923	8	3	14	15	4.5	17
December 0	12.132	3.211	9	3	15	17	4.9	18
Year 1								
January 0	13.254	3.508	10	3	17	19	5.4	20
February 0	14.377	3.805	11	4	18	20	5.9	22
March 0	15.391	4.073	11	4	19	22	6.3	23
April 0	16.514	4.370	12	4	21	23	6.7	25
May 0	17.600	4.658	13	4	22	25	7.2	27
June 0	18.723	4.955	14	5	24	27	7.6	28
July 0	19.809	5.242	15	5	25	28	8.1	30
August 0	20.932	5.539	15	5	26	30	8.5	32
September 0	22.054	5.836	16	6	28	31	9.0	33
October 0	23.140	6.124	17	6	29	33	9.4	35
November 0	24.263	6.421	18	6	30	34	9.9	37
December 0	25.349	6.709	19	6	32	36	10.3	38
Year 2								
January 0	26.472	7.006	20	7	33	37	10.8	40
February 0	27.595	7.303	20	7	34	39	11.3	42
March 0	28.609	7.571	21	7	36	40	11.7	43
April 0	29.732	7.868	22	7	37	42	12.1	45
May 0	30.818	8.156	23	8	38	44	12.6	47
June 0	31.941	8.453	24	8	40	45	13.0	48
July 0	33.027	8.741	25	8	41	47	13.5	50
August 0	34.150	9.038	25	9	43	48	13.9	52
September 0	35.272	9.335	26	9	44	50	14.4	53
October 0	36.358	9.622	27	9	45	51	14.8	55
November 0	37.481	9.919	28	9	47	53	15.3	57
December 0	38.567	10.207	29	10	48	55	15.7	58
Year 3								
January 0	39.690	10.504	30	10	50	56	16.2	60
February 0	40.813	10.801	30	10	51	58	16.7	62
March 0	41.827	11.070	31	11	52	59	17.1	63
April 0	42.950	11.367	32	11	54	61	17.5	65
May 0	44.036	11.654	33	11	55	62	18.0	67
June 0	45.159	11.951	34	11	57	64	18.4	68
July 0	46.245	12.239	35	12	58	65	18.9	70
August 0	47.368	12.536	35	12	59	67	19.3	72
September 0	48.490	12.833	36	12	61	68	19.8	73
October 0	49.576	13.121	37	12	62	70	20.2	75
November 0	50.699	13.418	38	13	64	72	20.7	77
December 0	51.785	13.705	39	13	65	73	21.1	78

TABLE IV.—MOTION OF ARGUMENTS FOR DAYS.

Days.	g	ω	θ	1	2	3	4	5	6	7	8	9
	′ ″	″	″									
1	0 42.23	0.09	0.05	0.119	0.036	0.010	0	0	0	0	0	0
2	1 24.47	0.17	0.10	0 238	0.072	0.019	0	0	0	0	0	0
3	2 6.70	0.26	0.15	0.357	0.109	0.029	0	0	0	0	0	0
4	2 48.93	0.35	0.20	0.476	0.145	0.039	0	0	0	0	0.1	0
5	3 31.17	0.43	0.25	0.595	0.181	0.048	0	0	0	0	0.1	0
6	4 13.40	0.52	0.30	0.714	0.217	0.058	0	0	0	0	0.1	0
7	4 55.63	0.61	0.36	0.833	0.253	0.067	0	0	0	0	0.1	0
8	5 37.87	0.69	0.41	0.951	0.290	0.077	0	0	0	0	0.1	0
9	6 20.10	0.78	0.46	1.070	0.326	0.086	0	0	0	0	0.1	0
10	7 2.33	0.87	0.51	1.189	0.362	0.096	0	0	1	1	0.1	1
11	7 44.57	0.95	0.56	1.308	0.398	0.105	0	0	1	1	0.2	1
12	8 26.80	1.04	0.61	1.427	0.434	0.115	0	0	1	1	0.2	1
13	9 9.03	1.13	0.66	1.546	0.471	0.125	0	0	1	1	0.2	1
14	9 51.27	1.21	0.71	1.665	0.507	0.134	0	0	1	1	0.2	1
15	10 33.50	1.30	0.76	1.784	0.543	0.144	0	0	1	1	0.2	1
16	11 15.73	1.39	0.81	1.903	0.579	0.153	0	0	1	1	0.2	1
17	11 57.97	1.47	0.86	2.022	0.616	0.163	0	0	1	1	0.2	1
18	12 40.20	1.56	0.91	2.141	0.652	0.173	0	0	1	1	0.3	1
19	13 22.43	1.65	0.97	2.260	0.688	0.182	1	0	1	1	0.3	1
20	14 4.67	1.74	1.02	2.378	0.724	0.192	1	0	1	1	0.3	1
21	14 46.90	1.82	1.07	2.497	0.760	0.201	1	0	1	1	0.3	1
22	15 29.14	1.91	1.12	2.616	0.797	0.211	1	0	1	1	0.3	1
23	16 11.37	2.00	1.17	2.735	0.833	0.220	1	0	1	1	0.3	1
24	16 53.60	2.08	1.22	2.854	0.869	0.230	1	0	1	1	0.3	1
25	17 35.84	2.17	1.27	2.973	0.905	0.240	1	0	1	1	0.4	1
26	18 18.07	2.26	1.32	3.092	0.941	0.249	1	0	1	1	0.4	1
27	19 0.30	2.34	1.37	3.211	0.978	0.259	1	0	1	1	0.4	1
28	19 42.54	2.43	1.42	3.330	1.014	0.268	1	0	1	1	0.4	1
29	20 24.77	2.52	1.47	3.449	1.050	0.278	1	0	1	1	0.4	1
30	21 7.00	2.60	1.52	3.568	1.086	0.288	1	0	1	1	0.4	1
31	21 49.24	2.69	1.57	3.687	1.123	0.297	1	0	1	1	0.5	1

TABLE V.—MOTION OF g FOR HOURS.

Hours.	g	Hours.	g	Hours.	g	Hours.	g
	″		″		″		″
0	0.00	6	10.56	12	21.12	18	31.67
1	1.76	7	12.32	13	22.88	19	33.43
2	3.52	8	14.08	14	24.64	20	35.19
3	5.28	9	15.84	15	26.40	21	36.95
4	7.04	10	17.60	16	28.16	22	38.71
5	8.80	11	19.36	17	29.92	23	40.47
6	10.56	12	21.12	18	31.67	24	42.23

The period of arguments 1 to 9 is 600.

In January and February of those years which, though divisible by 4, are not leap years, namely, 1700, 1800, 1900, 2100, etc., Table IV must be entered with a number 1 greater than the real day of the month.

TABLE VI.—Corrections of Arguments for Terms of Long Period.

Year.	g	1	2	3	Year.	g		1	2	3
	′ ″					′ ″				
1000	+36 51.00	—0.614	—1.024	+1.792	1550	+5 9.73		—0.200	—0.144	+0.252
1010	36 6.49	0.621	1.003	1.756	1560	4 49.70		0.153	0.134	0.236
1020	35 22.14	0.629	0.983	1.720	1570	4 30.32		0.106	0.125	0.219
1030	34 37.97	0.638	0.962	1.684	1580	4 11.57		0.060	0.117	0.205
1040	33 54.00	0.648	0.942	1.649	1590	3 53.48		—0.013	0.108	0.189
1050	+33 10.25	—0.659	—0.922	+1.614	1600	+3 36.03		+0.034	—0.100	+0.175
1060	32 26.71	0.672	0.902	1.579	1610	3 19.26		0.078	0.092	0.161
1070	31 43.41	0.686	0.882	1.544	1620	3 3.14		0.120	0.085	0.148
1080	31 0.37	0.702	0.862	1.509	1630	2 47.69		0.162	0.078	0.135
1090	30 17.61	0.718	0.842	1.474	1640	2 32.89		0.203	0.071	0.123
1100	+29 35.15	—0.735	—0.822	+1.439	1650	+2 18.77		+0.242	—0.064	+0.112
1110	28 53.01	0.752	0.803	1.405	1660	2 5.32		0.278	0.058	0.101
1120	28 11.20	0.770	0.783	1.370	1670	1 52.55		0.312	0.052	0.091
1130	27 29.71	0.790	0.764	1.337	1680	1 40.46		0.345	0.046	0.081
1140	26 48.56	0.809	0.745	1.304	1690	1 29.05		0.375	0.041	0.072
1150	+26 7.74	—0.829	—0.726	+1.270	1700	+1 18.32		+0.403	—0.036	+0.063
1160	25 27.27	0.848	0.707	1.237	1710	1 8.28		0.429	0.031	0.054
1170	24 47.16	0.866	0.689	1.206	1720	0 58.92		0.452	0.027	0.046
1180	24 7.42	0.883	0.670	1.172	1730	0 50.24		0.472	0.023	0.039
1190	23 28.06	0.899	0.652	1.141	1740	0 42.26		0.489	0.019	0.032
1200	+22 49.09	—0.914	—0.634	+1.110	1750	+0 34.96		+0.503	—0.015	+0.026
1210	22 10.51	0.927	0.616	1.078	1760	0 28.36		0.514	0.012	0.021
1220	21 32.34	0.939	0.598	1.046	1770	0 22.44		0.522	0.009	0.017
1230	20 54.57	0.950	0.581	1.017	1780	0 17.22		0.527	0.007	0.013
1240	20 17.23	0.960	0.564	0.986	1790	+0 12.68		0.529	0.005	0.010
						″				
1250	+19 40.31	—0.967	—0.547	+0.957	1800	+8.84		+0.528	—0.004	+0.007
1260	19 3.83	0.972	0.530	0.928	1801	8.49	—.35	0.528	0.004	0.007
1270	18 27.80	0.975	0.513	0.898	1802	8.15	.34	0.528	0.004	0.007
1280	17 52.21	0.977	0.497	0.870	1803	7.82	.33	0.527	0.004	0.007
1290	17 17.09	0.977	0.481	0.842	1804	7.50	.32	0.527	0.004	0.007
							.32			
1300	+16 42.43	—0.974	—0.465	+0.814	1805	+7.18		+0.527	—0.003	+0.006
1310	16 8.25	0.967	0.449	0.786	1806	6.87	—.31	0.526	0.003	0.006
1320	15 34.55	0.959	0.433	0.758	1807	6.57	.30	0.526	0.003	0.006
1330	15 1.34	0.948	0.417	0.730	1808	6.27	.30	0.525	0.003	0.006
1340	14 28.63	0.936	0.402	0.704	1809	5.98	.29	0.525	0.003	0.006
							.29			
1350	+13 56.44	—0.921	—0.387	+0.677	1810	+5.69		+0.524	—0.003	+0.005
1360	13 24.76	0.903	0.372	0.651	1811	5.41	—.28	0.523	0.003	0.005
1370	12 53.61	0.883	0.358	0.626	1812	5.14	.27	0.523	0.003	0.005
1380	12 22.99	0.861	0.344	0.602	1813	4.88	.26	0.522	0.003	0.005
1390	11 52.90	0.837	0.330	0.578	1814	4.62	.26	0.522	0.002	0.005
							.25			
1400	+11 23.35	—0.810	—0.317	+0.555	1815	+4.37		+0 521	—0.002	+0.004
1410	10 54.35	0.780	0.303	0.530	1816	4.13	—.24	0.520	0.002	0.004
1420	10 25.92	0.748	0.290	0.508	1817	3.89	.24	0.520	0.002	0.004
1430	9 58.04	0.714	0.277	0.485	1818	3.66	.23	0.519	0.002	0.004
1440	9 30.74	0.678	0.264	0.462	1819	3.44	.22	0.518	0.002	0.004
							.21			
1450	+ 9 4.01	—0.641	—0.252	+0.441	1820	+3.23		+0.517	—0.002	+0.003
1460	8 37.88	0.601	0.240	0.420	1821	3.02	—.21	0.516	0.002	0.003
1470	8 12.34	0.560	0.228	0.399	1822	2.82	.20	0.515	0.002	0.003
1480	7 47.39	0.518	0.216	0.378	1823	2.63	.19	0.514	0.002	0.003
1490	7 23.04	0.475	0.205	0.359	1824	2.44	.19	0.513	0.002	0.003
							.18			
1500	+ 6 59.29	—0.431	—0.194	+0.340	1825	+2.26		+0.512	—0.001	+0.002
1510	6 36.14	0.386	0.183	0.322	1826	2.09	—.17	0.511	0.001	0.002
1520	6 13.61	0.340	0.172	0.303	1827	1.92	.17	0.510	0.001	0.002
1530	5 51.69	0.293	0.162	0.284	1828	1.76	.16	0.509	0.001	0.002
1540	+ 5 30.40	—0.247	—0.153	+0.268	1829	+1.61	.15	—0.508	—0.001	+0.002
							—.15			

TABLE VI.—*Continued.*

Year.	g		1	2	3	Year.	g		1	2	3
	″						″				
1830	+1.46	—.14	+0.507	—0.001	+0.002	1885	+4.06	+.24	+0.404	—0.002	+0.003
1831	1.32	.13	0.506	0.001	0.002	1886	4.30	.24	0.402	0.002	0.004
1832	1.19	.12	0.505	0.001	0.002	1887	4.54	.25	0.399	0.002	0.004
1833	1.07	.12	0.503	0.001	0.002	1888	4.79	.26	0.397	0.002	0.004
1834	0.95	.11	0.502	0.001	0.002	1889	5.05	.27	0.394	0.002	0.004
1835	+0.84	—.10	+0.501	0	+0.001	1890	+5.32	+.27	+0.391	—0.003	+0.004
1836	0.74	.10	0.500	0	0.001	1891	5.59	.28	0.388	0.003	0.005
1837	0.64	.09	0.498	0	0.001	1892	5.87	.29	0.386	0.003	0.005
1838	0.55	.08	0.497	0	0.001	1893	6.16	.29	0.383	0.003	0.005
1839	0.47	.08	0.495	0	0	1894	6.45	.30	0.380	0.003	0.005
1840	+0.39	—.07	+0.494	0	0	1895	+6.75	+.31	+0.377	—0.003	+0.006
1841	0.32	.06	0.493	0	0	1896	7.06	.31	0.374	0.003	0.006
1842	0.26	.06	0.491	0	0	1897	7.37	.32	0.372	0.004	0.006
1843	0.20	.05	0.490	0	0	1898	7.69	.33	0.369	0.004	0.006
1844	0.15	.04	0.488	0	0	1899	8.02	.33	0.366	0.004	0.007
1845	+0.11	—.04	+0.487	0	0	1900	+8.35	+.33	+0.363	—0.004	+0.007
1846	0.07	.03	0.485	0	0	1901	8.68	.35	0.360	0.004	0.007
1847	0.04	.02	0.483	0	0	1902	9.03	.35	0.357	0.004	0.008
1848	0.02	.02	0.482	0	0	1903	9.38	.37	0.353	0.005	0.008
1849	0.00	.00	0.481	0	0	1904	9.75	.37	0.350	0.005	0.008
1850	0.00	.00	+0.479	0	0	1905	+10.12	+.38	+0.347	—0.005	+0.009
1851	0.00	.00	0.477	0	0	1906	10.50	.38	0.344	0.005	0.009
1852	0.00	+.01	0.476	0	0	1907	10.88	.39	0.340	0.005	0.010
1853	+0.01	.02	0.474	0	0	1908	11.27	.40	0.337	0.096	0.010
1854	0.03	.03	0.472	0	0	1909	11.67		0.334	0.006	9.010
1855	+0.06	+.04	+0.471	0	0	1910	+0′ 12.07″		—0.331	—0.006	+0.010
1856	0.10	.04	0.469	0	0	1920	0 16.47		0.298	0.008	0.013
1857	0.14	.05	0.467	0	0	1930	0 21.55		0.264	0.010	0.017
1858	0.19	.05	0.465	0	0	1940	0 27.30		0.229	0.012	0.021
1859	0.24	.06	0.463	0	0	1950	0 33.71		0.193	0.015	0.026
1860	+0.30	+.07	+0.461	0	0	1960	+0 40.79		+0.155	—0.018	+0.032
1861	0.37	.08	0.459	0	0	1970	0 48.53		0.115	0.022	0.039
1862	0.45	.08	0.457	0	0	1980	0 56.93		0.074	0.026	0.046
1863	0.53	.09	0.455	0	0	1990	1 5.98		0.032	0.030	0.053
1864	0.62	.09	0.453	0	0	2000	1 15.68		+0.010	0.035	0.061
1865	+0.71	+.10	+0.451	0	+0.001	2010	+1 26.03		—0.053	—0.040	+0.069
1866	0.81	.11	0.449	0	0.001	2020	1 37.03		0.096	0.045	0.078
1867	0.92	.12	0.446	0	0.001	2030	1 48.67		0.140	0.050	0.088
1868	1.04	.12	0.444	—0.001	0.001	2040	2 0.94		0.183	0.056	0.098
1869	1.16	.13	0.442	0.001	0.001	2050	2 13.85		0.226	0.062	0.108
1770	+1.29	+.14	+0.440	—0.001	+0.001	2060	+2 27.38		—0.268	—0.068	+0.119
1871	1.43	.14	0.438	0.001	0.001	2070	2 41.53		0.309	0.075	0.130
1872	1.57	.15	0.436	0.001	0.001	2080	2 56.29		0.349	0.082	0.142
1873	1.72	.16	0.433	0.001	0.001	2090	3 11.65		0.388	0.089	0.155
1874	1.88	.16	0.431	0.001	0.002	2100	3 27.61		0.427	0.096	0.168
1875	+2.04	+.17	+0.429	—0.001	+0.002	2110	+3 44.17		—0.464	—0.104	+0.182
1876	2.21	.18	0.426	0.001	0.002	2120	4 1.32		0.499	0.112	0.196
1877	2.39	.18	0.424	0.001	0.002	2130	4 19.07		0.532	0.120	0.210
1878	2.57	.19	0.422	0.001	0.002	2140	4 37.40		0.563	0.128	0.225
1879	+2.76	.20	+0.419	—0.001	+0.002	2150	4 56.31		0.593	0.137	0.240
1880	+2.96	+.20	+0.417	—0.002	+0.003	2160	+5 15.74		—0.620	—0.146	+0.256
1881	3.16	.21	0.414	0.002	0.003	2170	5 35.72		0.645	0.156	0.273
1882	3.37	.22	0.412	0.002	0.003	2180	5 56.24		0.668	0.165	0.289
1883	3.59	.23	0.409	0.002	0.003	2190	6 17.29		0.689	0.175	0.306
1884	+3.82	+.24	+0.407	—0.002	+0.003	2200	+6 38.88		—0.709	—0.184	+0.322

TABLE VII.—Equation of Centre and Principal Term of Log. r.

g	E		Log. r			g	E		Log. r		
	° ′ ″	″	1.26				° ′ ″	″	1.26		
0°	0 0 00.00	59.81	18915	1	360°	10°	0 59 26.71	58.70	22488	120	350°
10′	0 0 59.81	59.81	18916	3	50′	10′	1 0 25.41	58.67	22608	121	50′
20	0 1 59.62	59.80	18919	5	40	20	1 1 24.08	58.63	22729	123	40
30	0 2 59.42	59.81	18924	7	30	30	1 2 22.71	58.59	22852	125	30
40	0 3 59.23	59.80	18931	9	20	40	1 3 21.30	58.55	22977	127	20
50	0 4 59.03	59.79	18940	11	10	50	1 4 19.85	58.52	23104	130	10
1°	0 5 58.82	59.79	18951	13	359°	11°	1 5 18.37	58.48	23234	131	349°
10′	0 6 58.61	59.78	18964	15	50′	10′	1 6 16.85	58.43	23365	133	50′
20	0 7 58.39	59.79	18979	17	40	20	1 7 15.28	58.40	23498	135	40
30	0 8 58.18	59.78	18996	19	30	30	1 8 13.68	58.36	23633	137	30
40	0 9 57.96	59.78	19015	21	20	40	1 9 12.04	58.32	23770	139	20
50	0 10 57.74	59.77	19036	23	10	50	1 10 10.36	58.27	23909	142	10
2°	0 11 57.51	59.76	19059	25	358°	12°	1 11 8.63	58.23	24051	143	348°
10′	0 12 57.27	59.75	19084	27	50′	10′	1 12 6.86	58.18	24194	145	50′
20	0 13 57.02	59.74	19111	29	40	20	1 13 5.04	58.14	24339	146	40
30	0 14 56.76	59.73	19140	31	30	30	1 14 3.18	58.10	24485	148	30
40	0 15 56.49	59.73	19171	33	20	40	1 15 1.28	58.05	24633	151	20
50	0 16 56.22	59.71	19204	34	10	50	1 15 59.33	58.00	24784	153	10
3°	0 17 55.93	59.70	19238	37	357°	13°	1 16 57.33	57.96	24937	154	347°
10′	0 18 55.63	59.70	19275	39	50′	10′	1 17 55.29	57.91	25091	156	50′
20	0 19 55.33	59.68	19314	41	40	20	1 18 53.20	57.86	25247	158	40
30	0 20 55.01	59.67	19355	42	30	30	1 19 51.06	57.81	25405	161	30
40	0 21 54.68	59.66	19397	45	20	40	1 20 48.87	57.76	25566	162	20
50	0 22 54.34	59.64	19442	47	10	50	1 21 46.63	57.72	25728	164	10
4°	0 23 53.98	59.63	19489	49	356°	14°	1 22 44.35	57.66	25892	165	346°
10′	0 24 53.61	59.61	19538	51	50′	10′	1 23 42.01	57.62	26057	168	50′
20	0 25 53.22	59.59	19589	53	40	20	1 24 39.63	57.56	26225	170	40
30	0 26 52.81	59.58	19642	55	30	30	1 25 37.19	57.51	26395	171	30
40	0 27 52.39	59.56	19697	57	20	40	1 26 34.70	57.46	26566	174	20
50	0 28 51.95	59.54	19754	58	10	50	1 27 32.16	57.41	26740	176	10
5°	0 29 51.49	59.52	19812	60	355°	15°	1 28 29.57	57.35	26916	177	345°
10′	0 30 51.01	59.51	19872	62	50′	10′	1 29 26.92	57.30	27093	178	50′
20	0 31 50.52	59.49	19934	65	40	20	1 30 24.22	57.25	27271	180	40
30	0 32 50.01	59.47	19999	66	30	30	1 31 21.47	57.20	27451	183	30
40	0 33 49.48	59.45	20065	69	20	40	1 32 18.67	57.13	27634	185	20
50	0 34 48.93	59.43	20134	71	10	50	1 33 15.80	57.08	27819	188	10
6°	0 35 48.36	59.41	20205	73	354°	16°	1 34 12.88	57.02	28007	189	344°
10′	0 36 47.77	59.38	20278	74	50′	10′	1 35 9.90	56.97	28196	190	50′
20	0 37 47.15	59.36	20352	76	40	20	1 36 6.87	56.90	28386	192	40
30	0 38 46.51	59.33	20428	79	30	30	1 37 3.77	56.85	28578	194	30
40	0 39 45.84	59.31	20507	81	20	40	1 38 0.62	56.79	28772	196	20
50	0 40 45.15	59.29	20588	82	10	50	1 38 57.41	56.73	28968	199	10
7°	0 41 44.44	59.26	20670	85	353°	17°	1 39 54.14	56.67	29167	200	343°
10′	0 42 43.70	59.24	20755	86	50′	10′	1 40 50.81	56.62	29367	201	50′
20	0 43 42.94	59.21	20841	88	40	20	1 41 47.43	56.55	29568	203	40
30	0 44 42.15	59.18	20929	91	30	30	1 42 43.98	56.48	29771	205	30
40	0 45 41.33	59.15	21020	92	20	40	1 43 40.46	56.43	29976	207	20
50	0 46 40.48	59.13	21112	94	10	50	1 44 36.89	56.36	30183	209	10
8°	0 47 39.61	59.10	21206	96	352°	18°	1 45 33.25	56.30	30392	211	342°
10′	0 48 38.71	59.07	21302	98	50′	10′	1 46 29.55	56.23	30603	213	50′
20	0 49 37.78	59.04	21400	99	40	20	1 47 25.78	56.17	30816	214	40
30	0 50 36.82	59.01	21499	102	30	30	1 48 21.95	56.10	31030	216	30
40	0 51 35.83	58.98	21601	104	20	40	1 49 18.05	56.04	31246	219	20
50	0 52 34.81	58.95	21705	106	10	50	1 50 14.09	55.97	31465	221	10
9°	0 53 33.76	58.91	21811	108	351°	19°	1 51 10.06	55.90	31686	222	341°
10′	0 54 32.67	58.88	21919	110	50′	10′	1 52 5.96	55.84	31908	224	50′
20	0 55 31.55	58.84	22029	111	40	20	1 53 1.80	55.77	32132	226	40
30	0 56 30.39	58.81	22140	114	30	30	1 53 57.57	55.70	32358	227	30
40	0 57 29.20	58.77	22254	116	20	40	1 54 53.27	55.64	32585	229	20
50	0 58 27.97	58.74	22370	118	10	50	1 55 48.91	55.56	32814	231	10
10°	0 59 26.71		22488		350°	20°	1 56 44.47		33045		340°
			1.26		g				1.26		g

TABLE VII.—*Continued.*

g	E		Log. r		g	g	E		Log. r		g
	° ′ ″	″	1.26				° ′ ″	″	1.26		
20°	1 56 44.47		33045		340°	30°	2 49 51.28		50121		330°
10′	1 57 39.96	55.49	33278	233	50′	10′	2 50 41.63	50.35	50457	336	50′
20	1 58 35.39	55.43	33513	235	40	20	2 51 31.87	50.24	50795	338	40
30	1 59 30.74	55.35	33749	236	30	30	2 52 22.01	50.14	51134	339	30
40	2 0 26.02	55.28	33987	238	20	40	2 53 12.06	50.05	51475	341	20
50	2 1 21.23	55.21	34227	240	10	50	2 54 2.00	49.94	51818	343	10
21°	2 2 16.36	55.13	34468	241	339°	31°	2 54 51.83	49.83	52162	344	329°
10′	2 3 11.42	55.06	34711	243	50′	10′	2 55 41.56	49.73	52507	345	50′
20	2 4 6.41	54.99	34956	245	40	20	2 56 31.18	49.62	52854	347	40
30	2 5 1.33	54.92	35204	248	30	30	2 57 20.70	49.52	53203	349	30
40	2 5 56.17	54.84	35454	250	20	40	2 58 10.12	49.42	53553	350	20
50	2 6 50.93	54.76	35705	251	10	50	2 58 59.43	49.31	53905	352	10
22°	2 7 45.61	54.68	35957	252	338°	32°	2 59 48.63	49.20	54258	353	328°
10′	2 8 40.22	54.61	36211	254	50′	10′	3 0 37.73	49.10	54613	355	50′
20	2 9 34.76	54.54	36467	256	40	20	3 1 26.73	49.00	54970	357	40
30	2 10 29.22	54.46	36726	259	30	30	3 2 15.62	48.89	55329	359	30
40	2 11 23.60	54.38	36986	260	20	40	3 3 4.41	48.79	55689	360	20
50	2 12 17.90	54.30	37248	262	10	50	3 3 53.09	48.68	56050	361	10
23°	2 13 12.12	54.22	37511	263	337°	33°	3 4 41.66	48.57	56413	363	327°
10′	2 14 6.26	54.14	37776	265	50′	10′	3 5 30.12	48.46	56777	364	50′
20	2 15 0.32	54.06	38043	267	40	20	3 6 18.47	48.35	57143	366	40
30	2 15 54.30	53.98	38311	268	30	30	3 7 6.72	48.25	57510	367	30
40	2 16 48.20	53.90	38581	270	20	40	3 7 54.85	48.13	57879	369	20
50	2 17 42.02	53.82	38853	272	10	50	3 8 42.87	48.02	58249	370	10
24°	2 18 35.75	53.73	39128	275	336°	34°	3 9 30.78	47.91	58621	372	326°
10′	2 19 29.40	53.65	39404	276	50′	10	3 10 18.58	47.80	58994	373	50′
20	2 20 22.97	53.57	39681	277	40	20	3 11 6.26	47.68	59369	375	40
30	2 21 16.46	53.49	39960	279	30	30	3 11 53.83	47.57	59746	377	30
40	2 22 9.87	53.41	40241	281	20	40	3 12 41.29	47.46	60124	378	20
50	2 23 3.19	53.32	40524	283	10	50	3 13 28.64	47.35	60503	379	10
25°	2 23 56.42	53.23	40808	284	335°	35°	3 14 15.87	47.23	60884	381	325°
10′	2 24 49.57	53.15	41094	286	50′	10′	3 15 2.99	47.12	61266	382	50′
20	2 25 42.63	53.06	41382	288	40	20	3 15 50.00	47.01	61650	384	40
30	2 26 35.61	52.98	41671	289	30	30	3 16 36.89	46.89	62035	385	30
40	2 27 28.50	52.89	41962	291	20	40	3 17 23.67	46.78	62422	387	20
50	2 28 21.30	52.80	42255	293	10	50	3 18 10.33	46.66	62810	388	10
26°	2 29 14.01	52.71	42550	295	334°	36°	3 18 56.88	46.55	63200	390	324°
10′	2 30 6.63	52.62	42846	296	50′	10′	3 19 43.31	46.43	63592	392	50′
20	2 30 59.16	52.53	43144	298	40	20	3 20 29.63	46.32	63985	393	40
30	2 31 51.60	52.44	43444	300	30	30	3 21 15.83	46.20	64380	395	30
40	2 32 43.95	52.35	43745	301	20	40	3 22 1.91	46.08	64776	396	20
50	2 33 36.21	52.26	44048	303	10	50	3 22 47.88	45.97	65173	397	10
27°	2 34 28.38	52.17	44353	305	333°	37°	3 23 33.72	45.84	65572	399	323°
10′	2 35 20.46	52.08	44659	306	50′	10	3 24 19.44	45.72	65972	400	50′
20	2 36 12.44	51.98	44967	308	40	20	3 25 5.04	45.60	66373	401	40
30	2 37 4.34	51.90	45277	310	30	30	3 25 50.53	45.49	66776	403	30
40	2 37 56.14	51.80	45588	311	20	40	3 26 35.89	45.36	67180	404	20
50	2 38 47.85	51.71	45901	313	10	50	3 27 21.13	45.24	67585	405	10
28°	2 39 39.46	51.61	46216	315	332°	38°	3 28 6.25	45.12	67992	407	322°
10′	2 40 30.98	51.52	46532	316	50′	10′	3 28 51.25	45.00	68400	408	50′
20	2 41 22.40	51.42	46850	318	40	20	3 29 36.13	44.88	68810	410	40
30	2 42 13.72	51.32	47170	320	30	30	3 30 20.90	44.77	69221	411	30
40	2 43 4.95	51.23	47491	321	20	40	3 31 5.54	44.64	69633	412	20
50	2 43 56.08	51.13	47814	323	10	50	3 31 50.05	44.52	70047	414	10
29°	2 44 47.12	51.04	48138	324	331°	39°	3 32 34.45	44.40	70463	416	321°
10′	2 45 38.06	50.94	48464	326	50′	10′	3 33 18.72	44.27	70880	417	50′
20	2 46 28.90	50.84	48792	328	40	20	3 34 2.86	44.14	71298	418	40
30	2 47 19.64	50.74	49122	330	30	30	3 34 46.88	44.02	71717	419	30
40	2 48 10.29	50.65	49453	331	20	40	3 35 30.78	43.90	72138	420	20
50	2 49 0.83	50.54	49786	333	10	50	3 36 14.55	43.77	72560	422	10
30°	2 49 51.28	50.45	50121	335	330°	40°	3 36 58.19	43.64	72984	424	320°
			1.26		g				1.26		g

TABLE VII.—*Continued.*

g	*E*		Log. *r*			*g*	*E*		Log. *r*		
	° ′ ″	″	1.26				° ′ ″	″	1.27		
40°	3 36 58.19	43.52	72984	425	320°	50°	4 16 34.27	35.35	00687	496	310°
10′	3 37 41.71	43.39	73409	426	50′	10′	4 17 9.62	35.20	01183	498	50′
20	3 38 25.10	43.27	73835	427	40	20	4 17 44.82	35.06	01681	499	40
30	3 39 8.37	43.14	74262	429	30	30	4 18 19.88	34.92	02180	500	30
40	3 39 51.51	43.01	74691	430	20	40	4 18 54.80	34.77	02680	501	20
50	3 40 34.52	42.88	75121	432	10	50	4 19 29.57	34.62	03181	502	10
41°	3 41 17.40	42.76	75553	433	319°	51°	4 20 4.19	34.48	03683	503	309°
10′	3 42 0.16	42.63	75986	434	50′	10′	4 20 38.67	34.33	04186	504	50′
20	3 42 42.79	42.50	76420	436	40	20	4 21 13.00	34.18	04690	505	40
30	3 43 25.29	42.37	76856	437	30	30	4 21 47.18	34.04	05195	505	30
40	3 44 7.66	42.25	77293	438	20	40	4 22 21.22	33.89	05701	506	20
50	3 44 49.91	42.11	77731	440	10	50	4 22 55.11	33.74	06207	507	10
42°	3 45 32.02	41.99	78171	440	318°	52°	4 23 28.85	33.59	06714	508	308°
10′	3 46 14.01	41.85	78611	442	50′	10′	4 24 2.44	33.44	07222	510	50′
20	3 46 55.86	41.72	79053	443	40	20	4 24 35.88	33.30	07732	511	40
30	3 47 37.58	41.59	79496	445	30	30	4 25 9.18	33.14	08243	511	30
40	3 48 19.17	41.46	79941	446	20	40	4 25 42.32	33.00	08754	512	20
50	3 49 0.63	41.33	80387	448	10	50	4 26 15.32	32.84	09266	514	10
43°	3 49 41.96	41.20	80835	449	317°	53°	4 26 48.16	32.69	09780	514	307°
10′	3 50 23.16	41.06	81284	449	50′	10′	4 27 20.85	32.54	10294	515	50′
20	3 51 4.22	40.93	81733	450	40	20	4 27 53.39	32.40	10809	516	40
30	3 51 45.15	40.79	82183	452	30	30	4 28 25.79	32.24	11325	517	30
40	3 52 25.94	40.66	82635	454	20	40	4 28 58.03	32.09	11842	519	20
50	3 53 6.60	40.53	83089	455	10	50	4 29 30.12	31.94	12361	520	10
44°	3 53 47.13	40.39	83544	456	316°	54°	4 30 2.06	31.79	12881	520	306°
10′	3 54 27.52	40.26	84000	457	50′	10′	4 30 33.85	31.64	13401	521	50′
20	3 55 7.78	40.12	84457	457	40	20	4 31 5.49	31.49	13922	522	40
30	3 55 47.90	39.99	84914	459	30	30	4 31 36.98	31.33	14444	522	30
40	3 56 27.89	39.86	85373	461	20	40	4 32 8.31	31.19	14966	524	20
50	3 57 7.75	39.72	85834	463	10	50	4 32 39.50	31.03	15490	525	10
45°	3 57 47.47	39.58	86297	463	315°	55°	4 33 10.53	30.88	16015	525	305°
10′	3 58 27.05	39.45	86760	464	50′	10′	4 33 41.41	30.73	16540	527	50′
20	3 59 6.50	39.31	87224	466	40	20	4 34 12.14	30.57	17067	528	40
30	3 59 45.81	39.17	87690	466	30	30	4 34 42.71	30.42	17595	528	30
40	4 0 24.98	39.03	88156	468	20	40	4 35 13.13	30.27	18123	529	20
50	4 1 4.01	38.89	88624	469	10	50	4 35 43.40	30.12	18652	530	10
46°	4 1 42.90	38.75	89093	471	314°	56°	4 36 13.52	29.96	19182	530	304°
10′	4 2 21.65	38.62	89564	471	50′	10′	4 36 43.48	29.81	19712	531	50′
20	4 3 0.27	38.48	90035	472	40	20	4 37 13.29	29.65	20243	533	40
30	4 3 38.75	38.34	90507	473	30	30	4 37 42.94	29.49	20776	533	30
40	4 4 17.09	38.21	90980	475	20	40	4 38 12.43	29.34	21309	533	20
50	4 4 55.30	38.06	91455	476	10	50	4 38 41.77	29.19	21842	534	10
47°	4 5 33.36	37.93	91931	477	313°	57°	4 39 10.96	29.03	22376	535	303°
10′	4 6 11.29	37.78	92408	478	50′	10′	4 39 39.99	28.88	22911	537	50′
20	4 6 49.07	37.65	92886	479	40	20	4 40 8.87	28.72	23448	538	40
30	4 7 26.72	37.50	93365	480	30	30	4 40 37.59	28.57	23986	538	30
40	4 8 4.22	37.36	93845	482	20	40	4 41 6.16	28.41	24524	539	20
50	4 8 41.58	37.22	94327	483	10	50	4 41 34.57	28.26	25063	540	10
48°	4 9 18.80	37.08	94810	484	312°	58°	4 42 2.83	28.10	25603	540	302°
10′	4 9 55.88	36.93	95294	485	50′	10′	4 42 30.93	27.94	26143	541	50′
20	4 10 32.81	36.79	95779	486	40	20	4 42 58.87	27.79	26684	542	40
30	4 11 9.60	36.65	96265	487	30	30	4 43 26.66	27.63	27226	542	30
40	4 11 46.25	36.51	96752	488	20	40	4 43 54.29	27.47	27768	543	20
50	4 12 22.76	36.36	97240	490	10	50	4 44 21.76	27.31	28311	544	10
49°	4 12 59.12	36.22	97730	490	311°	59°	4 44 49.07	27.16	28855	544	301°
10′	4 13 35.34	36.07	98220	491	50′	10′	4 45 16.23	26.99	29399	545	50′
20	4 14 11.41	35.93	98711	492	40	20	4 45 43.22	26.84	29944	547	40
30	4 14 47.34	35.79	99203	493	30	30	4 46 10.06	26.69	30491	547	30
40	4 15 23.13	35.64	99696	495	20	40	4 46 36.75	26.52	31038	549	20
50	4 15 58.77	35.50	*00191	496	10	50	4 47 3.27	26.37	31587	550	10
50°	4 16 34.27		*00687		310°	60°	4 47 29.64		32137		300°
			*1.27		*g*				1.27		*g*

TABLE VII.—*Continued.*

g	E ° ′ ″	″	Log. r		g
			1.27		
60°	4 47 29.64		32137		300°
10′	4 47 55.85	26.21	32687	550	50′
20	4 48 21.90	26.05	33237	550	40
30	4 48 47.79	25.89	33787	550	30
40	4 49 13.53	25.74	34338	551	20
50	4 49 39.10	25.57	34890	552	10
61°	4 50 4.52	25.42	35443	553	299°
10′	4 50 29.78	25.26	35996	553	50′
20	4 50 54.88	25.10	36550	554	40
30	4 51 19.82	24.94	37105	555	30
40	4 51 44.60	24.78	37660	555	20
50	4 52 9.22	24.62	38216	556	10
62°	4 52 33.68	24.46	38773	557	298°
10′	4 52 57.98	24.30	39330	557	50′
20	4 53 22.11	24.13	39888	558	40
30	4 53 46.09	23.98	40446	558	30
40	4 54 9.90	23.81	41005	559	20
50	4 54 33 56	23.66	41565	560	10
63°	4 54 57.05	23.49	42126	561	297°
10′	4 55 20.38	23.33	42687	561	50′
20	4 55 43.56	23.18	43249	562	40
30	4 56 6.57	23.01	43811	562	30
40	4 56 29.42	22.85	44374	563	20
50	4 56 52.11	22.69	44938	564	10
64°	4 57 14.64	22.53	45503	565	296°
10′	4 57 37.01	22.37	46068	565	50′
20	4 57 59.21	22.20	46633	565	40
30	4 58 21.25	22.04	47199	566	30
40	4 58 43.13	21.88	47766	567	20
50	4 59 4.84	21.71	48333	567	10
65°	4 59 26.40	21.56	48901	568	295°
10′	4 59 47.79	21.39	49469	568	50′
20	5 0 9.02	21.23	50038	569	40
30	5 0 30.09	21.07	50607	569	30
40	5 0 50.99	20.90	51177	570	20
50	5 1 11.73	20.74	51747	570	10
66°	5 1 32.31	20.58	52318	571	294°
10′	5 1 52.72	20.41	52889	571	50′
20	5 2 12.97	20.25	53461	572	40
30	5 2 33.06	20.09	54033	572	30
40	5 2 52.98	19.92	54606	573	20
50	5 3 12.74	19.76	55179	573	10
67°	5 3 32.34	19.60	55753	574	293°
10′	5 3 51.77	19.43	56327	574	50′
20	5 4 11.04	19.27	56901	574	40
30	5 4 30.15	19.11	57476	575	30
40	5 4 49.09	18.94	58051	575	20
50	5 5 7.87	18.78	58627	576	10
68°	5 5 26.49	18.62	59204	577	292°
10	5 5 44.94	18.45	59781	577	50′
20	5 6 3.23	18.29	60358	577	40
30	5 6 21.35	18.12	60936	578	30
40	5 6 39.30	17.95	61514	578	20
50	5 6 57.09	17.79	62093	579	10
69°	5 7 14.72	17.63	62673	579	291°
10′	5 7 32.18	17.46	63252	579	50′
20	5 7 49.48	17.30	63832	580	40
30	5 8 6.61	17.13	64412	580	30
40	5 8 23.57	16.96	64993	581	20
50	5 8 40.37	16.80	65574	581	10
70°	5 8 57.01	16.64	66156	582	290°
			1.27		g

g	E ° ′ ″	″	Log. r		g
			1.27		
70°	5 8 57.01		66156		290°
10′	5 9 13.48	16.47	66738	582	50′
20	5 9 29.79	16.31	67320	582	40
30	5 9 45.93	16.14	67903	583	30
40	5 10 1.91	15.98	68487	584	20
50	5 10 17.72	15.81	69071	584	10
71°	5 10 33.36	15.64	69655	584	289°
10′	5 10 48.84	15.48	70239	584	50′
20	5 11 4.15	15.31	70824	585	40
30	5 11 19.30	15.15	71409	585	30
40	5 11 34.28	14.98	71994	585	20
50	5 11 49.09	14.81	72579	585	10
72°	5 12 3.74	14.65	73165	586	288°
10′	5 12 18.23	14.49	73751	586	50′
20	5 12 32.55	14.32	74337	586	40
30	5 12 46.70	14.15	74924	587	30
40	5 13 0.69	13.99	75511	587	20
50	5 13 14.51	13.82	76098	587	10
73°	5 13 28.17	13.66	76685	587	287°
10′	5 13 41.66	13.49	77273	588	50′
20	5 13 54.98	13.32	77861	588	40
30	5 14 8.13	13.15	78450	589	30
40	5 14 21.12	12.99	79039	589	20
50	5 14 33.94	12.82	79628	589	10
74°	5 14 46.60	12.66	80217	589	286°
10′	5 14 59.09	12.49	80807	590	50′
20	5 15 11.41	12.32	81397	590	40
30	5 15 23.57	12.16	81987	590	30
40	5 15 35.56	11.99	82577	590	20
50	5 15 47.39	11.83	83168	591	10
75°	5 15 59.05	11.66	83759	591	285°
10′	5 16 10.54	11.49	84350	591	50′
20	5 16 21.87	11.33	84941	591	40
30	5 16 33.03	11.16	85532	591	30
40	5 16 44.03	11.00	86124	592	20
50	5 16 54.86	10.83	86715	591	10
76°	5 17 5.52	10.66	87307	592	284°
10′	5 17 16.02	10.50	87899	592	50′
20	5 17 26.35	10.33	88491	592	40
30	5 17 36.51	10.16	89084	593	30
40	5 17 46.51	10.00	89677	593	20
50	5 17 56.34	9.83	90270	593	10
77°	5 18 6.00	9.66	90863	593	283°
10′	5 18 15.50	9.50	91456	593	50′
20	5 18 24.82	9.32	92049	593	40
30	5 18 33.99	9.17	92643	594	30
40	5 18 42.98	8.99	93237	594	20
50	5 18 51.81	8.83	93830	593	10
78°	5 19 0.47	8.66	94424	594	282°
10′	5 19 8.97	8.50	95018	594	50′
20	5 19 17.30	8.33	95612	594	40
30	5 19 25.46	8.16	96206	594	30
40	5 19 33.46	8.00	96800	594	20
50	5 19 41.29	7.83	97395	595	10
79°	5 19 48.95	7.66	97990	595	281°
10′	5 19 56.45	7.50	98585	595	50′
20	5 20 3.78	7.33	99179	594	40
30	5 20 10.94	7.16	99774	595	30
40	5 20 17.94	7.00	*00369	595	20
50	5 20 24.77	6.83	*00965	596	10
80°	5 20 31.44	6.67	*01560	595	280°
			*1.28		g

TABLE VII.—*Continued.*

g	E		Log. r		
	° ′ ″	″	1.28		
80°	5 20 31.44		01560		280°
10′	5 20 37.94	6.50	02155	595	50′
20	5 20 44.28	6.34	02750	595	40
30	5 20 50.45	6.17	03346	596	30
40	5 20 56.46	6.01	03941	595	20
50	5 21 2.30	5.84	04536	595	10
81°	5 21 7.96	5.66	05131	595	279°
10′	5 21 13.46	5.50	05726	595	50′
20	5 21 18.80	5.34	06322	596	40
30	5 21 23.97	5.17	06917	595	30
40	5 21 28.97	5.00	07512	595	20
50	5 21 33.81	4.84	08107	595	10
82°	5 21 38.48	4.67	08702	595	278°
10′	5 21 42.99	4.51	09297	595	50′
20	5 21 47.33	4.34	09893	596	40
30	5 21 51.50	4.17	10488	595	30
40	5 21 55.51	4.01	11083	595	20
50	5 21 59.36	3.85	11679	596	10
83°	5 22 3.04	3.68	12274	595	277°
10′	5 22 6.56	3.52	12869	595	50′
20	5 22 9.91	3.35	13465	596	40
30	5 22 13.09	3.18	14060	595	30
40	5 22 16.11	3.02	14655	595	20
50	5 22 18.97	2.86	15250	595	10
84°	5 22 21.66	2.69	15846	596	276°
10′	5 22 24.19	2.53	16441	595	50′
20	5 22 26.55	2.36	17036	595	40
30	5 22 28.75	2.20	17631	595	30
40	5 22 30.78	2.03	18226	595	20
50	5 22 32.64	1.86	18820	594	10
85°	5 22 34.34	1.70	19415	595	275°
10′	5 22 35.87	1.53	20010	595	50′
20	5 22 37.24	1.37	20604	594	40
30	5 22 38.44	1.20	21198	594	30
40	5 22 39.48	1.04	21792	594	20
50	5 22 40.35	0.87	22386	594	10
86°	5 22 41.06	0.71	22980	594	274°
10′	5 22 41.60	0.54	23574	594	50′
20	5 22 41.99	0.39	24167	593	40
30	5 22 42.21	0.22	24761	594	30
40	5 22 42.26	0.05	25354	593	20
50	5 22 42.15	0.11	25948	594	10
87°	5 22 41.88	0.27	26541	593	273°
10′	5 22 41.45	0.43	27134	593	50′
20	5 22 40.85	0.60	27726	592	40
30	5 22 40.09	0.76	28319	593	30
40	5 22 39.16	0.93	28911	592	20
50	5 22 38.07	1.09	29504	593	10
88°	5 22 36.82	1.25	30096	592	272°
10′	5 22 35.40	1.42	30688	592	50′
20	5 22 33.82	1.58	31280	592	40
30	5 22 32.08	1.74	31872	592	30
40	5 22 30.17	1.91	32464	592	20
50	5 22 28.10	2.07	33055	591	10
89°	5 22 25.86	2.24	33647	592	271°
10′	5 22 23.47	2.39	34238	591	50′
20	5 22 20.91	2.56	34828	590	40
30	5 22 18.19	2.72	35418	590	30
40	5 22 15.31	2.88	36008	590	20
50	5 22 12.26	3.05	36597	589	10
90°	5 22 9.05	3.21	37187	590	270°
			1.28		g

g	E		Log. r		
	° ′ ″	″	1.28		
90°	5 22 9.05		37187		270°
10′	5 22 5.68	3.37	37776	589	50
20	5 22 2.15	3.53	38365	589	40
30	5 21 58.45	3.70	38954	589	30
40	5 21 54.59	3.86	39543	589	20
50	5 21 50.58	4.01	40131	588	10
91°	5 21 46.40	4.18	40719	588	269°
10′	5 21 42.06	4.34	41307	588	50
20	5 21 37.57	4.49	41894	587	40
30	5 21 32.91	4.66	42482	588	30
40	5 21 28.09	4.82	43069	587	20
50	5 21 23.11	4.98	43655	586	10
92°	5 21 17.97	5.14	44242	587	268°
10′	5 21 12.67	5.30	44828	586	50′
20	5 21 7.20	5.47	45414	586	40
30	5 21 1.58	5.62	46000	586	30
40	5 20 55.79	5.79	46585	585	20
50	5 20 49.85	5.94	47170	585	10
93°	5 20 43.74	6.11	47755	585	267°
10′	5 20 37.48	6.26	48339	584	50′
20	5 20 31.05	6.43	48923	584	40
30	5 20 24.47	6.58	49507	584	30
40	5 20 17.72	6.75	50090	583	20
50	5 20 10.82	6.90	50673	583	10
94°	5 20 3.76	7.06	51256	583	266°
10′	5 19 56.54	7.22	51838	582	50′
20	5 19 49.16	7.38	52420	582	40
30	5 19 41.62	7.54	53002	582	30
40	5 19 33.93	7.69	53583	581	20
50	5 19 26.07	7.86	54163	580	10
95°	5 19 18.06	8.01	54743	580	265°
10′	5 19 9.89	8.17	55323	580	50′
20	5 19 1.56	8.33	55902	579	40
30	5 18 53.06	8.50	56481	579	30
40	5 18 44.42	8.64	57060	579	20
50	5 18 35.61	8.81	57638	578	10
96°	5 18 26.65	8.96	58216	578	264°
10′	5 18 17.53	9.12	58794	578	50′
20	5 18 8.26	9.27	59371	577	40
30	5 17 58.83	9.43	59948	577	30
40	5 17 49.24	9.59	60525	577	20
50	5 17 39.50	9.74	61101	576	10
97°	5 17 29.60	9.90	61677	576	263°
10′	5 17 19.54	10.06	62252	575	50′
20	5 17 9.33	10.21	62827	575	40
30	5 16 58.96	10.37	63401	574	30
40	5 16 48.44	10.52	63975	574	20
50	5 16 37.76	10.68	64548	573	10
98°	5 16 26.92	10.84	65120	572	262°
10′	5 16 15.93	10.99	65692	572	50′
20	5 16 4.78	11.15	66264	572	40
30	5 15 53.48	11.30	66836	572	30
40	5 15 42.02	11.46	67407	571	20
50	5 15 30.41	11.61	67977	570	10
99°	5 15 18.64	11.77	68547	570	261°
10′	5 15 6.72	11.92	69117	570	50′
20	5 14 54.64	12.08	69686	569	40
30	5 14 42.41	12.23	70255	569	30
40	5 14 30.03	12.38	70823	568	20
50	5 14 17.49	12.54	71391	568	10
100°	5 14 4.80	12.69	71958	567	260°
			1.28		g

TABLE VII.—*Continued.*

g	E		Log. r			g	E		Log. r		
	° ′ ″	″	1.23				° ′ ″	″	1.29		
100°	5 14 4.80		71958		260°	110°	4 56 49.88		04888		250°
10′	5 13 51.96	12.84	72525	567	50′	10′	4 56 28.20	21.68	05416	528	50′
20	5 13 38.96	13.00	73091	566	40	20	4 56 6.39	21.81	05944	528	40
30	5 13 25.81	13.15	73656	565	30	30	4 55 44.44	21.95	06471	527	30
40	5 13 12.51	13.30	74221	565	20	40	4 55 22.34	22.10	06997	526	20
50	5 12 59.06	13.45	74785	564	10	50	4 55 0.10	22.24	07522	525	10
101°	5 12 45.45	13.61	75348	563	259°	111°	4 54 37.72	22.38	08047	525	249°
10′	5 12 31.69	13.76	75911	563	50′	10′	4 54 15.21	22.51	08571	524	50′
20	5 12 17.78	13.91	76474	563	40	20	4 53 52.55	22.66	09094	523	40
30	5 12 3.72	14.06	77036	562	30	30	4 53 29.76	22.79	09616	522	30
40	5 11 49.50	14.22	77598	562	20	40	4 53 6.83	22.93	10137	521	20
50	5 11 35.14	14.36	78159	561	10	50	4 52 43.76	23.07	10658	521	10
102°	5 11 20.62	14.52	78720	561	258°	112°	4 52 20.55	23.21	11178	520	248°
10′	5 11 5.95	14.67	79280	560	50′	10′	4 51 57.20	23.35	11697	519	50′
20	5 10 51.13	14.82	79839	559	40	20	4 51 33.72	23.48	12215	518	40
30	5 10 36.16	14.97	80398	559	30	30	4 51 10.10	23.62	12732	517	30
40	5 10 21.04	15.12	80956	558	20	40	4 50 46.34	23.76	13248	516	20
50	5 10 5.77	15.27	81514	558	10	50	4 50 22.44	23.90	13764	516	10
103°	5 9 50.35	15.42	82071	557	257°	113°	4 49 58.41	24.03	14279	515	247°
10′	5 9 34.78	15.57	82628	557	50	10′	4 49 34.24	24.17	14793	514	50′
20	5 9 19.06	15.72	83184	556	40	20	4 49 9.94	24.30	15306	513	40
30	5 9 3.19	15.87	83739	555	30	30	4 48 45.50	24.44	15818	512	30
40	5 8 47.17	16.02	84294	555	20	40	4 48 20.92	24.58	16329	511	20
50	5 8 31.01	16.16	84849	555	10	50	4 47 56.21	24.71	16840	511	10
104°	5 8 14.69	16.32	85403	554	256°	114°	4 47 31.36	24.85	17350	510	246°
10′	5 7 58.22	16.47	85956	553	50′	10′	4 47 6.38	24.98	17859	509	50′
20	5 7 41.61	16.61	86508	552	40	20	4 46 41.26	25.12	18367	508	40
30	5 7 24.85	16.76	87059	551	30	30	4 46 16.00	25.26	18874	507	30
40	5 7 7.94	16.91	87610	551	20	40	4 45 50.61	25.39	19380	506	20
50	5 6 50.88	17.06	88160	550	10	50	4 45 25.09	25.52	19886	506	10
105°	5 6 33.67	17.21	88709	549	255°	115°	4 44 59.44	25.65	20391	505	245°
10′	5 6 16.32	17.35	89258	549	50′	10′	4 44 33.66	25.78	20895	504	50′
20	5 5 58.82	17.50	89806	548	40	20	4 44 7.74	25.92	21398	503	40
30	5 5 41.17	17.65	90354	548	30	30	4 43 41.69	26.05	21900	502	30
40	5 5 23.37	17.80	90902	548	20	40	4 43 15.50	26.19	22401	501	20
50	5 5 5.42	17.95	91449	547	10	50	4 42 49.19	26.31	22901	500	10
106°	5 4 47.33	18.09	91995	546	254°	116°	4 42 22.74	26.45	23401	500	244°
10′	5 4 29.10	18 23	92540	545	50′	10′	4 41 56.16	26.58	23899	498	50′
20	5 4 10.72	18.38	93085	545	40	20	4 41 29.45	26.71	24397	498	40
30	5 3 52.19	18.53	93629	544	30	30	4 41 2.60	26.85	24894	497	30
40	5 3 33.52	18.67	94172	543	20	40	4 40 35.62	26.98	25390	496	20
50	5 3 14.70	18.82	94715	543	10	50	4 40 8.52	27.10	25885	495	10
107°	5 2 55.74	18.96	95257	542	253°	117°	4 39 41.28	27.24	26380	495	243°
10′	5 2 36.63	19.11	95798	541	50′	10′	4 39 13.91	27.37	26874	494	50′
20	5 2 17.38	19.25	96338	540	40	20	4 38 46.41	27.50	27366	492	40
30	5 1 57.99	19.39	96878	540	30	30	4 38 18.78	27.63	27857	491	30
40	5 1 38.45	19.54	97417	539	20	40	4 37 51.02	27.76	28348	491	20
50	5 1 18.77	19.68	97955	538	10	50	4 37 23.13	27.89	28837	489	10
108°	5 0 58.94	19.83	98493	538	252°	118°	4 36 55.12	28.01	29325	488	242°
10′	5 0 38.97	19.97	99030	537	50′	10′	4 36 26.98	28.14	29812	487	50′
20	5 0 18.86	20.11	99566	536	40	20	4 35 58.71	28.27	30299	487	40
30	4 59 58.60	20.26	*00101	535	30	30	4 35 30.31	28.40	30785	486	30
40	4 59 38.21	20.39	*00636	535	20	40	4 35 1.78	28.53	31270	485	20
50	4 59 17.67	20.54	*01170	534	10	50	4 34 33.13	28.65	31754	484	10
109°	4 58 56.98	20.69	*01703	533	251°	119°	4 34 4.35	28.78	32237	483	241°
10′	4 58 36.15	20.83	*02236	533	50′	10′	4 33 35.44	28.91	32719	482	50′
20	4 58 15.18	20.97	*02768	532	40	20	4 33 6.41	29.03	33200	481	40
30	4 57 54.07	21.11	*03299	531	30	30	4 32 37.25	29.16	33681	481	30
40	4 57 32.81	21.26	*03829	530	20	40	4 32 7.96	29.29	34161	480	20
50	4 57 11.42	21.39	*04359	530	10	50	4 31 38.55	29.41	34640	479	10
110°	4 56 49.88	21.54	*04888	529	250°	120°	4 31 9.01	29.54	35117	477	240°
			*1.29		g				1.29		g

TABLE VII.—*Continued.*

g	E		Log. r			g	E		Log. r		
			1.29						1.29		
120°	4 31 9.01		35117		240°	130°	3 57 57.75		61903		230°
10′	4 30 39.35	29.66	35593	476	50′	10′	3 57 21.10	36.65	62317	414	50′
20	4 30 9.56	29.79	36069	476	40	20	3 56 44.35	36.75	62729	412	40
30	4 29 39.65	29.91	36544	475	30	30	3 56 7.48	36.87	63140	411	30
40	4 29 9.61	30.04	37018	474	20	40	3 55 30.51	36.97	63550	410	20
50	4 28 39.45	30.16	37491	473	10	50	3 54 53.43	37.08	63959	409	10
121°	4 28 9.16	30.29	37963	472	239°	131°	3 54 16.25	37.18	64366	407	229°
10′	4 27 38.75	30.41	38434	471	50′	10′	3 53 38.97	37.28	64773	407	50′
20	4 27 8.22	30.53	38904	470	40	20	3 53 1.58	37.39	65179	406	40
30	4 26 37.56	30.66	39372	468	30	30	3 52 24.08	37.50	65583	404	30
40	4 26 6.78	30.78	39840	468	20	40	3 51 46.48	37.60	65987	404	26
50	4 25 35.88	30.90	40306	466	10	50	3 51 8.78	37.70	66389	402	10
122°	4 25 4.85	31.03	40771	465	238°	132°	3 50 30.97	37.81	66790	401	228°
10′	4 24 33.71	31.14	41236	465	50′	10′	3 49 53.06	37.91	67190	400	50′
20	4 24 2.44	31.27	41700	464	40	20	3 49 15.04	38.02	67588	398	40
30	4 23 31.05	31.39	42163	463	30	30	3 48 36.92	38.12	67985	397	30
40	4 22 59.54	31.51	42625	462	20	40	3 47 58.70	38.22	68382	397	20
50	4 22 27.91	31.63	43086	461	10	50	3 47 20.38	38.32	68777	395	10
123°	4 21 56.16	31.75	43546	460	237°	133°	3 46 41.95	38.43	69171	394	227°
10′	4 21 24.29	31.87	44005	459	50′	10′	3 46 3.42	38.53	69564	393	50′
20	4 20 52.29	32.00	44463	458	40	20	3 45 24.79	38.63	69955	391	40
30	4 20 20.18	32.11	44919	456	30	30	3 44 46.06	38.73	70345	390	30
40	4 19 47.94	32.24	45375	456	20	40	3 44 7.23	38.83	70734	389	20
50	4 19 15.59	32.35	45829	454	10	50	3 43 28.30	38.93	71122	388	10
124°	4 18 43.12	32.47	46282	453	236°	134°	3 42 49.27	39.03	71509	387	226°
10′	4 18 10.53	32.59	46735	453	50′	10′	3 42 10.14	39.13	71895	386	50′
20	4 17 37.83	32.70	47186	451	40	20	3 41 30.91	39.23	72279	384	40
30	4 17 5.01	32.82	47636	450	30	30	3 40 51.58	39.33	72662	383	30
40	4 16 32.07	32.94	48086	450	20	40	3 40 12.15	39.43	73044	382	20
50	4 15 59.01	33.06	48534	448	10	50	3 39 32.62	39.53	73425	381	10
125°	4 15 25.84	33.17	48981	447	235°	135°	3 38 53.00	39.62	73805	380	225°
10′	4 14 52.55	33.29	49428	447	50′	10′	3 38 13.28	39.72	74184	379	50′
20	4 14 19.14	33.41	49873	445	40	20	3 37 33.46	39.82	74561	377	40
30	4 13 45.62	33.52	50317	444	30	30	3 36 53.54	39.92	74937	376	30
40	4 13 11.98	33.64	50761	444	20	40	3 36 13.52	40.02	75312	375	20
50	4 12 38.22	33.76	51204	443	10	50	3 35 33.41	40.11	75686	374	10
126°	4 12 4.35	33.87	51645	441	234°	136°	3 34 53.21	40.20	76059	373	224°
10′	4 11 30.37	33.98	52085	440	50′	10′	3 34 12.90	40.31	76430	371	50′
20	4 10 56.27	34.10	52524	439	40	20	3 33 32.50	40.40	76800	370	40
30	4 10 22.06	34.21	52962	438	30	30	3 32 52.01	40.49	77169	369	30
40	4 9 47.73	34.33	53399	437	20	40	3 32 11.42	40.59	77537	368	20
50	4 9 13.29	34.44	53834	435	10	50	3 31 30.73	40.69	77903	366	10
127°	4 8 38.74	34.55	54268	434	233°	137°	3 30 49.95	40.78	78268	365	223°
10′	4 8 4.07	34.67	54702	434	50′	10′	3 30 9.08	40.87	78632	364	50′
20	4 7 29.29	34.78	55134	432	40	20	3 29 28.11	40.97	78995	363	40
30	4 6 54.40	34.89	55565	431	30	30	3 28 47.05	41.06	79356	361	30
40	4 6 19.40	35.00	55995	430	20	40	3 28 5.90	41.15	79716	360	20
50	4 5 44.29	35.11	56424	429	10	50	3 27 24.66	41.24	80075	359	10
128°	4 5 9.06	35.23	56852	428	232°	138°	3 26 43.32	41.34	80433	358	222°
10′	4 4 33.72	35.34	57279	427	50′	10′	3 26 1.89	41.43	80789	356	50
20	4 3 58.27	35.45	57705	426	40	20	3 25 20.37	41.52	81144	355	40
30	4 3 22.72	35.55	58130	425	30	30	3 24 38.76	41.61	81498	354	30
40	4 2 47.05	35.67	58554	424	20	40	3 23 57.05	41.71	81851	353	20
50	4 2 11.27	35.78	58977	423	10	50	3 23 15.26	41.79	82202	351	10
129°	4 1 35.38	35.89	59399	422	231°	139°	3 22 33.38	41.88	82552	350	221°
10′	4 0 59.38	36.00	59820	421	50′	10′	3 21 51.41	41.97	82901	349	50′
20	4 0 23.27	36.11	60239	419	40	20	3 21 9.35	42.06	83249	348	40
30	3 59 47.05	36.22	60657	418	30	30	3 20 27.20	42.15	83595	346	30
40	3 59 10.73	36.32	61074	417	20	40	3 19 44.96	42.24	83940	345	20
50	3 58 34.29	36.44	61489	415	10	50	3 19 2.63	42.33	84284	344	10
130°	3 57 57.75	36.54	61903	414	230°	140°	3 18 20.22	42.41	84627	343	220°
			1.29		g				1.29		g

TABLE VII.—*Continued.*

g	E		Log. r			g	E		Log. r		
	° ′ ″	″	1.29				° ′ ″	″	1.30		
140°	3 18 20.22		84627		220°	150°	2 33 26.93		02798		210°
10′	3 17 37.72	42.50	84968	341	50′	10′	2 32 39.79	47.14	03060	262	50′
20	3 16 55.13	42.59	85308	340	40	20	2 31 52.59	47.20	03320	260	40
30	3 16 12.45	42.68	85647	339	30	30	2 31 5.32	47.27	03579	259	30
40	3 15 29.68	42.77	85985	338	20	40	2 30 17.98	47.34	03837	258	20
50	3 14 46.83	42.85	86321	336	10	50	2 29 30.58	47.40	04093	256	10
141°	3 14 3.89	42.94	86656	335	219°	151°	2 28 43.11	47.47	04348	255	209°
10′	3 13 20.87	43.02	86990	334	50′	10′	2 27 55.58	47.53	04602	254	50′
20	3 12 37.76	43.11	87322	332	40	20	2 27 7.98	47.60	04854	252	40
30	3 11 54.56	43.20	87653	331	30	30	2 26 20.32	47.66	05105	251	30
40	3 11 11.28	43.28	87983	330	20	40	2 25 32.59	47.73	05355	250	20
50	3 10 27.92	43.36	88312	329	10	50	2 24 44.80	47.79	05604	249	10
142°	3 9 44.47	43.45	88639	327	218°	152°	2 23 56.95	47.85	05851	247	208°
10′	3 9 0.94	43.53	88965	326	50′	10′	2 23 9.04	47.91	06096	245	50′
20	3 8 17.32	43.62	89290	325	40	20	2 22 21.06	47.98	06340	244	40
30	3 7 33.62	43.70	89614	324	30	30	2 21 33.02	48.04	06583	243	30
40	3 6 49.83	43.79	89936	322	20	40	2 20 44.92	48.10	06824	241	20
50	3 6 5.97	43.86	90257	321	10	50	2 19 56.76	48.16	07064	240	10
143°	3 5 22.02	43.95	90577	320	217°	153°	2 19 8.54	48.22	07302	238	207°
10′	3 4 37.99	44.03	90895	318	50′	10′	2 18 20.26	48.28	07539	237	50′
20	3 3 53.88	44.11	91212	317	40	20	2 17 31.92	48.34	07775	236	40
30	3 3 9.69	44.19	91528	316	30	30	2 16 43.51	48.41	08009	234	30
40	3 2 25.42	44.27	91842	314	20	40	2 15 55.05	48.46	08242	233	20
50	3 1 41.08	44.34	92155	313	10	50	2 15 6.53	48.52	08473	231	10
144°	3 0 56.65	44.43	92466	311	216°	154°	2 14 17.95	48.58	08703	230	206°
10′	3 0 12.14	44.51	92776	310	50′	10′	2 13 29.31	48.64	08931	228	50′
20	2 59 27.55	44.59	93085	309	40	20	2 12 40.62	48.69	09158	227	40
30	2 58 42.89	44.66	93393	308	30	30	2 11 51.86	48.76	09384	226	30
40	2 57 58.14	44.75	93700	307	20	40	2 11 3.05	48.81	09608	224	20
50	2 57 13.32	44.82	94005	305	10	50	2 10 14.18	48.87	09831	223	10
145°	2 56 28.42	44.90	94309	304	215°	155°	2 9 25.26	48.92	10053	222	205°
10	2 55 43.44	44.98	94612	303	50′	10′	2 8 36.28	48.98	10273	220	50′
20	2 54 58.39	45.05	94913	301	40	20	2 7 47.24	49.04	10492	219	40
30	2 54 13.26	45.13	95213	300	30	30	2 6 58.15	49.09	10709	217	30
40	2 53 28.05	45.21	95511	298	20	40	2 6 9.01	49.14	10925	216	20
50	2 52 42.77	45.28	95808	297	10	50	2 5 19.81	49.20	11139	214	10
146°	2 51 57.41	45.36	96104	296	214°	156°	2 4 30.55	49.26	11352	213	204°
10′	2 51 11.97	45.44	96398	294	50′	10′	2 3 41.24	49.31	11564	212	50′
20	2 50 26.46	45.51	96691	293	40	20	2 2 51.88	49.36	11774	210	40
30	2 49 40.87	45.59	96983	292	30	30	2 2 2.47	49.41	11983	209	30
40	2 48 55.21	45.66	97273	290	20	40	2 1 13.00	49.47	12191	208	20
50	2 48 9.48	45.73	97562	289	10	50	2 0 23.48	49.52	12397	206	10
147°	2 47 23.67	45.81	97850	288	213°	157°	1 59 33.91	49.57	12602	205	203°
10′	2 46 37.79	45.88	98136	286	50′	10′	1 58 44.29	49.62	12805	203	50′
20	2 45 51.83	45.96	98421	285	40	20	1 57 54.61	49.68	13007	202	40
30	2 45 5.80	46.03	98705	284	30	30	1 57 4.88	49.73	13207	200	30
40	2 44 19.70	46.10	98988	283	20	40	1 56 15.10	49.78	13405	198	20
50	2 43 33.53	46.17	99269	281	10	50	1 55 25.28	49.82	13602	197	10
148°	2 42 47.29	46.24	99549	280	212°	158°	1 54 35.40	49.88	13798	196	202°
10′	2 42 0.98	46.31	99827	278	50′	10′	1 53 45.47	49.93	13992	194	50′
20	2 41 14.59	46.39	*00104	277	40	20	1 52 55.50	49.97	14185	193	40
30	2 40 28.14	46.45	*00380	276	30	30	1 52 5.48	50.02	14377	192	30
40	2 39 41.61	46.53	*00654	274	20	40	1 51 15.41	50.07	14567	190	20
50	2 38 55.01	46.60	*00926	272	10	50	1 50 25.29	50.12	14756	189	10
149°	2 38 8.35	46.66	*01197	271	211°	159°	1 49 35.12	50.17	14944	188	201°
10′	2 37 21.62	46.73	*01467	270	50′	10′	1 48 44.91	50.21	15130	186	50′
20	2 36 34.81	46.81	*01736	269	40	20	1 47 54.65	50.26	15315	185	40
30	2 35 47.94	46.87	*02004	268	30	30	1 47 4.34	50.31	15498	183	30
40	2 35 1.01	46.93	*02270	266	20	40	1 46 13.98	50.36	15680	182	20
50	2 34 14.00	47.01	*02535	265	10	50	1 45 23.58	50.40	15860	180	10
150°	2 33 26.93	47.07	*02798	263	210°	160°	1 44 33.14	50.44	16039	179	200°
			*1.30		g				1.30		

TABLE VII.—*Continued.*

g	E		Log. r		g	g	E		Log. r		g
	° ′ ″	″	1.30				° ′ ″	″	1.30		
160°	1 44 33.14	50.49	16039	177	200°	170°	0 52 57.32	52.51	24087	89	190°
10′	1 43 42.65	50.54	16216	176	50′	10′	0 52 4.81	52.53	24176	88	50′
20	1 42 52.11	50.58	16392	174	40	20	0 51 12.28	52.56	24264	86	40
30	1 42 1.53	50.62	16566	173	30	30	0 50 19.72	52.58	24350	84	30
40	1 41 10.91	50.67	16739	171	20	40	0 49 27.14	52.60	24434	83	20
50	1 40 20.24	50.71	16910	170	10	50	0 48 34.54	52.62	24517	82	10
161°	1 39 29.53	50.75	17080	168	199°	171°	0 47 41.92	52.64	24599	80	189°
10′	1 38 38.78	50.80	17248	167	50′	10′	0 46 49.28	52.66	24679	79	50′
20	1 37 47.98	50.84	17415	166	40	20	0 45 56.62	52.69	24758	77	40
30	1 36 57.14	50.89	17581	164	30	30	0 45 3.93	52.70	24835	75	30
40	1 36 6.25	50.92	17745	163	20	40	0 44 11.23	52.72	24910	74	20
50	1 35 15.33	50.96	17908	162	10	50	0 43 18.51	52.74	24984	73	10
162°	1 34 24.37	51.00	18070	160	198°	172°	0 42 25.77	52.76	25057	71	188°
10′	1 33 33.37	51.04	18230	158	50′	10′	0 41 33.01	52.77	25128	70	50′
20	1 32 42.33	51.09	18388	157	40	20	0 40 40.24	52.79	25198	69	40
30	1 31 51.24	51.12	18545	155	30	30	0 39 47.45	52.81	25267	67	30
40	1 31 0.12	51.16	18700	154	20	40	0 38 54.64	52.82	25334	65	20
50	1 30 8.96	51.20	18854	153	10	50	0 38 1.82	52.84	25399	64	10
163°	1 29 17.76	51.24	19007	151	197°	173°	0 37 8.98	52.86	25463	62	187°
10′	1 28 26.52	51.27	19158	150	50′	10′	0 36 16.12	52.87	25525	61	50′
20	1 27 35.25	51.32	19308	148	40	20	0 35 23.25	52.89	25586	59	40
30	1 26 43.93	51.35	19456	147	30	30	0 34 30.36	52.91	25645	58	30
40	1 25 52.58	51.39	19603	146	20	40	0 33 37.45	52.92	25703	56	20
50	1 25 1.19	51.42	19749	144	10	50	0 32 44.53	52.93	25759	55	10
164°	1 24 9.77	51.46	19893	142	196°	174°	0 31 51.60	52.94	25814	53	186°
10′	1 23 18.31	51.50	20035	141	50′	10′	0 30 58.66	52.96	25867	52	50′
20	1 22 26.81	51.53	20176	140	40	20	0 30 5.70	52.97	25919	50	40
30	1 21 35.28	51.57	20316	138	30	30	0 29 12.73	52.98	25969	48	30
40	1 20 43.71	51.60	20454	136	20	40	0 28 19.75	52.99	26017	47	20
50	1 19 52.11	51.64	20590	135	10	50	0 27 26.76	53.00	26064	46	10
165°	1 19 0.47	51.67	20725	133	195°	175°	0 26 33.76	53.01	26110	44	185°
10′	1 18 8.80	51.70	20858	132	50′	10′	0 25 40.75	53.03	26154	43	50′
20	1 17 17.10	51.74	20990	131	40	20	0 24 47.72	53.04	26197	42	40
30	1 16 25.36	51.77	21121	129	30	30	0 23 54.68	53.04	26239	40	30
40	1 15 33.59	51.80	21250	127	20	40	0 23 1.64	53.06	26279	39	20
50	1 14 41.79	51.83	21377	126	10	50	0 22 8.58	53.07	26318	37	10
166°	1 13 49.96	51.86	21503	125	194°	176°	0 21 15.51	53.08	26355	35	184°
10′	1 12 58.10	51.90	21628	123	50′	10′	0 20 22.43	53.08	26390	34	50′
20	1 12 6.20	51.93	21751	122	40	20	0 19 29.35	53.10	26424	32	40
30	1 11 14.27	51.96	21873	120	30	30	0 18 36.25	53.10	26456	30	30
40	1 10 22.31	51.98	21993	119	20	40	0 17 43.15	53.12	26486	29	20
50	1 9 30.33	52.02	22112	118	10	50	0 16 50.03	53.12	26515	28	10
167°	1 8 38.31	52.04	22230	116	193°	177°	0 15 56.91	53.13	26543	26	183°
10′	1 7 46.27	52.08	22346	114	50′	10′	0 15 3.78	53.13	26569	25	50′
20	1 6 54.19	52.11	22460	113	40	20	0 14 10.65	53.14	26594	23	40
30	1 6 2.08	52.14	22573	111	30	30	0 13 17.51	53.15	26617	22	30
40	1 5 9.94	52.16	22684	109	20	40	0 12 24.36	53.15	26639	20	20
50	1 4 17.78	52.19	22793	108	10	50	0 11 31.21	53.15	26659	19	10
168°	1 3 25.59	52.22	22901	107	192°	178°	0 10 38.06	53.16	26678	17	182°
10′	1 2 33.37	52.24	23008	105	50′	10′	0 9 44.90	53.16	26695	16	50′
20	1 1 41.13	52.27	23113	104	40	20	0 8 51.74	53.16	26711	15	40
30	1 0 48.86	52.29	23217	103	30	30	0 7 58.58	53.17	26726	13	30
40	0 59 56.57	52.32	23320	101	20	40	0 7 5.41	53.17	26739	11	20
50	0 59 4.25	52.35	23421	100	10	50	0 6 12.24	53.17	26750	10	10
169°	0 58 11.90	52.37	23521	98	191°	179°	0 5 19.07	53.18	26760	8	181°
10′	0 57 19.53	52.40	23619	97	50′	10′	0 4 25.89	53.17	26768	7	50′
20	0 56 27.13	52.42	23716	95	40	20	0 3 32.72	53.18	26775	5	40
30	0 55 34.71	52.44	23811	93	30	30	0 2 39.54	53.18	26780	4	30
40	0 54 42.27	52.46	23904	92	20	40	0 1 46.36	53.18	26784	2	20
50	0 53 49.81	52.49	23996	91	10	50	0 0 53.18	53.18	26786	1	10
170°	0 52 57.32		24087		190°	180°	0 0 0.00		26787		180°
			1.30		g				1.30		g

TABLE VIII, Arg. 1.—Action of Jupiter.

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
0	55.03		0.14	4.42	0.10	0.11	2331	164	134
1	55.58	+0.55	0.13	4.42	0.10	0.12	2331	166	134
2	56.13	0.55	0.12	4.41	0.10	0.12	2330	167	134
3	56.68	0.55	0.11	4.41	0.10	0.12	2330	169	134
4	57.23	0.55	0.11	4.41	0.10	0.12	2330	171	135
5	57.78	0.55	0.10	4.40	0.10	0.13	2329	172	135
6	58.32	+0.54	0.09	4.40	0.11	0.13	2328	174	135
7	58.87	0.55	0.09	4.39	0.11	0.13	2328	176	135
8	59.42	0.55	0.08	4.39	0.11	0.13	2327	177	135
9	59.97	0.55	0.08	4.38	0.11	0.14	2326	179	136
10	60.51	0.54	0.08	4.38	0.11	0.14	2324	180	136
11	61.06	+0.55	0.07	4.37	0.11	0.14	2323	182	136
12	61.60	0.54	0.07	4.37	0.11	0.15	2321	184	137
13	62.15	0.55	0.07	4.36	0.11	0.15	2320	185	137
14	62.69	0.54	0.08	4.36	0.11	0.15	2318	187	137
15	63.23	0.54	0.08	4.35	0.11	0.15	2316	188	137
16	63.78	+0.55	0.08	4.35	0.11	0.16	2314	190	137
17	64.32	0.54	0.09	4.34	0.11	0.16	2312	191	138
18	64.86	0.54	0.09	4.34	0.11	0.16	2310	193	138
19	65.40	0.54	0.10	4.33	0.11	0.17	2308	194	138
20	65.93	0.53	0.11	4.33	0.11	0.17	2305	196	138
21	66.47	+0.54	0.12	4.32	0.11	0.17	2303	198	139
22	67.00	0.53	0.13	4.32	0.11	0.17	2300	199	139
23	67.54	0.54	0.14	4.31	0.11	0.18	2297	200	139
24	68.07	0.53	0.15	4.30	0.11	0.18	2294	202	140
25	68.60	0.53	0.16	4.30	0.10	0.18	2291	203	140
26	69.14	+0.54	0.18	4.29	0.10	0.19	2288	205	140
27	69.66	0.52	0.20	4.29	0.10	0.19	2284	206	140
28	70.19	0.53	0.21	4.28	0.10	0.19	2281	208	140
29	70.72	0.53	0.23	4.28	0.10	0.20	2277	209	141
30	71.24	0.52	0.25	4.27	0.10	0.20	2274	210	141
31	71.76	+0.52	0.27	4.27	0.10	0.20	2270	212	141
32	72.28	0.52	0.29	4.26	0.10	0.21	2266	213	141
33	72.80	0.52	0.31	4.26	0.10	0.21	2262	214	142
34	73.32	0.52	0.33	4.25	0.10	0.21	2258	216	142
35	73.83	0.51	0.35	4.25	0.10	0.22	2254	217	142
36	74.34	+0.51	0.38	4.24	0.10	0.22	2249	218	142
37	74.85	0.51	0.40	4.24	0.10	0.22	2245	219	143
38	75.36	0.51	0.43	4.23	0.10	0.23	2240	221	143
39	75.87	0.51	0.46	4.23	0.09	0.23	2236	222	143
40	76.37	0.50	0.49	4.22	0.09	0.23	2231	223	143
41	76.87	+0.50	0.52	4.22	0.09	0.23	2226	224	144
42	77.37	0.50	0.55	4.22	0.09	0.24	2221	225	144
43	77.87	0.50	0.58	4.21	0.09	0.24	2216	227	144
44	78.37	0.50	0.61	4.21	0.09	0.24	2210	228	144
45	78.86	0.49	0.64	4.21	0.09	0.25	2205	229	144
46	79.35	+0.49	0.68	4.20	0.09	0.25	2200	230	145
47	79.84	0.49	0.71	4.20	0.09	0.25	2194	231	145
48	80.32	0.48	0.75	4.20	0.09	0.26	2188	232	145
49	80.80	0.48	0.79	4.20	0.09	0.26	2182	233	145
50	81.28	0.48	0.82	4.20	0.08	0.26	2176	234	146
51	81.76	+0.48	0.86	4.19	0.08	0.26	2170	235	146
52	82.23	0.47	0.90	4.19	0.08	0.26	2164	236	146
53	82.70	0.47	0.94	4.19	0.08	0.27	2158	237	146
54	83.17	0.47	0.98	4.19	0.08	0.27	2151	238	146
55	83.63	0.46	1.02	4.19	0.07	0.27	2145	239	146
56	84.10	+0.47	1.07	4.19	0.07	0.28	2138	240	146
57	84.55	0.45	1.11	4.19	0.07	0.28	2132	241	146
58	85.01	0.46	1.16	4.19	0.07	0.28	2125	242	147
59	85.46	0.45	1.20	4.19	0.07	0.28	2118	243	147
60	85.91	0.45	1.25	4.19	0.07	0.29	2111	244	147

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	"	"	"	"	"	"			
60	85.91	+0.45	1.25	4.19	0.07	0.29	2111	244	147
61	86.36	0.44	1.30	4.19	0.07	0.29	2104	244	147
62	86.80	0.44	1.34	4.20	0.06	0.29	2097	245	148
63	87.24	0.44	1.39	4.20	0.06	0.29	2090	246	148
64	87.68	0.43	1.44	4.20	0.06	0.29	2082	247	148
65	88.11	+0.42	1.49	4.20	0.06	0.30	2075	248	148
66	88.53	0.43	1.54	4.21	0.06	0.30	2067	248	148
67	88.96	0.42	1.60	4.21	0.06	0.30	2059	249	148
68	89.38	0.42	1.65	4.21	0.05	0.30	2052	250	148
69	89.80	0.41	1.70	4.22	0.05	0.31	2044	250	148
70	90.21	+0.41	1.75	4.22	0.05	0.31	2036	251	149
71	90.62	0.41	1.81	4.23	0.05	0.31	2028	252	149
72	91.03	0.40	1.86	4.23	0.05	0.31	2020	252	149
73	91.43	0.40	1.92	4.24	0.05	0.31	2012	253	149
74	91.83	0.39	1.98	4.24	0.04	0.32	2003	253	149
75	92.22	+0.40	2.03	4.25	0.04	0.32	1995	254	149
76	92.62	0.33	2.09	4.26	0.04	0.32	1986	254	149
77	93.00	0.38	2.15	4.26	0.04	0.32	1978	255	149
78	93.38	0.38	2.21	4.27	0.04	0.32	1969	255	149
79	93.76	0.38	2.27	4.28	0.04	0.32	1960	256	149
80	94.14	+0.37	2.33	4.29	0.04	0.32	1952	256	149
81	94.51	0.36	2.39	4.30	0.03	0.32	1943	257	149
82	94.87	0.36	2.45	4.31	0.03	0.33	1934	257	149
83	95.23	0.36	2.51	4.32	0.03	0.33	1925	257	149
84	95.59	0.36	2.57	4.33	0.03	0.33	1916	258	149
85	95.95	+0.35	2.64	4.34	0.03	0.33	1906	258	149
86	96.30	0.34	2.70	4.35	0.03	0.33	1897	258	149
87	96.64	0.34	2.76	4.36	0.03	0.33	1888	259	149
88	96.98	0.34	2.83	4.37	0.03	0.83	1878	259	149
89	97.32	0.33	2.89	4.38	0.02	0.33	1869	259	149
90	97.65	+0.32	2.96	4.39	0.02	0.33	1859	260	148
91	97.97	0.32	3.02	4.41	0.02	0.33	1850	260	148
92	98.29	0.32	3.09	4.42	0.02	0.33	1840	260	148
93	98.61	0.31	3.15	4.43	0.02	0.33	1830	260	148
94	98.92	0.31	3.22	4.45	0.02	0.33	1820	260	148
95	99.23	+0.30	3.29	4.46	0.02	0.33	1810	261	148
96	99.53	0.30	3.35	4.48	0.02	0.34	1800	261	148
97	99.83	0.29	3.42	4.49	0.02	0.34	1790	261	148
98	100.12	0.29	3.49	4.51	0.01	0.34	1780	261	147
99	100.41	0.29	3.56	4.52	0.01	0.34	1770	261	147
100	100.70	+0.27	3.63	4.54	0.01	0.34	1760	261	147
101	100.97	0.28	3.69	4.56	0.01	0.34	1750	261	147
102	101.25	0.27	3.76	4.57	0.01	0.34	1739	261	147
103	101.52	0.26	3.83	4.59	0.01	0.34	1729	261	146
104	101.78	0.26	3.90	4.61	0.01	0.34	1718	261	146
105	102.04	+0.25	3.97	4.63	0.01	0.34	1708	261	146
106	102.29	0.25	4.04	4.65	0.01	0.34	1697	261	145
107	102.54	0.24	4.11	4.67	0.01	0.34	1686	261	145
108	102.78	0.24	4.18	4.69	0.01	0.34	1676	261	145
109	103.02	0.23	4.25	4.71	0.01	0.34	1665	261	145
110	103.25	+0.23	4.32	4.73	0.01	0.34	1654	261	144
111	103.48	0.22	4.39	4.75	0.01	0.34	1643	261	144
112	103.70	0.22	4.46	4.77	0.01	0.34	1632	261	144
113	103.92	0.21	4.53	4.79	0.01	0.34	1622	260	143
114	104.13	0.21	4.60	4.81	0.01	0.34	1611	260	143
115	104.34	+0.20	4.67	4.83	0.01	0.33	1600	260	142
116	104.54	0.20	4.74	4.85	0.01	0.33	1588	260	142
117	104.74	0.19	4.81	4.87	0.01	0.33	1577	260	142
118	104.93	0.18	4.88	4.90	0.01	0.33	1566	259	141
119	105.11	0.18	4.95	4.92	0.01	0.33	1555	259	141
120	105.29		5.01	4.94	0.02	0.33	1544	259	140

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
120	105.29	+0.17	5.01	4.94	0.02	0.33	1544	259	140
121	105.46	0.17	5.08	4.97	0.02	0.33	1533	259	140
122	105.63	0.16	5.15	4.99	0.02	0.33	1521	258	139
123	105.79	0.16	5.22	5.01	0.02	0.33	1510	258	139
124	105.95	0.15	5.29	5.04	0.02	0.33	1499	258	138
125	106.10	+0.15	5.36	5.06	0.02	0.33	1487	257	138
126	106.25	0.14	5.43	5.09	0.02	0.33	1476	257	137
127	106.39	0.14	5.50	5.11	0.02	0.33	1464	257	137
128	106.53	0.12	5.57	5.14	0.02	0.33	1453	256	136
129	106.65	0.13	5.64	5.16	0.02	0.33	1441	256	135
130	106.78	+0.11	5.71	5.19	0.02	0.32	1430	256	135
131	106.89	0.12	5.78	5.21	0.03	0.32	1418	255	134
132	107.01	0.10	5.85	5.24	0.03	0.32	1407	255	134
133	107.11	0.11	5.91	5.26	0.03	0.32	1395	254	133
134	107.22	0.09	5.98	5.29	0.03	0.32	1384	254	132
135	107.31	+0.09	6.05	5.32	0.03	0.32	1372	254	132
136	107.40	0.09	6.11	5.34	0.04	0.32	1360	253	131
137	107.49	0.08	6.18	5.37	0.04	0.32	1349	253	131
138	107.57	0.07	6.25	5.40	0.04	0.32	1337	252	130
139	107.64	0.06	6.31	5.42	0.04	0.32	1325	252	129
140	107.70	+0.06	6.38	5.45	0.04	0.32	1314	251	129
141	107.76	0.06	6.44	5.48	0.05	0.31	1302	251	128
142	107.82	0.05	6.51	5.51	0.05	0.31	1290	250	127
143	107.87	0.04	6.57	5.53	0.05	0.31	1278	250	126
144	107.91	0.04	6.64	5.56	0.05	0.31	1267	249	126
145	107.95	+0.03	6.70	5.59	0.05	0.31	1255	249	125
146	107.98	0.03	6.76	5.61	0.06	0.31	1243	248	124
147	108.01	0.02	6.83	5.64	0.06	0.31	1231	248	123
148	108.03	0.02	6.89	5.67	0.06	0.31	1220	247	123
149	108.05	0.00	6.95	5.69	0.06	0.31	1208	247	122
150	108.05	+0.01	7.01	5.72	0.07	0.31	1196	246	121
151	108.06	−0.01	7.08	5.75	0.07	0.31	1184	246	120
152	108.05	0.01	7.14	5.78	0.07	0.31	1172	245	119
153	108.04	0.01	7.20	5.80	0.07	0.30	1161	244	118
154	108.03	0.02	7.25	5.83	0.08	0.30	1149	244	118
155	108.01	−0.03	7.31	5.86	0.08	0.30	1137	243	117
156	107.98	0.03	7.37	5.88	0.08	0.30	1126	243	116
157	107.95	0.04	7.43	5.91	0.09	0.30	1114	242	115
158	107.91	0.04	7.49	5.94	0.09	0.30	1102	241	114
159	107.87	0.05	7.54	5.97	0.09	0.30	1090	241	113
160	107.82	−0.05	7.60	5.99	0.10	0.30	1079	240	112
161	107.77	0.07	7.66	6.02	0.10	0.30	1067	240	112
162	107.70	0.06	7.71	6.04	0.10	0.30	1055	239	111
163	107.64	0.08	7.77	6.07	0.11	0.30	1044	238	110
164	107.56	0.07	7.82	6.10	0.11	0.29	1032	238	109
165	107.49	−0.09	7.87	6.12	0.11	0.29	1020	237	108
166	107.40	0.09	7.93	6.15	0.12	0.29	1009	236	107
167	107.31	0.09	7.98	6.17	0.12	0.29	997	236	106
168	107.22	0.11	8.03	6.20	0.12	0.29	986	235	105
169	107.11	0.10	8.08	6.22	0.13	0.29	974	234	104
170	107.01	−0.12	8.13	6.25	0.13	0.29	963	234	103
171	106.89	0.12	8.18	6.27	0.14	0.29	951	233	102
172	106.77	0.12	8.23	6.29	0.14	0.29	940	232	101
173	106.65	0.13	8.28	6.32	0.14	0.29	928	232	100
174	106.52	0.14	8.33	6.34	0.15	0.29	917	231	99
175	106.38	−0.14	8.37	6.36	0.15	0.29	906	230	98
176	106.24	0.14	8.42	6.39	0.15	0.29	894	230	97
177	106.10	0.16	8.46	6.41	0.16	0.29	883	229	96
178	105.94	0.16	8.51	6.43	0.16	0.29	872	228	95
179	105.78	0.16	8.55	6.45	0.17	0.29	861	228	94
180	105.62		8.60	6.47	0.17	0.29	850	227	93

TABLE VIII, Arg. 1.—*Continued.*

Arg.	$(v.c.0)$	Diff	$(v.s.1)$	$(v.c.1)$	$(v.s.2)$	$(v.c.2)$	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$
	"	"	"	"	"	"			
180	105.62		8.60	6.47	0.17	0.29	850	227	93
181	105.45	—0.17	8.64	6.50	0.17	0.29	839	226	92
182	105.27	0.18	8.68	6.52	0.18	0.29	828	226	91
183	105.09	0.18	8.72	6.54	0.18	0.29	816	225	90
184	104.90	0.19	8.76	6.56	0.19	0.29	805	224	89
185	104.71	0.19	8.80	6.57	0.19	0.29	794	224	88
186	104.51	—0.20	8.84	6.59	0.19	0.29	784	223	87
187	104.31	0.20	8.88	6.61	0.20	0.29	773	222	86
188	104.10	0.21	8.92	6.63	0.20	0.29	762	222	85
189	103.89	0.21	8.96	6.65	0.21	0.29	751	221	84
190	103.67	0.22	8.99	6.67	0.21	0.29	740	221	83
191	103.44	—0.23	9.03	6.68	0.21	0.29	730	220	82
192	103.21	0.23	9.07	6.70	0.22	0.29	719	219	81
193	102.98	0.23	9.10	6.71	0.22	0.29	708	219	80
194	102.74	0.24	9.13	6.73	0.22	0.29	698	218	79
195	102.49	0.25	9.17	6.74	0.23	0.29	688	217	78
196	102.24	—0.25	9.20	6.76	0.23	0.29	677	216	77
197	101.98	0.26	9.23	6.77	0.24	0.29	667	216	76
198	101.72	0.26	9.26	6.78	0.24	0.29	657	215	75
199	101.45	0.27	9.29	6.79	0.24	0.29	646	215	74
200	101.18	0.27	9.32	6.81	0.25	0.29	636	214	73
201	100.90	—0.28	9.35	6.82	0.25	0.29	626	213	72
202	100.62	0.28	9.38	6.83	0.25	0.29	616	213	71
203	100.33	0.29	9.41	6.84	0.26	0.29	606	212	70
204	100.04	0.29	9.43	6.85	0.26	0.29	596	211	69
205	99.74	0.30	9.46	6.86	0.26	0.29	586	211	68
206	99.44	—0.30	9.48	6.86	0.27	0.29	577	210	67
207	99.13	0.31	9.51	6.87	0.27	0.29	567	210	66
208	98.82	0.31	9.53	6.88	0.27	0.29	557	209	65
209	98.50	0.32	9.55	6.88	0.28	0.29	548	208	64
210	98.18	0.32	9.58	6.89	0.28	0.29	538	208	63
211	97.85	—0.33	9.60	6.89	0.28	0.29	529	207	62
212	97.52	0.33	9.62	6.90	0.29	0.29	520	206	61
213	97.18	0.34	9.64	6.90	0.29	0.29	510	206	60
214	96.84	0.34	9.66	6.90	0.29	0.29	501	205	59
215	96.49	0.35	9.68	6.91	0.30	0.29	492	205	58
216	96.14	—0.35	9.70	6.91	0.30	0.29	483	204	57
217	95.79	0.35	9.71	6.91	0.30	0.29	474	204	56
218	95.43	0.36	9.73	6.91	0.31	0.29	464	203	55
219	95.06	0.37	9.75	6.91	0.31	0.30	456	202	54
220	94.69	0.37	9.76	6.91	0.31	0.30	448	202	53
221	94.32	—0.37	9.78	6.90	0.31	0.30	439	201	52
222	93.94	0.38	9.79	6.90	0.32	0.30	430	201	51
223	93.56	0.38	9.81	6.90	0.32	0.30	422	200	50
224	93.17	0.39	9.82	6.90	0.32	0.30	413	200	49
225	92.78	0.39	9.83	6.89	0.33	0.30	405	199	48
226	92.39	—0.39	9.85	6.88	0.33	0.30	398	198	47
227	91.99	0.40	9.86	6.88	0.33	0.30	390	198	46
228	91.59	0.40	9.87	6.87	0.33	0.30	382	197	46
229	91.18	0.41	9.88	6.86	0.34	0.30	374	197	45
230	90.77	0.41	9.89	6.85	0.34	0.30	365	196	44
231	90.36	—0.41	9.90	6.84	0.34	0.30	357	196	43
232	89.94	0.42	9.91	6.83	0.34	0.30	349	195	42
233	89.51	0.43	9.91	6.82	0.34	0.30	342	194	41
234	89.09	0.42	9.92	6.81	0.35	0.30	334	194	41
235	88.65	0.44	9.93	6.80	0.35	0.30	327	194	40
236	88.22	—0.43	9.94	6.79	0.35	0.30	320	193	39
237	87.78	0.44	9.94	6.78	0.35	0.30	312	192	38
238	87.34	0.44	9.95	6.76	0.35	0.30	305	192	38
239	86.89	0.45	9.95	6.74	0.36	0.30	298	191	37
240	86.44	0.45	9.96	6.73	0.36	0.31	291	191	36

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
240	86.44		9.96	6.73	0.36	0.31	291	191	36
241	85.99	—0.45	9.96	6.71	0.36	0.31	284	190	35
242	85.53	0.46	9.96	6.69	0.36	0 31	278	190	35
243	85.08	0.45	9.97	6.67	0.36	0.31	271	189	34
244	84.61	0.47	9.97	6.66	0.37	0.31	265	189	33
245	84.15	0.46	9.97	6.64	0.37	0.31	258	188	33
246	83.68	—0.47	9.97	6.61	0.37	0.31	252	188	32
247	83.21	0.47	9.98	6.59	0.37	0.31	246	187	31
248	82.73	0.48	9.98	6.57	0.37	0.31	240	187	31
249	82.25	0.48	9.98	6.55	0.37	0.31	234	186	30
250	81.77	0.48	9.98	6.53	0.37	0.31	228	186	29
251	81.28	—0.49	9.98	6.51	0.38	0.31	222	185	29
252	80.80	0.48	9.98	6.48	0.38	0.31	216	185	28
253	80.30	0.50	9.97	6.46	0.38	0.31	211	184	28
254	79.81	0.49	9.97	6.43	0.38	0.31	205	184	27
255	79.31	0.50	9.97	6.40	0.38	0.31	200	183	27
256	78.82	—0.49	9.97	6.37	0.38	0.30	195	183	26
257	78.31	0.51	9.96	6.34	0.38	0.30	190	182	26
258	77.81	0.50	9.96	6.31	0.38	0.30	184	182	25
259	77.30	0.51	9.96	6.28	0.38	0.30	179	181	25
260	76.79	0.51	9.95	6.25	0.38	0.30	175	181	24
261	76.28	—0.51	9.95	6.22	0.38	0.30	170	180	24
262	75.77	0.51	9.95	6.19	0.38	0.30	165	180	24
263	75.25	0.52	9.94	6.16	0.38	0.30	161	179	23
264	74.73	0.52	9.94	6.13	0.38	0.30	156	179	23
265	74.21	0.52	9.93	6.10	0.38	0.30	152	178	22
266	73.69	—0.52	9.93	6.06	0.38	0.30	148	178	22
267	73.16	0.53	9.92	6.03	0.38	0.30	144	177	22
268	72.64	0.52	9.92	5.99	0.38	0.30	140	177	21
269	72.11	0.53	9.91	5.96	0.38	0.30	137	176	21
270	71.57	0.54	9.90	5.92	0.39	0.30	133	176	21
271	71.04	—0.53	9.90	5.88	0.39	0.30	129	175	20
272	70.51	0.53	9.89	5.85	0.39	0.30	126	175	20
273	69.97	0.54	9.88	5.81	0.39	0.30	122	174	20
274	69.43	0.54	9.88	5.77	0.39	0.30	119	174	20
275	68.89	0.54	9.87	5.73	0.39	0.30	116	174	20
276	68.35	—0.54	9.86	5.69	0.39	0.29	113	173	19
277	67.81	0.54	9.85	5.65	0.39	0.29	110	172	19
278	67.26	0.55	9.84	5.61	0.39	0.29	107	172	19
279	66.72	0.54	9.84	5.57	0.39	0.29	105	171	19
280	66.17	0.55	9.83	5.52	0.39	0.29	102	171	19
281	65.62	—0.55	9.82	5.48	0.39	0.29	100	170	19
282	65.07	0.55	9.81	5.44	0.39	0.29	97	170	18
283	64.52	0.55	9.80	5.40	0.39	0.29	95	169	18
284	63.97	0.55	9.79	5.35	0.39	0.29	93	169	18
285	63.41	0.56	9.78	5.31	0.39	0.28	91	168	18
286	62.86	—0.55	9.77	5.26	0.39	0.28	90	168	18
287	62.31	0.55	9.77	5.22	0.39	0.28	88	167	18
288	61.75	0.56	9.76	5.17	0.39	0.28	86	167	18
289	61.19	0.56	9.75	5.13	0.38	0.28	85	166	18
290	60.64	0.55	9.74	5.08	0.38	0.28	84	166	18
291	60.08	—0.56	9.73	5.03	0.38	0.27	82	165	19
292	59.52	0.56	9.72	4.99	0.38	0.27	81	164	19
293	58.96	0.56	9.71	4.94	0.38	0.27	80	164	19
294	58.40	0.56	9.70	4.89	0.38	0.27	80	163	19
295	57.84	0.56	9.69	4.84	0.38	0.27	79	163	19
296	57.28	—0.56	9.68	4.80	0.38	0.27	78	162	19
297	56.72	0.56	9.67	4.75	0.38	0.26	78	162	19
298	56.16	0.56	9.66	4.70	0.38	0.26	77	161	20
299	55.60	0.56	9.65	4.65	0.38	0.26	77	161	20
300	55.04	0.56	9.64	4.60	0.38	0.26	77	160	20

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(*ρ.c.*0)	(*ρ.s.*1)	(*ρ.c.*1)
	"	"	"	"	"	"			
300	55.04	−0.56	9.64	4.60	0.38	0.26	77	160	20
301	54.48	0.56	9.63	4.55	0.38	0.26	77	159	20
302	53.92	0.56	9.62	4.50	0.38	0.25	77	159	20
303	53.36	0.56	9.61	4.45	0.38	0.25	77	158	21
304	52.80	0.56	9.60	4.40	0.38	0.25	78	158	21
305	52.24	−0.56	9.59	4.35	0.38	0.25	78	157	21
306	51.68	0.56	9.58	4.30	0.38	0.25	79	156	22
307	51.12	0.56	9.57	4.25	0.38	0.24	80	156	22
308	50.56	0.56	9.56	4.20	0.38	0.24	81	155	22
309	50.00	0.56	9.55	4.15	0.38	0.24	82	154	23
310	49.44	−0.56	9.54	4.10	0.38	0.24	83	154	23
311	48.88	0.55	9.53	4.05	0.38	0.23	84	153	24
312	48.33	0.56	9.52	4.01	0.38	0.23	85	152	24
313	47.77	0.55	9.51	3.94	0.38	0.23	87	152	24
314	47.22	0.56	9.50	3.89	0.38	0.23	88	151	25
315	46.66	−0.55	9.49	3.84	0.38	0.23	90	150	25
316	46.11	0.55	9.48	3.79	0.38	0.22	92	150	26
317	45.56	0.55	9.47	3.74	0.38	0.22	94	149	26
318	45.01	0.55	9.46	3.69	0.38	0.22	96	148	27
319	44.46	0.55	9.45	3.63	0.38	0.22	98	148	27
320	43.91	−0.55	9.44	3.58	0.38	0.21	100	147	28
321	43.36	0.55	9.43	3.53	0.38	0.21	103	146	28
322	42.81	0.54	9.42	3.48	0.38	0.21	105	146	29
323	42.27	0.55	9.41	3.43	0.38	0.21	108	145	29
324	41.72	0.54	9.40	3.38	0.38	0.20	111	144	30
325	41.18	−0.54	9.40	3.33	0.38	0.20	114	144	30
326	40.64	0.54	9.39	3.28	0.38	0.20	117	143	31
327	40.10	0.54	9.38	3.23	0.38	0.20	120	142	32
328	39.56	0.53	9.37	3.18	0.38	0.19	123	141	32
329	39.03	0.54	9.36	3.14	0.37	0.19	127	140	33
330	38.49	−0.53	9.35	3.07	0.37	0.19	130	140	34
331	37.96	0.53	9.34	3.02	0.37	0.19	134	139	34
332	37.43	0.53	9.33	2.98	0.37	0.18	138	138	35
333	36.90	0.53	9.32	2.93	0.37	0.18	141	137	36
334	36.37	0.52	9.31	2.88	0.37	0.18	145	136	36
335	35.85	−0.52	9.30	2.83	0.37	0.18	150	136	37
336	35.33	0.52	9.29	2.78	0.37	0.17	154	135	38
337	34.81	0.52	9.29	2.73	0.37	0.17	158	134	38
338	34.29	0.51	9.28	2.69	0.37	0.17	162	133	39
339	33.78	0.52	9.27	2.64	0.37	0.17	167	132	40
340	33.26	−0.51	9.26	2.59	0.37	0.17	172	132	41
341	32.75	0.51	9.25	2.54	0.37	0.16	176	131	42
342	32.24	0.50	9.24	2.50	0.37	0.16	181	130	42
343	31.74	0.51	9.23	2.45	0.37	0.16	186	129	43
344	31.23	0.50	9.22	2.41	0.37	0.16	191	128	44
345	30.73	−0.49	9.21	2.36	0.37	0.15	197	127	45
346	30.24	0.50	9.20	2.31	0.37	0.15	202	126	46
347	29.74	0.49	9.20	2.27	0.37	0.15	207	126	46
348	29.25	0.49	9.19	2.23	0.37	0.15	213	125	47
349	28.76	0.49	9.18	2.18	0.37	0.15	218	124	48
350	28.27	−0.48	9.17	2.14	0.37	0.14	224	123	49
351	27.79	0.48	9.16	2.09	0.37	0.14	230	122	50
352	27.31	0.48	9.15	2.05	0.37	0.14	236	121	51
353	26.83	0.47	9.14	2.01	0.37	0.14	242	120	51
354	26.36	0.47	9.13	1.97	0.37	0.14	248	119	52
355	25.89	−0.47	9.12	1.93	0.37	0.14	255	118	53
356	25.42	0.46	9.12	1.89	0.37	0.13	261	117	54
357	24.96	0.46	9.11	1.85	0.37	0.13	268	116	55
358	24.50	0.46	9.10	1.81	0.37	0.13	274	115	56
359	24.04	0.46	9.09	1.77	0.37	0.13	281	114	57
360	23.58		9.08	1.73	0.38	0.13	288	113	58

TABLE VIII, ARG. 1.—*Continued.*

Arg.	($v.c.0$)	Diff.	($v.s.1$)	($v.c.1$)	($v.s.2$)	($v.c.2$)	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)
	"	"	"	"	"	"			
360	23.58		9.08	1.73	0.38	0.13	288	113	58
361	23.13	—0.45	9.07	1.70	0.38	0.13	294	112	58
362	22.69	0.44	9.06	1.66	0.38	0.13	301	111	59
363	22.24	0.45	9.05	1.62	0.38	0.13	308	110	60
364	21.80	0.44	9.04	1.59	0.38	0.12	316	109	61
365	21.37	0.43	9.03	1.55	0.37	0.12	323	108	62
366	20.94	—0.43	9.02	1.52	0.37	0.12	330	107	63
367	20.51	0.43	9.01	1.48	0.37	0.12	338	106	64
368	20.08	0.43	9.00	1.45	0.37	0.12	345	105	65
369	19.66	0.42	8.99	1.42	0.37	0.11	353	104	66
370	19.24	0.42	8.98	1.39	0.37	0.11	361	103	66
371	18.83	—0.41	8.97	1.36	0.37	0.11	369	102	67
372	18.42	0.41	8.96	1.32	0.37	0.11	377	101	68
373	18.02	0.40	8.95	1.29	0.37	0.11	385	100	69
374	17.62	0.40	8.94	1.27	0.37	0.11	393	99	70
375	17.22	0.40	8.93	1.24	0.37	0.11	401	98	71
376	16.83	—0.39	8.92	1.21	0.37	0.11	410	97	72
377	16.44	0.39	8.91	1.18	0.37	0.11	418	96	73
378	16.06	0.38	8.89	1.16	0.37	0.11	426	95	74
379	15.68	0.38	8.88	1.13	0.37	0.11	435	94	75
380	15.30	0.38	8.87	1.11	0.37	0.10	444	93	76
381	14.93	—0.37	8.86	1.08	0.37	0.10	452	92	77
382	14.57	0.36	8.85	1.06	0.37	0.10	461	91	78
383	14.21	0.36	8.83	1.04	0.37	0.10	470	90	78
384	13.85	0.36	8.82	1.02	0.37	0.10	479	89	79
385	13.50	0.35	8.81	1.00	0.37	0.10	488	88	80
386	13.15	—0.35	8.79	0.98	0.37	0.10	497	87	81
387	12.80	0.35	8.78	0.96	0.37	0.10	506	86	82
388	12.47	0.33	8.77	0.94	0.37	0.10	514	85	83
389	12.13	0.34	8.75	0.92	0.37	0.10	525	84	84
390	11.81	0.32	8.74	0.90	0.37	0.10	534	82	85
391	11.48	—0.33	8.72	0.88	0.37	0.10	544	81	86
392	11.16	0.32	8.71	0.87	0.37	0.10	554	80	86
393	10.85	0.31	8.70	0.85	0.36	0.10	564	79	87
394	10.54	0.31	8.68	0.84	0.36	0.10	573	78	88
395	10.23	0.31	8.67	0.83	0.36	0.10	583	77	89
396	9.93	—0.30	8.65	0.82	0.36	0.10	593	76	90
397	9.64	0.29	8.63	0.80	0.36	0.10	603	75	91
398	9.35	0.29	8.62	0.79	0.36	0.10	613	74	92
399	9.07	0.28	8.60	0.78	0.36	0.10	623	73	93
400	8.79	0.28	8.58	0.77	0.36	0.10	633	72	93
401	8.52	—0.27	8.57	0.76	0.36	0.10	643	71	94
402	8.25	0.27	8.55	0.76	0.36	0.10	653	70	95
403	7.98	0.27	8.53	0.75	0.36	0.10	664	69	96
404	7.73	0.25	8.51	0.74	0.35	0.10	674	68	97
405	7.47	0.26	8.49	0.74	0.35	0.10	684	67	98
406	7.22	—0.25	8.47	0.74	0.35	0.10	695	66	98
407	6.98	0.24	8.46	0.73	0.35	0.10	706	65	99
408	6.75	0.23	8.44	0.73	0.35	0.10	716	64	100
409	6.51	0.24	8.42	0.73	0.34	0.10	727	63	101
410	6.29	0.22	8.39	0.73	0.34	0.10	737	62	101
411	6.07	—0.22	8.37	0.73	0.34	0.10	748	61	102
412	5.85	0.22	8.35	0.73	0.34	0.10	759	60	103
413	5.64	0.21	8.33	0.73	0.34	0.10	770	59	104
414	5.44	0.20	8.31	0.73	0.34	0.10	781	58	104
415	5.24	0.20	8.28	0.73	0.34	0.10	792	57	105
416	5.04	—0.20	8.26	0.74	0.33	0.10	803	56	106
417	4.86	0.18	8.24	0.74	0.33	0.10	814	55	107
418	4.67	0.19	8.21	0.74	0.33	0.10	825	54	107
419	4.50	0.17	8.19	0.75	0.33	0.10	836	53	108
420	4.33	0.17	8.17	0.76	0.33	0.10	847	52	109

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	"	"	"	"	"	"			
420	4.33	−0.17	8.17	0.76	0.33	0.10	847	52	109
421	4.16	0.16	8.14	0.76	0.32	0.10	858	51	110
422	4.00	0.15	8.11	0.77	0.32	0.11	870	50	110
423	3.85	0.15	8.09	0.78	0.32	0.11	881	50	111
424	3.70	0.14	8.06	0.79	0.32	0.11	892	49	111
425	3.56	−0.14	8.04	0.80	0.32	0.11	904	48	112
426	3.42	0.13	8.01	0.81	0.31	0.11	916	47	113
427	3.29	0.13	7.98	0.82	0.31	0.11	927	46	113
428	3.16	0.12	7.95	0.83	0.31	0.11	938	45	114
429	3.04	0.11	7.92	0.85	0.31	0.11	950	44	114
430	2.93	−0.11	7.89	0.86	0.31	0.11	961	44	115
431	2.82	0.10	7.86	0.87	0.30	0.11	973	43	116
432	2.72	0.10	7.83	0.89	0.30	0.11	984	42	116
433	2.62	0.09	7.80	0.90	0.30	0.11	996	41	117
434	2.53	0.08	7.77	0.92	0.29	0.11	1008	40	118
435	2.45	−0.08	7.74	0.94	0.29	0.11	1019	40	118
436	2.37	0.08	7.71	0.95	0.29	0.12	1031	39	119
437	2.29	0.06	7.67	0.97	0.29	0.12	1043	38	119
438	2.23	0.07	7.64	0.99	0.28	0.12	1054	38	120
439	2.16	0.05	7.60	1.01	0.28	0.12	1066	37	120
440	2.11	−0.05	7.57	1.03	0.28	0.12	1078	36	121
441	2.06	0.04	7.54	1.05	0.28	0.12	1089	35	121
442	2.02	0.04	7.50	1.07	0.27	0.12	1100	35	122
443	1.98	0.03	7.47	1.09	0.27	0.12	1113	34	122
444	1.95	0.03	7.43	1.11	0.27	0.12	1124	33	122
445	1.92	−0.02	7.39	1.14	0.26	0.12	1137	33	123
446	1.90	0.02	7.36	1.16	0.26	0.12	1149	32	123
447	1.88	0.01	7.32	1.18	0.26	0.12	1160	32	124
448	1.87	0.00	7.28	1.21	0.26	0.12	1172	31	124
449	1.87	0.00	7.24	1.23	0.25	0.12	1184	30	125
450	1.87	+0.01	7.20	1.26	0.25	0.12	1196	30	125
451	1.88	0.02	7.16	1.28	0.25	0.12	1208	30	125
452	1.90	0.02	7.12	1.31	0.24	0.12	1220	29	126
453	1.92	0.03	7.08	1.33	0.24	0.12	1232	28	126
454	1.95	0.03	7.04	1.36	0.24	0.12	1243	28	127
455	1.98	+0.03	7.00	1.39	0.23	0.12	1255	28	127
456	2.01	0.05	6.96	1.42	0.23	0.12	1267	27	127
457	2.06	0.05	6.91	1.45	0.23	0.12	1279	27	128
458	2.11	0.05	6.87	1.47	0.22	0.12	1291	26	128
459	2.16	0.07	6.83	1.50	0.22	0.12	1302	26	128
460	2.23	+0.06	6.78	1.53	0.22	0.12	1314	26	128
461	2.29	0.08	6.74	1.56	0.21	0.12	1326	25	129
462	2.37	0.07	6.69	1.59	0.21	0.12	1338	25	129
463	2.44	0.09	6.65	1.62	0.21	0.12	1350	25	129
464	2.53	0.09	6.60	1.65	0.20	0.12	1361	25	130
465	2.62	+0.09	6.56	1.68	0.20	0.12	1373	24	130
466	2.71	0.11	6.51	1.72	0.20	0.12	1385	24	130
467	2.82	0.10	6.46	1.75	0.19	0.13	1396	24	130
468	2.92	0.12	6.41	1.78	0.19	0.13	1408	24	130
469	3.04	0.11	6.37	1.81	0.19	0.13	1420	23	131
470	3.15	+0.13	6.32	1.84	0.18	0.13	1431	23	131
471	3.28	0.13	6.27	1.88	0.18	0.13	1443	23	131
472	3.41	0.14	6.22	1.91	0.18	0.12	1455	23	131
473	3.55	0.14	6.17	1.94	0.17	0.12	1466	23	131
474	3.69	0.14	6.12	1.98	0.17	0.12	1478	23	132
475	3.83	+0.15	6.07	2.01	0.17	0.12	1489	23	132
476	3.98	0.16	6.02	2.04	0.16	0.12	1501	23	132
477	4.14	0.17	5.96	2.07	0.16	0.12	1512	23	132
478	4.31	0.17	5.91	2.11	0.16	0.12	1524	23	132
479	4.48	0.17	5.86	2.14	0.15	0.12	1535	23	132
480	4.65		5.81	2.18	0.15	0.12	1546	23	132

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
480	4.65	+0.18	5.81	2.18	0.15	0.12	1546	23	132
481	4.83	0.19	5.75	2.21	0.15	0.12	1558	23	132
482	5.02	0.19	5.70	2.24	0.15	0.12	1569	23	132
483	5.21	0.19	5.65	2.28	0.14	0.12	1580	23	132
484	5.40	0.21	5.59	2.31	0.14	0.12	1591	23	133
485	5.61	+0.20	5.54	2.35	0.14	0.12	1602	24	133
486	5.81	0.22	5.48	2.38	0.13	0.12	1613	24	133
487	6.03	0.21	5.43	2.42	0.13	0.12	1624	24	133
488	6.24	0.23	5.37	2.45	0.13	0.11	1635	24	134
489	6.47	0.23	5.32	2.48	0.12	0.11	1646	25	134
490	6.70	+0.23	5.26	2.52	0.12	0.11	1657	25	133
491	6.93	0.24	5.20	2.55	0.12	0.11	1668	25	133
492	7.17	0.24	5.15	2.59	0.12	0.11	1679	25	133
493	7.41	0.25	5.09	2.62	0.11	0.11	1690	25	133
494	7.66	0.26	5.03	2.66	0.11	0.11	1700	26	133
495	7.92	+0.26	4.98	2.69	0.11	0.11	1711	27	133
496	8.18	0.26	4.92	2.72	0.11	0.11	1721	27	133
497	8.44	0.27	4.86	2.76	0.10	0.11	1732	28	133
498	8.71	0.28	4.80	2.79	0.10	0.11	1742	28	133
499	8.99	0.27	4.75	2.83	0.10	0.11	1753	28	133
500	9.26	+0.29	4.69	2.86	0.10	0.11	1763	29	133
501	9.55	0.29	4.63	2.89	0.10	0.11	1774	30	133
502	9.84	0.29	4.57	2.93	0.09	0.10	1784	30	133
503	10.13	0.30	4.51	2.96	0.09	0.10	1794	31	133
504	10.43	0.31	4.44	2.99	0.09	0.10	1804	31	133
505	10.74	+0.31	4.39	3.02	0.09	0.10	1814	32	133
506	11.05	0.31	4.33	3.06	0.09	0.10	1824	32	133
507	11.36	0.32	4.28	3.09	0.08	0.10	1834	33	133
508	11.68	0.32	4.22	3.12	0.08	0.10	1844	34	133
509	12.00	0.33	4.16	3.15	0.08	0.10	1853	35	133
510	12.33	+0.33	4.10	3.18	0.08	0.09	1863	35	133
511	12.66	0.34	4.04	3.21	0.08	0.09	1873	36	133
512	13.00	0.34	3.98	3.25	0.07	0.09	1882	37	133
513	13.34	0.34	3.92	3.28	0.07	0.09	1892	38	133
514	13.68	0.35	3.86	3.31	0.07	0.09	1901	39	133
515	14.03	+0.36	3.80	3.34	0.07	0.09	1910	40	133
516	14.39	0.36	3.74	3.37	0.07	0.09	1920	40	133
517	14.75	0.36	3.68	3.40	0.07	0.09	1929	41	133
518	15.11	0.37	3.62	3.43	0.07	0.09	1938	42	133
519	15.48	0.37	3.56	3.46	0.07	0.08	1947	43	133
520	15.85	+0.38	3.50	3.49	0.06	0.08	1956	44	132
521	16.23	0.38	3.44	3.51	0.06	0.08	1964	45	132
522	16.61	0.38	3.38	3.54	0.06	0.08	1973	46	132
523	16.99	0.39	3.32	3.57	0.06	0.08	1982	47	132
524	17.38	0.39	3.27	3.60	0.06	0.08	1990	48	132
525	17.77	+0.40	3.21	3.62	0.06	0.08	1999	49	132
526	18.17	0.40	3.15	3.65	0.06	0.08	2007	50	132
527	18.57	0.40	3.09	3.67	0.06	0.08	2016	51	132
528	18.97	0.41	3.03	3.70	0.06	0.07	2024	52	132
529	19.38	0.41	2.97	3.73	0.05	0.07	2032	53	132
530	19.79	+0.42	2.91	3.75	0.05	0.07	2040	55	132
531	20.21	0.41	2.86	3.77	0.05	0.07	2048	56	132
532	20.62	0.43	2.80	3.80	0.05	0.07	2056	57	132
533	21.05	0.42	2.75	3.82	0.05	0.07	2063	58	132
534	21.47	0.43	2.69	3.84	0.05	0.07	2071	59	131
535	21.90	+0.44	2.63	3.87	0.05	0.07	2079	61	131
536	22.34	0.43	2.57	3.89	0.05	0.07	2086	62	131
537	22.77	0.44	2.52	3.91	0.05	0.07	2094	63	131
538	23.21	0.45	2.46	3.93	0.05	0.07	2101	64	131
539	23.66	0.44	2.40	3.96	0.05	0.06	2108	66	131
540	24.10		2.35	3.98	0.05	0.06	2115	67	131

TABLE VIII, Arg. 1.—*Concluded.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	"	"	"	"	"	"			
540	24.10	+0.46	2.35	3.98	0.05	0.06	2115	67	131
541	24.56	0.45	2.29	4.00	0.05	0.06	2122	68	131
542	25.01	0.46	2.24	4.02	0.05	0.06	2129	70	131
543	25.47	0.46	2.18	4.04	0.05	0.06	2135	71	130
544	25.93	0.46	2.13	4.06	0.05	0.06	2142	73	130
545	26.39	+0.46	2.08	4.07	0.05	0.06	2149	74	130
546	26.85	0.47	2.02	4.09	0.05	0.06	2155	75	130
547	27.32	0.47	1.97	4.11	0.05	0.06	2162	77	130
548	27.79	0.48	1.92	4.13	0.05	0.06	2168	78	130
549	28.27	0.48	1.87	4.14	0.05	0.06	2174	80	130
550	28.75	+0.48	1.82	4.16	0.05	0.06	2180	81	130
551	29.23	0.48	1.77	4.17	0.05	0.06	2186	83	130
552	29.71	0.49	1.72	4.19	0.06	0.06	2191	84	130
553	30.20	0.49	1.67	4.20	0.06	0.06	2197	86	130
554	30.69	0.49	1.62	4.22	0.06	0.06	2203	87	130
555	31.18	+0.49	1.57	4.23	0.06	0.06	2208	89	130
556	31.67	0.49	1.52	4.25	0.06	0.06	2214	90	130
557	32.16	0.50	1.48	4.26	0.06	0.06	2219	92	130
558	32.66	0.50	1.43	4.27	0.06	0.06	2224	94	130
559	33.16	0.51	1.38	4.28	0.06	0.06	2229	95	130
560	33.67	+0.50	1.34	4.29	0.06	0.06	2234	97	130
561	34.17	0.51	1.29	4.31	0.06	0.06	2238	98	130
562	34.68	0.51	1.25	4.32	0.07	0.06	2243	100	130
563	35.19	0.51	1.20	4.32	0.07	0.06	2248	101	130
564	35.70	0.51	1.16	4.33	0.07	0.06	2252	103	130
565	36.21	+0.52	1.12	4.34	0.07	0.06	2256	105	130
566	36.73	0.52	1.08	4.35	0.07	0.06	2261	106	130
567	37.25	0.51	1.03	4.36	0.07	0.06	2265	108	130
568	37.76	0.53	0.99	4.37	0.07	0.06	2269	110	130
569	38.29	0.52	0.96	4.38	0.07	0.06	2272	111	130
570	38.81	+0.53	0.92	4.38	0.07	0.06	2276	113	130
571	39.34	0.52	0.88	4.39	0.07	0.06	2280	115	130
572	39.86	0.53	0.84	4.40	0.07	0.07	2283	116	130
573	40.39	0.53	0.80	4.40	0.08	0.07	2287	118	130
574	40.92	0.53	0.77	4.41	0.08	0.07	2290	120	131
575	41.45	+0.53	0.73	4.41	0.08	0.07	2293	121	131
576	41.98	0.54	0.70	4.42	0.08	0.07	2296	123	131
577	42.52	0.53	0.67	4.42	0.08	0.07	2299	125	131
578	43.05	0.54	0.63	4.42	0.08	0.07	2302	126	131
579	43.59	0.53	0.60	4.43	0.08	0.07	2304	128	131
580	44.12	+0.54	0.57	4.43	0.08	0.07	2307	130	131
581	44.66	0.54	0.54	4.43	0.08	0.08	2309	132	131
582	45.20	0.54	0.51	4.44	0.09	0.08	2312	133	131
583	45.74	0.54	0.48	4.44	0.09	0.08	2314	135	131
584	46.28	0.55	0.46	4.44	0.09	0.08	2316	137	132
585	46.83	+0.54	0.43	4.44	0.09	0.08	2318	138	132
586	47.37	0.54	0.40	4.44	0.09	0.08	2319	140	132
587	47.91	0.55	0.38	4.44	0.09	0.09	2321	142	132
588	48.46	0.54	0.36	4.44	0.09	0.09	2322	144	132
589	49.00	0.55	0.33	4.44	0.09	0.09	2324	145	132
590	49.55	+0.55	0.31	4.44	0.09	0.09	2325	147	132
591	50.10	0.54	0.29	4.44	0.09	0.09	2326	149	132
592	50.64	0.55	0.27	4.44	0.10	0.09	2327	150	133
593	51.19	0.55	0.25	4.44	0.10	0.10	2328	152	133
594	51.74	0.55	0.23	4.43	0.10	0.10	2329	154	133
595	52.29	+0.55	0.22	4.43	0.10	0.10	2330	155	133
596	52.84	0.54	0.20	4.43	0.10	0.10	2330	157	133
597	53.38	0.55	0.18	4.43	0.10	0.11	2330	159	133
598	53.93	0.55	0.17	4.43	0.10	0.11	2331	161	134
599	54.48	0.55	0.16	4.42	0.10	0.11	2331	162	134
600	55.03		0.14	4.42	0.10	0.11	2331	164	134

TABLE IX, Arg. 2.—Action of Saturn.

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec. var.	(v.c.1)	Diff.	Sec. var.	(v.s.2)	Diff.	Sec. var.	(v.c.2)	Diff.	Sec. var.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
0	38.54	+0.09	139.94	+1.44	2.53	293.78	+0.03	0.56	182.62	+1.18	1.77	244.67	−0.57	0.12
1	38.63	0.09	141.38	1.44	2.52	293.81	+0.02	0.55	183.80	1.16	1.76	244.10	0.58	0.12
2	38.72	0.09	142.82	1.44	2.50	293.83	0.00	0.54	184.96	1.16	1.74	243.52	0.59	0.12
3	38.81	0.09	144.26	1.44	2.49	293.83	−0.01	0.53	186.12	1.15	1.73	242.93	0.61	0.11
4	38.90	0.08	145.70	1.44	2.48	293.82	0.03	0.52	187.27	1.15	1.71	242.32	0.61	0.11
5	38.98	+0.09	147.14	+1.44	2.47	293.79	−0.04	0.51	188.42	+1.14	1.70	241.71	−0.62	0.11
6	39.07	0.09	148.58	1.43	2.46	293.75	0.05	0.50	189.56	1.13	1.69	241.09	0.63	0.11
7	39.16	0.08	150.01	1.44	2.45	293.70	0.07	0.49	190.69	1.13	1.67	240.46	0.65	0.11
8	39.24	0.09	151.45	1.44	2.44	293.63	0.09	0.48	191.82	1.12	1.66	239.81	0.66	0.10
9	39.33	0.08	152.89	1.43	2.43	293.54	0.10	0.47	192.94	1.11	1.64	239.15	0.67	0.10
10	39.41	+0.09	154.32	+1.44	2.42	293.44	−0.11	0.46	194.05	+1.11	1.63	238.48	−0.68	0.10
11	39.50	0.08	155.76	1.43	2.41	293.33	0.13	0.45	195.16	1.10	1.61	237.80	0.69	0.10
12	39.58	0.09	157.19	1.43	2.40	293.20	0.14	0.44	196.26	1.09	1.60	237.11	0.70	0.10
13	39.67	0.08	158.62	1.43	2.39	293.06	0.16	0.43	197.35	1.08	1.58	236.41	0.72	0.10
14	39.75	0.09	160.05	1.43	2.38	292.90	0.17	0.42	198.43	1.08	1.57	235.69	0.73	0.10
15	39.84	+0.08	161.48	+1.42	2.36	292.73	−0.18	0.42	199.51	+1.07	1.55	234.96	−0.74	0.10
16	39.92	0.09	162.90	1.42	2.35	292.55	0.21	0.41	200.58	1.06	1.53	234.22	0.74	0.09
17	40.01	0.09	164.32	1.42	2.34	292.34	0.21	0.40	201.64	1.05	1.52	233.48	0.75	0.09
18	40.10	0.08	165.74	1.42	2.33	292.13	0.23	0.39	202.69	1.05	1.50	232.73	0.77	0.09
19	40.18	0.09	167.16	1.42	2.32	291.90	0.25	0.38	203.74	1.04	1.49	231.96	0.78	0.09
20	40.27	+0.08	168.58	+1.41	2.31	291.65	−0.26	0.37	204.78	+1.03	1.47	231.18	−0.79	0.09
21	40.35	0.09	169.99	1.41	2.30	291.39	0.27	0.36	205.81	1.02	1.46	230.39	0.79	0.09
22	40.44	0.08	171.40	1.41	2.29	291.12	0.29	0.35	206.83	1.01	1.44	229.60	0.80	0.09
23	40.52	0.10	172.81	1.41	2.27	290.83	0.30	0.35	207.84	1.01	1.43	228.80	0.82	0.10
24	40.62	0.09	174.22	1.40	2.26	290.53	0.31	0.34	208.85	1.00	1.41	227.98	0.83	0.10
25	40.71	+0.09	175.62	+1.40	2.25	290.22	−0.33	0.33	209.85	+0.99	1.40	227.15	−0.84	0.10
26	40.80	0.09	177.02	1.40	2.24	289.89	0.34	0.32	210.84	0.98	1.39	226.31	0.85	0.10
27	40.89	0.09	178.42	1.39	2.23	289.55	0.36	0.31	211.82	0.97	1.37	225.46	0.86	0.10
28	40.98	0.09	179.81	1.39	2.21	289.19	0.37	0.30	212.79	0.97	1.36	224.60	0.86	0.11
29	41.07	0.09	181.20	1.39	2.20	288.82	0.38	0.30	213.76	0.95	1.34	223.74	0.87	0.11
30	41.16	+0.09	182.59	+1.38	2.19	288.44	−0.40	0.29	214.71	+0.95	1.33	222.87	−0.88	0.11
31	41.25	0.10	183.97	1.38	2.18	288.04	0.41	0.28	215.66	0.94	1.32	221.99	0.89	0.11
32	41.35	0.09	185.35	1.38	2.17	287.63	0.43	0.28	216.60	0.93	1.30	221.10	0.90	0.11
33	41.44	0.10	186.73	1.37	2.15	287.20	0.44	0.27	217.53	0.91	1.29	220.20	0.92	0.12
34	41.54	0.10	188.10	1.37	2.14	286.76	0.46	0.26	218.44	0.91	1.27	219.28	0.92	0.12
35	41.64	+0.10	189.47	+1.36	2.13	286.30	−0.46	0.26	219.35	+0.90	1.26	218.36	−0.93	0.12
36	41.74	0.09	190.83	1.36	2.11	285.84	0.49	0.25	220.25	0.89	1.25	217.43	0.94	0.12
37	41.83	0.10	192.19	1.36	2.10	285.35	0.49	0.24	221.14	0.89	1.23	216.49	0.94	0.12
38	41.93	0.10	193.55	1.35	2.09	284.86	0.51	0.23	222.03	0.87	1.22	215.55	0.96	0.13
39	42.03	0.10	194.90	1.35	2.07	284.35	0.51	0.23	222.90	0.86	1.20	214.59	0.97	0.13
40	42.13	+0.11	196.25	+1.34	2.06	283.84	−0.53	0.22	223.76	+0.85	1.19	213.62	−0.97	0.13
41	42.24	0.10	197.59	1.34	2.05	283.31	0.55	0.22	224.61	0.84	1.18	212.65	0.98	0.13
42	42.34	0.10	198.93	1.33	2.03	282.76	0.56	0.21	225.45	0.83	1.16	211.67	0.99	0.14
43	42.44	0.11	200.26	1.33	2.02	282.20	0.57	0.21	226.28	0.83	1.15	210.68	1.00	0.14
44	42.55	0.11	201.59	1.32	2.00	281.63	0.59	0.20	227.11	0.81	1.13	209.68	1.01	0.14
45	42.66	+0.11	202.91	+1.32	1.99	281.04	−0.60	0.20	227.92	+0.80	1.12	208.67	−1.01	0.15
46	42.77	0.10	204.23	1.31	1.97	280.44	0.62	0.19	228.72	0.79	1.10	207.66	1.02	0.15
47	42.87	0.11	205.54	1.31	1.96	279.82	0.62	0.19	229.51	0.78	1.09	206.64	1.03	0.15
48	42.98	0.11	206.85	1.30	1.94	279.20	0.64	0.18	230.29	0.78	1.07	205.61	1.04	0.15
49	43.09	0.11	208.15	1.29	1.93	278.56	0.65	0.18	231.07	0.76	1.06	204.57	1.05	0.16
50	43.20	+0.12	209.44	+1.29	1.92	277.91	−0.67	0.17	231.83	+0.75	1.04	203.52	−1.05	0.16
51	43.32	0.11	210.73	1.28	1.91	277.24	0.68	0.17	232.58	0.74	1.03	202.47	1.06	0.17
52	43.43	0.11	212.01	1.28	1.89	276.56	0.69	0.16	233.32	0.73	1.01	201.41	1.07	0.17
53	43.54	0.11	213.29	1.28	1.88	275.87	0.70	0.16	234.05	0.72	1.00	200.34	1.08	0.18
54	43.65	0.12	214.57	1.27	1.87	275.17	0.72	0.16	234.77	0.71	0.99	199.26	1.08	0.18
55	43.77	+0.11	215.84	+1.26	1.85	274.45	−0.73	0.15	235.48	+0.70	0.97	198.18	−1.09	0.19
56	43.88	0.12	217.10	1.25	1.84	273.72	0.74	0.15	236.18	0.69	0.96	197.09	1.09	0.20
57	44.00	0.12	218.35	1.25	1.82	272.98	0.75	0.15	236.87	0.67	0.95	196.00	1.10	0.20
58	44.12	0.12	219.60	1.24	1.81	272.23	0.77	0.15	237.54	0.66	0.94	194.90	1.11	0.21
59	44.24	0.11	220.84	1.23	1.79	271.46	0.77	0.14	238.20	0.65	0.92	193.79	1.12	0.21
60	44.35		222.07		1.78	270.69		0.14	238.85		0.91	192.67		0.22

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″	″							
0	11.40	14.53	1.58	1.68	1678	182	1513	711	306	137	85
1	11.52	14.48	1.59	1.66	1679	182	1526	711	302	137	85
2	11.64	14.43	1.60	1.65	1680	183	1539	710	298	137	85
3	11.75	14.38	1.61	1.64	1680	184	1551	709	295	137	85
4	11.87	14.33	1.62	1.63	1681	185	1564	708	291	137	85
5	11.98	14.28	1.63	1.62	1682	187	1577	707	288	137	85
6	12.10	14.22	1.64	1.61	1682	188	1590	706	284	137	85
7	12.21	14.16	1.64	1.60	1683	190	1603	704	281	138	84
8	12.32	14.10	1.65	1.59	1683	191	1615	703	278	138	84
9	12.43	14.04	1.66	1.58	1682	193	1628	701	274	138	84
10	12.54	13.97	1.67	1.56	1682	195	1641	700	271	138	84
11	12.65	13.91	1.68	1.55	1681	197	1654	699	267	138	84
12	12.76	13.84	1.69	1.54	1680	199	1666	698	264	138	84
13	12.87	13.77	1.69	1.53	1679	201	1679	696	260	138	84
14	12.97	13.70	1.70	1.52	1678	203	1691	695	257	137	83
15	13.07	13.63	1.71	1.50	1677	206	1704	694	253	137	83
16	13.18	13.56	1.71	1.49	1675	208	1716	692	250	137	83
17	13.28	13.49	1.72	1.48	1674	211	1729	691	246	137	83
18	13.38	13.41	1.73	1.47	1672	214	1741	689	243	137	82
19	13.48	13.33	1.74	1.45	1670	217	1754	687	239	137	82
20	13.58	13.25	1.74	1.44	1668	220	1766	685	236	137	82
21	13.68	13.17	1.75	1.43	1666	223	1778	683	233	137	82
22	13.78	13.09	1.75	1.42	1664	227	1790	681	230	137	81
23	13.87	13.01	1.76	1.40	1662	230	1802	679	226	137	81
24	13.97	12.92	1.76	1.39	1660	234	1814	677	223	137	81
25	14.06	12.83	1.77	1.37	1658	237	1826	675	220	137	81
26	14.15	12.74	1.78	1.36	1656	241	1838	673	217	137	81
27	14.24	12.65	1.78	1.35	1653	245	1850	671	214	137	80
28	14.33	12.56	1.79	1.34	1651	249	1862	668	211	137	80
29	14.41	12.47	1.79	1.32	1648	253	1874	666	208	137	80
30	14.50	12.37	1.79	1.31	1645	258	1886	664	205	137	79
31	14.58	12.28	1.80	1.30	1642	262	1898	662	202	137	78
32	14.66	12.19	1.80	1.28	1638	267	1910	659	199	137	78
33	14.74	12.09	1.80	1.27	1635	272	1922	657	196	137	78
34	14.82	11.99	1.81	1.25	1631	277	1934	654	193	137	78
35	14.90	11.89	1.81	1.24	1627	282	1946	652	190	137	77
36	14.97	11.79	1.81	1.22	1623	287	1958	650	187	137	77
37	15.05	11.69	1.81	1.21	1619	292	1970	647	184	137	77
38	15.12	11.59	1.82	1.20	1615	297	1982	645	181	137	76
39	15.19	11.49	1.82	1.18	1611	302	1993	642	179	137	76
40	15.26	11.38	1.82	1.17	1608	308	2005	640	176	137	76
41	15.33	11.28	1.82	1.15	1602	313	2016	637	173	137	76
42	15.39	11.17	1.82	1.14	1597	319	2028	635	171	137	76
43	15.46	11.06	1.82	1.13	1592	325	2039	632	168	137	75
44	15.52	10.95	1.82	1.11	1587	331	2051	630	166	137	74
45	15.58	10.84	1.82	1.10	1582	337	2062	627	163	137	74
46	15.64	10.73	1.82	1.08	1577	343	2073	624	160	137	74
47	15.70	10.62	1.82	1.07	1572	349	2084	621	158	137	73
48	15.76	10.51	1.82	1.06	1566	355	2095	619	155	137	73
49	15.81	10.40	1.82	1.04	1561	362	2106	616	153	137	73
50	15.86	10.28	1.82	1.03	1556	368	2117	613	150	137	72
51	15.91	10.17	1.82	1.01	1550	375	2128	610	148	137	72
52	15.96	10.05	1.82	1.00	1545	381	2138	607	145	137	72
53	16.01	9.94	1.81	0.98	1539	388	2149	604	143	137	72
54	16.05	9.82	1.81	0.97	1533	394	2159	601	140	137	72
55	16.09	9.70	1.81	0.96	1527	401	2170	598	138	137	71
56	16.13	9.59	1.81	0.94	1521	408	2180	595	135	137	71
57	16.17	9.47	1.80	0.93	1515	415	2191	592	133	137	71
58	16.20	9.35	1.80	0.92	1509	422	2201	589	131	137	71
59	16.23	9.23	1.80	0.90	1502	430	2212	586	129	137	71
60	16.26	9.11	1.80	0.89	1496	437	2222	583	127	137	71

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec. var.	(*v.c.*1)	Diff.	Sec. var.	(*v.s.*2)	Diff.	Sec. var.	(*v.c.*2)	Diff.	Sec. var.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
60	44.35	+0.12	222.07	+1.23	1.78	270.69	−0.80	0.14	238.85	+0.64	0.91	192.67	−1.12	0.22
61	44.47	0.12	223.30	1.22	1.76	269.89	0.80	0.13	239.49	0.63	0.90	191.55	1.13	0.23
62	44.59	0.12	224.52	1.21	1.75	269.09	0.81	0.13	240.12	0.62	0.98	190.42	1.13	0.23
63	44.71	0.12	225.73	1.21	1.73	268.28	0.83	0.13	240.74	0.61	0.87	189.29	1.15	0.24
64	44.83	0.12	226.94	1.20	1.72	267.45	0.84	0.13	241.35	0.60	0.86	188.14	1.15	0.25
65	44.95	+0.12	228.14	+1.19	1.70	266.61	−0.85	0.12	241.95	+0.58	0.84	186.99	−1.16	0.25
66	45.07	0.12	229.33	1.18	1.69	265.76	0.87	0.12	242.53	0.57	0.83	185.83	1.16	0.26
67	45.19	0.12	230.51	1.18	1.67	264.89	0.87	0.12	243.10	0.57	0.82	184.67	1.16	0.27
68	45.31	0.13	231.69	1.17	1.66	264.02	0.89	0.12	243.67	0.55	0.81	183.51	1.17	0.28
69	45.44	0.12	232.86	1.16	1.64	263.13	0.90	0.11	244.22	0.54	0.79	182.34	1.18	0.28
70	45.56	+0.12	234.02	+1.15	1.63	262.23	−0.91	0.11	244.76	+0.52	0.78	181.16	−1.19	0.29
71	45.68	0.12	235.17	1.14	1.62	261.32	0.93	0.11	245.28	0.51	0.77	179.97	1.19	0.30
72	45.80	0.12	236.31	1.14	1.60	260.39	0.93	0.11	245.79	0.50	0.75	178.78	1.19	0.31
73	45.92	0.12	237.45	1.13	1.59	259.46	0.94	0.11	246.29	0.49	0.74	177.59	1.19	0.32
74	46.04	0.12	238.58	1.12	1.57	258.52	0.96	0.11	246.78	0.48	0.73	176.40	1.20	0.33
75	46.16	+0.13	239.70	+1.11	1.56	257.56	−0.97	0.11	247.26	+0.47	0.71	175.20	−1.21	0.34
76	46.29	0.12	240.81	1.10	1.55	256.59	0.98	0.11	247.73	0.45	0.70	173.99	1.21	0.34
77	46.41	0.12	241.91	1.09	1.53	255.61	0.99	0.11	248.18	9.44	0.69	172.78	1.22	0.35
78	46.53	0.12	243.00	1.09	1.52	254.62	1.00	0.11	248.62	0.43	0.68	171.56	1.22	0.36
79	46.65	0.12	244.09	1.08	1.50	253.62	1.01	0.11	249.05	0.42	0.66	170.34	1.23	0.37
80	46.77	+0.12	245.17	+1.07	1.49	252.61	−1.02	0.11	249.47	+0.40	0.65	169.11	−1.23	0.38
81	46.89	0.12	246.24	1.06	1.48	251.59	1.03	0.11	249.87	0.39	0.64	167.88	1.23	0.39
82	47.01	0.12	247.30	1.05	1.46	250.56	1.05	0.11	250.26	0.38	0.63	166.65	1.24	0.40
83	47.13	0.11	248.35	1.03	1.45	249.51	1.06	0.11	250.64	0.37	0.62	165.41	1.24	0.41
84	47.24	0.12	249.38	1.03	1.43	248.45	1.07	0.11	251.01	0.36	0.61	164.17	1.25	0.42
85	47.36	+0.12	250.41	+1.02	1.42	247.38	−1.08	0.11	251.37	+0.34	0.60	162.92	−1.25	0.43
86	47.48	0.11	251.43	1.01	1.40	246.30	1.09	0.12	251.71	0.33	0.58	161.67	1.25	0.43
87	47.59	0.12	252.44	1.00	1.39	245.21	1.09	0.12	252.04	0.32	0.57	160.42	1.26	0.44
88	47.71	0.11	253.44	0.99	1.38	244.12	1.11	0.12	252.36	0.31	0.56	159.16	1.26	0.45
89	47.82	0.12	254.43	0.98	1.36	243.01	1.12	0.12	252.67	0.29	0.55	157.90	1.27	0.46
90	47.94	+0.11	255.41	+0.97	1.35	241.89	−1.13	0.12	252.96	+0.28	0.54	156.63	−1.27	0.47
91	48.05	0.11	256.38	0.96	1.33	240.76	1.14	0.12	253.24	0.26	0.53	155.36	1.27	0.48
92	48.16	0.11	257.34	0.95	1.32	239.62	1.14	0.12	253.50	0.25	0.52	154.09	1.27	0.49
93	48.27	0.11	258.29	0.94	1.30	238.48	1.15	0.13	253.75	0.23	0.51	152.82	1.27	0.50
94	48.38	0.11	259.23	0.93	1.29	237.33	1.17	0.13	253.98	0.23	0.50	151.55	1.28	0.51
95	48.49	+0.10	260.16	+0.92	1.28	236.16	−1.18	0.13	254.21	+0.22	0.49	150.27	−1.28	0.52
96	48.59	0.11	261.08	0.91	1.26	234.98	1.19	0.13	254.43	0.20	0.48	148.99	1.28	0.54
97	48.70	0.10	261.99	0.89	1.25	233.79	1.19	0.13	254.63	0.19	0.47	147.71	1.29	0.55
98	48.80	0.11	262.88	0.88	1.23	232.60	1.21	0.13	254.82	0.17	0.46	146.42	1.29	0.56
99	48.91	0.10	263.76	0.87	1.22	231.39	1.21	0.14	254.99	0.16	0.45	145.13	1.29	0.57
100	49.01	+0.10	264.63	+0.86	1.20	230.18	−1.22	0.14	255.15	+0.15	0.44	143.84	−1.29	0.58
101	49.11	0.10	265.49	0.85	1.19	228.96	1.23	0.14	255.30	0.13	0.43	142.55	1.29	0.59
102	49.21	0.10	266.34	0.84	1.17	227.73	1.23	0.15	255.43	0.12	0.42	141.26	1.29	0.60
103	49.31	0.09	267.18	0.82	1.16	226.50	1.25	0.15	255.55	0.11	0.41	139.97	1.30	0.61
104	49.40	0.10	268.00	0.81	1.14	225.25	1.26	0.16	255.66	0.09	0.40	138.67	1.30	0.62
105	49.50	+0.09	268.81	+0.80	1.13	223.99	−1.26	0.16	255.75	+0.08	0.39	137.37	−1.30	0.63
106	49.59	0.09	269.61	0.80	1.11	222.73	1.27	0.16	255.83	0.07	0.39	136.07	1.30	0.65
107	49.68	0.09	270.41	0.78	1.10	221.46	1.28	0.17	255.90	0.05	0.38	134.77	1.30	0.66
108	49.77	0.08	271.19	0.77	1.09	220.18	1.30	0.17	255.95	0.04	0.37	133.47	1.50	0.67
109	49.85	0.09	271.96	0.75	1.07	218.88	1.30	0.18	255.99	0.03	0.36	132.17	1.30	0.68
110	49.94	+0.08	272.71	+0.74	1.06	217.58	−1.30	0.18	256.02	+0.02	0.35	130.87	−1.30	0.69
111	50.02	0.09	273.45	0.73	1.05	216.28	1.31	0.18	256.04	0.00	0.34	129.57	1.31	0.70
112	50.11	0.08	274.18	0.72	1.03	214.97	1.32	0.19	256.04	−0.01	0.33	128.26	1.30	0.71
113	50.19	0.07	274.90	0.70	1.02	213.65	1.32	0.19	256.03	0.03	0.33	126.96	1.30	0.73
114	50.26	0.08	275.60	0.69	1.00	212.33	1.34	0.20	256.00	0.04	0.32	125.66	1.30	0.74
115	50.34	+0.07	276.29	+0.68	0.99	210.99	−1.34	0.20	255.96	−0.05	0.31	124.36	−1.31	0.75
116	50.41	0.07	276.97	0.66	0.98	209.65	1.34	0.21	255.91	0.07	0.30	123.05	1.30	0.76
117	50.48	0.07	277.63	0.65	0.97	208.31	1.35	0.21	255.84	0.08	0.29	121.75	1.30	0.77
118	50.55	0.07	278.28	0.64	0.96	206.96	1.36	0.22	255.76	0.09	0.28	120.45	1.30	0.78
119	50.62	0.07	278.92	0.63	0.94	205.60	1.37	0.22	255.67	0.11	0.28	119.15	1.30	0.80
120	50.69		279.55		0.93	204.23		0.23	255.56		0.27	117.85		0.81

TABLE IX, ARG. 2.—*Continued.*

Arg.	$(v.s.3)$	$(v.c.3)$	$(v.s.4)$	$(v.c.4)$	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	$(\rho.s.3)$	$(\rho.c.3)$
	"	"	"	"							
60	16.26	9.11	1.80	0.89	1496	437	2222	583	127	137	71
61	16.29	9.00	1.79	0.88	1489	445	2233	580	125	137	70
62	16.32	8.88	1.79	0.87	1483	453	2243	577	123	137	70
63	16.35	8.76	1.78	0.85	1477	460	2254	574	121	137	70
64	16.38	8.64	1.78	0.84	1470	468	2264	570	119	137	69
65	16.41	8.52	1.78	0.83	1464	476	2275	567	117	136	69
66	16.43	8.40	1.77	0.81	1457	484	2285	564	115	136	68
67	16.45	8.28	1.77	0.80	1451	492	2295	560	113	136	68
68	16.47	8.16	1.76	0.79	1444	501	2306	557	112	136	67
69	16.49	8.04	1.75	0.78	1438	509	2316	553	110	136	67
70	16.50	7.91	1.75	0.76	1431	518	2326	550	108	136	66
71	16.51	7.79	1.74	0.75	1424	527	2336	546	107	136	66
72	16.52	7.67	1.74	0.74	1417	536	2346	543	105	136	66
73	16.53	7.55	1.73	0.73	1410	545	2356	540	103	135	65
74	16.54	7.43	1.72	0.71	1403	554	2365	536	102	135	65
75	16.54	7.31	1.72	0.70	1396	563	2375	533	100	135	64
76	16.54	7.19	1.71	0.69	1389	572	2384	530	99	135	64
77	16.54	7.07	1.70	0.68	1382	581	2394	526	98	135	64
78	16.54	6.95	1.70	0.67	1374	590	2403	523	96	135	63
79	16.53	6.83	1.69	0.65	1367	599	2413	519	95	135	63
80	16.53	6.71	1.68	0.64	1360	608	2422	516	94	135	62
81	16.52	6.60	1.67	0.63	1353	618	2431	512	93	135	62
82	16.52	6.48	1.66	0.62	1346	627	2440	508	92	134	61
83	16.51	6.36	1.65	0.61	1339	637	2449	505	91	134	61
84	16.50	6.24	1.64	0.60	1332	646	2457	501	89	134	60
85	16.48	6.13	1.63	0.59	1325	656	2466	498	88	134	60
86	16.46	6.01	1.62	0.58	1318	664	2475	494	87	134	59
87	16.44	5.89	1.61	0.57	1311	676	2483	490	86	134	59
88	16.42	5.78	1.60	0.56	1304	684	2492	487	85	134	58
89	16.40	5.66	1.59	0.55	1296	696	2500	483	84	133	58
90	16.38	5.55	1.58	0.54	1289	706	2509	480	84	133	57
91	16.35	5.44	1.57	0.53	1281	717	2517	476	83	133	57
92	16.33	5.32	1.56	0.52	1273	727	2525	472	82	133	56
93	16.30	5.21	1.55	0.51	1266	738	2534	469	82	132	56
94	16.27	5.09	1.54	0.50	1258	749	2542	465	81	132	55
95	16.24	4.98	1.53	0.50	1250	760	2550	461	81	132	55
96	16.20	4.87	1.52	0.49	1242	771	2558	457	80	131	54
97	16.17	4.76	1.51	0.48	1234	782	2566	454	80	131	54
98	16.13	4.64	1.50	0.48	1226	793	2574	450	80	131	53
99	16.09	4.54	1.49	0.47	1218	804	2582	446	79	130	53
100	16.05	4.44	1.48	0.46	1210	815	2590	442	79	130	52
101	16.01	4.33	1.46	0.46	1202	826	2598	438	79	130	51
102	15.96	4.23	1.45	0.45	1194	838	2605	435	79	129	51
103	15.92	4.13	1.44	0.44	1186	849	2613	431	78	129	50
104	15.87	4.02	1.43	0.44	1178	861	2620	428	78	129	50
105	15.83	3.92	1.42	0.43	1170	872	2627	424	78	128	49
106	15.78	3.82	1.41	0.43	1162	883	2634	420	78	128	49
107	15.73	3.72	1.39	0.42	1154	895	2641	416	78	128	48
108	15.68	3.62	1.38	0.42	1146	907	2647	413	78	127	48
109	15.62	3.52	1.37	0.41	1138	919	2654	409	78	127	47
110	15.56	3.43	1.36	0.41	1130	931	2661	405	78	126	46
111	15.51	3.34	1.34	0.40	1122	943	2668	402	78	126	46
112	15.45	3.24	1.33	0.40	1114	955	2674	398	79	125	45
113	15.39	3.15	1.32	0.40	1106	967	2681	394	79	125	45
114	15.33	3.06	1.31	0.39	1098	979	2687	391	79	124	44
115	15.27	2.97	1.29	0.39	1090	991	2693	387	80	124	44
116	15.21	2.88	1.28	0.38	1082	1003	2699	383	80	123	44
117	15.14	2.80	1.26	0.38	1074	1016	2705	380	80	122	43
118	15.08	2.71	1.25	0.38	1066	1028	2711	376	81	122	43
119	15.01	2.62	1.24	0.38	1059	1041	2716	373	81	121	42
120	14.94	2.54	1.23	0.37	1051	1053	2722	370	82	120	42

TABLE IX, ARG. 2.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec.var.	(v.c.1)	Diff.	Sec.var.	(v.s.2)	Diff.	Sec.var.	(v.c.2)	Diff.	Sec.var.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
120	50.69	+0.07	279.55	+0.61	0.93	204.23	−1.37	0.23	255.56	−0.12	0.27	117.85	−1.30	0.81
121	50.76	0.06	280.16	0.60	0.92	202.86	1.38	0.24	255.44	0.13	0.26	116.55	1.30	0.82
122	50.82	0.06	280.76	0.59	0.90	201.48	1.38	0.24	255.31	0.15	0.26	115.25	1.30	0.84
123	50.88	0.05	281.35	0.57	0.89	200.10	1.39	0.25	255.16	0.16	0.25	113.95	1.30	0.85
124	50.93	0.05	281.92	0.56	0.88	198.71	1.40	0.25	254.00	0.17	0.25	112.65	1.29	0.87
125	50.98	+0.05	282.48	+0.54	0.87	197.31	−1.40	0.26	254.83	−0.19	0.24	111.36	−1.29	0.88
126	51.03	0.05	283.02	0.53	0.85	195.91	1.41	0.27	254.64	0.20	0.23	110.07	1.30	0.89
127	51.08	0.05	283 55	0.52	0.84	194.50	1.41	0.27	254.44	0.21	0.23	108.77	1.29	0.91
128	51.13	0.04	284.07	0.50	0.83	193.09	1.42	0.28	254.23	0.23	0.22	107.48	1.29	0.92
129	51.17	0.05	284.57	0.49	0.81	191.67	1.42	0.28	254.00	0.24	0.22	106.19	1.28	0.94
130	51.22	+0.04	285.06	+0.47	0.80	190.25	−1.43	0.29	253.76	−0.25	0.21	104.91	−1.28	0.95
131	51.26	0.04	285.53	0.46	0.79	188.82	1.43	0.30	253.51	0.27	0.21	103.63	1.28	0.96
132	51.30	0.04	285.99	0.45	0.78	187.39	1.44	0.31	253.24	0.28	0.20	102.35	1.28	0.98
133	51.34	0.03	286.44	0.43	0.76	185.95	1.44	0.32	252.96	0.29	0.20	101.07	1.28	0.99
134	51.37	0.03	286.87	0.42	0.75	184.51	1.44	0.33	252.67	0.31	0.19	99.79	1.27	1.01
135	51.40	+0.03	287.29	+0.40	0.74	183.07	−1.45	0.33	252.36	−0.32	0.19	98.52	−1.27	1.02
136	51.43	0.02	287.69	0.39	0.73	181.62	1.45	0.34	252.04	0.33	0.18	97.25	1.26	1.03
137	51.45	0.02	288.08	0.37	0.72	180.17	1.45	0.35	251.71	0.35	0.18	95.99	1.26	1.05
138	51.47	0.02	288.45	0.36	0.70	178.72	1.46	0.36	251.36	0.36	0.17	94.73	1.26	1.06
139	51.49	0.02	288.81	0.35	0.69	177.26	1.46	0.37	251.00	0.37	0.17	93.47	1.26	1.08
140	51.51	+0.01	289.16	+0.33	0.68	175.80	−1.47	0.38	250.63	−0.39	0.16	92.21	−1.25	1.09
141	51.52	0.01	289.49	0.31	0.67	174.33	1.47	0.39	250.24	0.40	0.16	90.96	1.25	1.10
142	51.53	0.01	289.80	0.30	0.66	172.86	1.47	0.40	249.84	0.41	0.15	89.71	1.25	1.12
143	51.54	0.01	290.10	0.29	0.65	171.39	1.47	0.41	249.43	0.42	0.15	88.46	1.24	1.13
144	51.55	0.00	290.39	0.27	0.63	169.92	1.48	0.42	249.01	0.44	0.14	87.22	1.24	1.15
145	51.55	0.00	290.66	+0.26	0.62	168.44	−1.48	0.42	248.57	−0.45	0.14	85.98	−1.23	1.16
146	51.55	0.00	290.92	0.24	0.61	166.96	1.48	0.43	248.12	0.46	0.14	84.75	1.23	1.17
147	51.55	−0.01	291.16	0.22	0.60	165.48	1.48	0.44	247.66	0.48	0.13	83.52	1.23	1.19
148	51.54	0.01	291.38	0.21	0.58	164.00	1.48	0.45	247.18	0.49	0.13	82.29	1.22	1.20
149	51.53	0.01	291.59	0.20	0.57	162.52	1.49	0.46	246.69	0.50	0.12	81.07	1.22	1.22
150	51.52	−0.02	291.79	+0.18	0.56	161.03	−1.49	0.47	246.19	−0.51	0.12	79.85	−1.21	1.23
151	51.50	0.01	291.97	0.16	0.55	159.54	1.49	0.48	245.68	0.53	0.12	78.64	1.20	1.24
152	51.49	0.02	292.13	0.15	0.54	158.05	1.49	0.49	245.15	0.54	0.12	77.44	1.20	1.26
153	51.47	0.02	292.28	0.14	0.53	156.56	1.49	0.50	244.61	0.55	0.11	76.24	1.20	1.27
154	51.45	0.02	292.42	0.12	0.52	155.07	1.49	0.51	244.06	0.56	0.11	75.04	1.19	1.29
155	51.43	−0.03	292.54	+0.10	0.51	153.58	−1.49	0.52	243.50	−0.58	0.11	73.85	−1.18	1.30
156	51.40	0.03	292.64	0.09	0.50	152.09	1.49	0.54	242.92	0.59	0.11	72.67	1.18	1.31
157	51.37	0.03	292.73	0.07	0.49	150.60	1.49	0.55	242.33	0.60	0.11	71.49	1.17	1.33
158	51.34	0.04	292.80	0.06	0.48	149.11	1.49	0.56	241.73	0.61	0.10	70.32	1.17	1.34
159	51.30	0.04	292.86	0.04	0.47	147.62	1.50	0.57	241.12	0.63	0.10	69.15	1.16	1.36
160	51.26	−0.04	292.90	+0.03	0.46	146.12	−1.49	0.58	240.49	−0.64	0.10	67.99	−1.15	1.37
161	51.22	0.05	292.93	0.01	0.45	144.63	1.50	0.59	239.85	0.65	0.10	66.84	1.15	1.39
162	51.17	0.05	292.94	0.00	0.44	143.13	1.50	0.60	239.20	0.66	0.10	65.69	1.14	1.40
163	51.12	0.05	292.94	−0.02	0.43	141.63	1.49	0.61	238.54	0.67	0.10	64.55	1.14	1.42
164	51.07	0.06	292.92	0.04	0.42	140.14	1.50	0.62	237.87	0.69	0.10	63.41	1.13	1.43
165	51.01	−0.06	292.88	−0.05	0.41	138.64	−1.49	0.64	237.18	−0.70	0.09	62.28	−1.12	1.45
166	50.95	0.06	292.83	0.07	0.40	137.15	1.49	0.65	236.48	0.71	0.09	61.16	1.11	1.47
167	50.89	0.07	292.76	0.08	0.40	135.66	1.49	0.66	235.77	0.72	0.09	60.05	1.11	1.48
168	50.82	0.06	292.68	0.10	0.39	134.17	1.49	0.67	235.05	0.73	0.09	58.94	1.10	1.50
169	50.76	0.07	292.58	0.11	0.38	132.68	1.48	0.68	234.32	0.74	0.09	57.84	1.09	1.51
170	50.69	−0.07	292.47	−0.13	0.37	131.20	−1.49	0.69	233.58	−0.76	0.09	56.75	−1.08	1.53
171	50.62	0.08	292.34	0.15	0.36	129.71	1.48	0.70	232.82	0.77	0.09	55.67	1.08	1.54
172	50.54	0.08	292.19	0.16	0.35	128.23	1.48	0.71	232.05	0.78	0.09	54.59	1.07	1.56
173	50.46	0.09	292.03	0.17	0.35	126.75	1.48	0.73	231.27	0.79	0.10	53.52	1.07	1.57
174	50.37	0.09	291.86	0.19	0.34	125.27	1.48	0.74	230.48	0.80	0.10	52.45	1.05	1.59
175	50.28	−0.09	291.67	−0.21	0.33	123.79	−1.47	0.75	229.68	−0.81	0.10	51.40	−1.05	1.60
176	50.19	0.09	291.46	0.22	0.32	122.32	1.47	0.76	228.87	0.82	0.10	50.35	1.04	1.61
177	50.10	0.09	291.24	0.23	0.31	120.85	1.47	0.77	228.05	0.83	0.10	49.31	1.03	1.63
178	50.01	0.10	291.01	0.25	0.31	119.38	1.47	0.79	227.22	0.85	0.11	48.28	1.02	1.64
179	49.91	0.10	290.76	0.27	0.30	118.91	1.46	0.80	226.37	0.86	0.11	47.26	1.01	1.66
180	49.81		290.49		0.29	116.45		0.81	225.51		0.11	46.25		1.67

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	"	"	"	"							
120	14.94	2.54	1.23	0.37	1051	1053	2722	370	82	120	42
121	14.87	2.46	1.22	0.37	1043	1065	2727	366	82	120	42
122	14.80	2.38	1.20	0.37	1035	1078	2732	362	83	120	41
123	14.73	2.30	1.19	0.37	1028	1090	2737	359	83	119	41
124	14.66	2.22	1.18	0.37	1020	1103	2742	355	84	119	40
125	14.59	2.15	1.17	0.37	1012	1116	2747	352	85	119	40
126	14.52	2.07	1.15	0.37	1004	1129	2751	348	85	118	39
127	14.44	2.00	1.14	0.37	996	1141	2756	344	86	118	39
128	14.37	1.93	1.13	0.37	989	1154	2760	341	87	117	38
129	14.29	1.86	1.11	0.37	981	1167	2765	337	88	117	38
130	14.21	1.79	1.10	0.37	973	1180	2769	334	89	116	37
131	14.13	1.72	1.09	0.37	965	1193	2773	330	90	115	37
132	14.06	1.66	1.07	0.37	958	1206	2777	327	91	115	36
133	13.98	1.59	1.06	0.37	950	1219	2781	324	92	114	36
134	13.90	1.53	1.05	0.38	942	1232	2785	321	93	113	35
135	13.82	1.47	1.04	0.38	935	1245	2789	318	94	112	35
136	13.74	1.41	1.02	0.38	927	1258	2793	314	95	112	35
137	13.66	1.35	1.01	0.38	920	1271	2796	311	96	111	34
138	13.57	1.30	1.00	0.39	912	1284	2800	308	97	110	34
139	13.49	1.24	0.99	0.39	905	1298	2803	305	99	109	33
140	13.40	1.19	0.97	0.39	897	1311	2806	302	100	108	33
141	13.32	1.14	0.96	0.40	889	1325	2809	298	101	108	33
142	13.24	1.09	0.95	0.40	882	1338	2811	295	103	107	32
143	13.15	1.04	0.94	0.41	875	1352	2814	292	104	106	32
144	13.07	0.99	0.93	0.41	867	1366	2816	289	106	105	32
145	12.98	0.94	0.91	0.41	859	1379	2819	286	107	105	31
146	12.90	0.90	0.90	0.42	852	1393	2821	282	109	104	31
147	12.81	0.86	0.89	0.42	844	1407	2823	279	110	103	31
148	12.73	0.82	0.88	0.43	837	1420	2825	276	112	102	30
149	12.64	0.78	0.87	0.44	829	1433	2826	273	113	101	30
150	12.55	0.74	0.86	0.44	822	1447	2828	270	115	100	30
151	12.46	0.70	0.85	0.45	815	1461	2829	267	117	99	30
152	12.38	0.67	0.84	0.46	807	1474	2831	264	118	98	29
153	12.29	0.64	0.83	0.46	800	1488	2832	261	120	97	29
154	12.20	0.61	0.82	0.47	793	1502	2834	258	122	96	29
155	12.11	0.58	0.81	0.48	786	1515	2835	256	124	95	28
156	12.03	0.55	0.80	0.49	778	1529	2836	253	126	94	28
157	11.94	0.52	0.79	0.49	771	1543	2837	250	128	93	28
158	11.86	0.50	0.78	0.50	764	1556	2837	247	129	92	28
159	11.77	0.47	0.77	0.51	756	1569	2838	244	131	91	27
160	11.68	0.45	0.76	0.52	749	1583	2838	242	133	90	27
161	11.59	0.43	0.75	0.52	742	1597	2838	239	135	90	27
162	11.51	0.41	0.74	0.53	735	1611	2838	236	137	89	27
163	11.42	0.40	0.73	0.54	728	1624	2837	233	139	88	27
164	11.33	0.38	0.72	0.55	721	1638	2837	230	141	87	27
165	11.24	0.37	0.71	0.56	714	1652	2836	228	143	86	27
166	11.16	0.36	0.71	0.57	707	1665	2835	225	145	85	27
167	11.07	0.35	0.70	0.58	700	1679	2834	222	147	84	27
168	10.99	0.34	0.69	0.59	693	1693	2833	219	149	83	27
169	10.91	0.33	0.68	0.60	686	1706	2833	216	151	82	27
170	10.82	0.32	0.68	0.61	679	1720	2831	214	153	82	27
171	10.74	0.32	0.67	0.62	672	1734	2829	211	156	81	27
172	10.66	0.31	0.66	0.63	665	1747	2828	208	158	80	27
173	10.57	0.31	0.66	0.64	659	1761	2826	206	160	79	27
174	10.49	0.31	0.65	0.65	652	1775	2824	204	162	78	28
175	10.41	0.31	0.65	0.66	645	1788	2822	202	165	77	28
176	10.33	0.31	0.64	0.67	638	1802	2820	199	167	76	28
177	10.25	0.31	0.63	0.68	632	1815	2817	197	170	75	28
178	10.17	0.32	0.63	0.69	625	1829	2815	195	173	74	28
179	10.09	0.32	0.63	0.70	618	1842	2812	192	175	73	28
180	10.01	0.33	0.62	0.71	611	1855	2810	190	178	72	28

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec.var.	(v.c.1)	Diff.	Sec.var.	(v.s.2)	Diff.	Sec.var.	(v.c.2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
180	49.81	−0.10	290.49	−0.29	0.29	116.45	−1.46	0.81	225.51	−0.87	0.11	46.25	−1.01	1.67
181	49.71	0.11	290.20	0.30	0.28	114.99	1.46	0.82	224.64	0.87	0.11	45.24	1.00	1.68
182	49.60	0.11	289.90	0.31	0.28	113.53	1.45	0.84	223.77	0.88	0.11	44.24	0.99	1.70
183	49.49	0.12	289.59	0.33	0.27	112.08	1.45	0.85	222.89	0.90	0.12	43.25	0.97	1.71
184	49.37	0.12	289.26	0.35	0.26	110.63	1.45	0.86	221.99	0.91	0.12	42.28	0.97	1.73
185	49.25	−0.12	288.91	−0.36	0.26	109.18	−1.44	0.88	221.08	−0.92	0.12	41.31	−0.96	1.74
186	49.13	0.13	288.55	0.38	0.25	107.74	1.44	0.89	220.16	0.93	0.12	40.35	0.95	1.75
187	49.00	0.13	288.17	0.39	0.24	106.30	1.43	0.90	219.23	0.94	0.12	39.40	0.94	1.77
188	48.87	0.13	287.78	0.41	0.24	104.87	1.43	0.91	218.29	0.95	0.13	38.46	0.93	1.78
189	48.74	0.13	287.37	0.42	0.23	103.44	1.42	0.93	217.34	0.96	0.13	37.53	0.92	1.80
190	48.61	−0.14	286.95	−0.44	0.22	102.02	−1.42	0.94	216.38	−0.97	0.13	36.61	−0.91	1.81
191	48.47	0.14	286.51	0.46	0 21	100.60	1.41	0.95	215.41	0.97	0.13	35.70	0.90	1.82
192	48.33	0.14	286.05	0.47	0.21	99.19	1.41	0.97	214.44	0.98	0.14	34.80	0.89	1.84
193	48.19	0.14	285.58	0.48	0.20	97.78	1.40	0.98	213.46	1.00	0.14	33.91	0.89	1.85
194	48.05	0.15	285.10	0.50	0.20	96.38	1.40	1.00	212.46	1.01	0.14	33.02	0.87	1.87
195	47.90	−0.15	284.60	−0.51	0.19	94.98	−1.39	1.01	211.45	−1.02	0.15	32.15	−0.86	1.88
196	47.75	0.15	284.09	0.53	0.19	93.59	1.39	1.02	210.43	1.03	0.15	31.29	0.85	1.90
197	47.60	0.16	283.56	0.55	0.18	92.20	1.38	1.04	209.40	1.03	0.15	30.44	0.84	1.91
198	47.44	0.16	283.01	0.56	0.18	90.82	1.38	1.05	208.37	1.04	0.15	29.60	0.83	1 93
199	47.28	0.16	282.45	0.57	0.17	89.44	1.37	1.07	207.33	1.05	0.16	28.77	0.82	1.94
200	47.12	−0.17	281.88	−0.59	0.17	88.07	−1.36	1.08	206.28	−1.06	0.16	27.95	−0.81	1.96
201	46.95	0.17	281.29	0.61	0.17	86.71	1.35	1.09	205.22	1.07	0.17	27.14	0.80	1.97
202	46.78	0.17	280.68	0.62	0.16	85.36	1.35	1.11	204.15	1.08	0.17	26.34	0.79	1.99
203	46.61	0.17	280.06	0.63	0.16	84.01	1.35	1.12	203.07	1.09	0.18	25.55	0.77	2.00
204	46.44	0.18	279.43	0.65	0.16	82.66	1.34	1.14	201.98	1.09	0.18	24.78	0.77	2.01
205	46.26	−0.18	278.78	−0.66	0.15	81.32	−1.33	1.15	200.89	−1.10	0.19	24.01	−0.75	2.03
206	46.08	0.18	278.12	0.68	0.15	79.99	1.32	1.16	199.79	1.11	0.20	23.26	0.74	2.04
207	45.90	0.19	277.44	0.69	0.15	78.67	1.31	1.18	198.68	1.12	0.20	22.52	0.73	2.05
208	45.71	0.19	276.75	0.71	0.14	77.36	1.31	1.19	197.56	1.13	0.21	21.79	0.72	2.06
209	45.52	0.19	276.04	0.72	0.14	76.05	1.30	1.21	196.43	1.13	0.21	21.07	0.71	2.08
210	45.33	−0.19	275.32	−0.74	0.14	74.75	−1.29	1.22	195.30	−1.14	0.22	20.36	−0.70	2.09
211	45.14	0.20	274.58	0.75	0.13	73.46	1.28	1.23	194.16	1.15	0.23	19.66	0.69	2.10
212	44.94	0.20	273.83	0.77	0.13	72.18	1.27	1.25	193.01	1.16	0.23	18.97	0.67	2.12
213	44.74	0.20	273.06	0.78	0.13	70.91	1.27	1.26	191.85	1.16	0.24	18.30	0.66	2.13
214	44.54	0.20	272.28	0.79	0.13	69.64	1.27	1.28	190.69	1.17	0.25	17.64	0.64	2.14
215	44.34	−0.21	271.49	−0.81	0.12	68.37	−1.25	1.29	189.52	−1.18	0.26	17.00	−0.63	2.16
216	44.13	0.21	270.68	0.82	0.12	67.12	1.24	1.31	188.34	1.19	0.27	16.37	0.63	2.17
217	43.92	0.21	269.86	0.83	0.12	65.88	1.23	1.32	187.15	1.19	0.28	15.74	0.61	2.18
218	43.71	0.21	269.03	0.85	0.12	64.65	1.23	1.34	185.96	1.20	0.28	15.13	0.60	2.19
219	43.50	0.22	268.18	0.86	0.12	63.42	1.22	1.35	184.76	1.20	0.29	14.53	0.58	2.21
220	43.28	−0.22	267.32	−0.88	0.11	62.20	−1.21	1.37	183.56	−1.21	0.29	13.95	−0.57	2.22
221	43.06	0.22	266.44	0.89	0.11	60.99	1.19	1.38	182.35	1.21	0.30	13.38	0.56	2.23
222	42.84	0.22	265.55	0.90	0.11	59.80	1.19	1.40	181.14	1.22	0.31	12.82	0.55	2.25
223	42.62	0.22	264.65	0.92	0.11	58.61	1.18	1.41	179.92	1.23	0.32	12.27	0.54	2.26
224	42.40	0.23	263.73	0.93	0.11	57.43	1.17	1.43	178.69	1.23	0.33	11.73	0.52	2.27
225	42.17	−0.23	262.80	−0.95	0.11	56.26	−1.16	1.44	177.46	−1.24	0.34	11.21	−0.51	2.29
226	41.94	0.23	261.85	0.96	0.11	55.10	1.14	1.45	176.22	1.25	0.34	10.70	0.50	2.30
227	41.71	0.23	260.89	0.97	0.11	53.96	1.14	1.47	174.97	1.25	0.35	10.20	0.48	2.31
228	41.48	0.24	259.92	0.98	0.11	52.82	1.13	1.48	173.72	1.26	0 36	9.72	0.47	2.32
229	41.24	0.24	258.94	1.00	0.11	51.69	1.12	1.50	172.46	1.27	0.37	9.25	0.46	2.34
230	41.00	−0.24	257.94	−1.01	0.11	50.57	−1.11	1.51	171.19	−1.27	0.38	8.79	−0.44	2.35
231	40.76	0.24	256.93	1.01	0.11	49.46	1.09	1.52	169.92	1.27	0.39	8.35	0.43	2.36
232	40.52	0.24	255.92	1.03	0.11	48.37	1.09	1.54	168.65	1.28	0.40	7.92	0.42	2.37
233	40.28	0.25	254.89	1.04	0.11	47.28	1.08	1.55	167.37	1.28	0.41	7.50	0.41	2.38
234	40.03	0.25	253.85	1.06	0.11	46.20	1.07	1.57	166.09	1.29	0.42	7.09	0.39	2.39
235	39.78	−0.25	252.79	−1.07	0.11	45.13	−1.05	1.58	164.80	−1.29	0.42	6.70	−0.38	2.41
236	39.53	0.25	251.72	1.08	0.11	44.08	1.03	1.60	163.51	1.29	0.43	6.32	0.36	2.42
237	39.28	0.25	250.64	1.09	0.12	43.05	1.03	1.61	162.22	1.30	0.44	5.96	0.35	2.43
238	39.03	0.26	249.55	1.10	0.12	42.02	1.02	1.63	160.92	1.30	0.45	5.61	0.34	2.44
239	38.77	0.26	248.45	1.12	0.12	41.00	1.02	1.64	159.62	1.31	0.46	5.27	0.32	2.45
240	38.51		247.33		0.12	39.98		1.66	158.31		0.47	4.95		2.46

TABLE IX, Arg. 2.—*Continued.*

Arg.	($v.s.3$)	($v.c.3$)	($v.s.4$)	($v.c.4$)	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)	($\rho.s.2$)	($\rho.c.2$)	($\rho.s.3$)	($\rho.c.3$)
	"	"	"	"							
180	10.01	0.33	0.62	0.71	611	1855	2810	190	178	72	28
181	9.93	0.34	0.62	0.72	604	1869	2807	188	180	71	28
182	9.85	0.35	0.61	0.73	598	1882	2804	186	183	70	28
183	9.78	0.36	0.61	0.75	591	1895	2801	184	185	69	28
184	9.70	0.37	0.61	0.76	584	1908	2798	182	188	68	28
185	9.63	0.38	0.60	0.77	577	1921	2794	180	190	67	29
186	9.55	0.39	0.60	0.78	571	1935	2791	178	192	66	29
187	9.47	0.41	0.60	0.79	564	1948	2787	176	195	65	29
188	9.40	0.42	0.60	0.80	557	1961	2783	174	197	64	29
189	9.33	0.44	0.59	0.81	551	1974	2779	172	200	63	30
190	9.26	0.45	0.59	0.83	544	1987	2775	171	202	62	30
191	9.19	0.47	0.59	0.84	538	2000	2771	169	205	61	30
192	9.12	0.50	0.59	0.85	531	2013	2766	167	207	60	30
193	9.05	0.52	0.59	0.86	525	2025	2762	165	210	59	31
194	8.98	0.54	0.59	0.87	519	2038	2757	163	212	58	31
195	8.91	0.56	0.59	0.88	512	2051	2752	161	215	57	32
196	8.84	0.59	0.59	0.90	506	2064	2747	159	218	56	32
197	8.78	0.61	0.59	0.91	500	2076	2742	157	220	55	32
198	8.71	0.64	0.59	0.92	494	2089	2737	156	223	54	33
199	8.65	0.66	0.59	0.93	488	2101	2731	154	225	53	33
200	8.59	0.69	0.59	0.94	482	2113	2726	152	228	52	34
201	8.53	0.72	0.59	0.95	476	2126	2720	151	231	51	34
202	8.47	0.75	0.59	0.97	470	2138	2714	150	233	50	35
203	8.41	0.78	0.59	0.98	464	2150	2708	148	236	49	35
204	8.36	0.81	0.60	0.99	459	2162	2702	147	239	48	36
205	8.30	0.84	0.60	1.00	453	2174	2695	146	242	47	36
206	8.25	0.87	0.60	1.01	447	2186	2689	144	244	47	37
207	8.19	0.90	0.61	1.02	441	2198	2682	142	247	46	37
208	8.14	0.93	0.61	1.03	436	2210	2675	141	250	45	38
209	8.08	0.96	0.61	1.04	430	2222	2668	140	253	44	39
210	8.03	1.00	0.62	1.05	425	2234	2661	138	256	43	39
211	7.98	1.03	0.62	1.06	419	2245	2654	137	259	43	40
212	7.93	1.06	0.62	1.07	413	2257	2646	135	262	42	41
213	7.88	1.10	0.63	1.08	408	2268	2639	134	265	41	42
214	7.84	1.13	0.63	1.09	402	2280	2631	133	268	40	43
215	7.79	1.17	0.64	1.10	397	2291	2624	132	271	39	43
216	7.74	1.21	0.64	1.11	392	2302	2616	130	274	39	44
217	7.70	1.24	0.65	1.12	386	2313	2608	129	277	38	44
218	7.65	1.28	0.66	1.13	380	2324	2600	128	280	37	45
219	7.61	1.32	0.66	1.14	375	2335	2592	127	283	36	46
220	7.57	1.35	0.67	1.15	370	2346	2584	126	286	35	46
221	7.53	1.39	0.68	1.16	364	2357	2575	125	289	35	47
222	7.49	1.43	0.69	1.17	359	2367	2567	125	292	34	47
223	7.45	1.46	0.69	1.18	354	2378	2558	124	295	33	48
224	7.41	1.50	0.70	1.19	349	2389	2550	123	298	32	49
225	7.38	1.54	0.71	1.20	344	2399	2541	122	301	32	50
226	7.34	1.58	0.71	1.20	339	2409	2532	121	304	31	50
227	7.31	1.62	0.72	1.21	334	2420	2523	121	307	30	51
228	7.28	1.66	0.73	1.22	329	2430	2513	120	310	30	52
229	7.25	1.70	0.74	1.23	324	2440	2504	119	313	29	53
230	7.22	1.74	0.75	1.24	319	2450	2494	118	316	28	54
231	7.19	1.78	0.75	1.24	314	2460	2484	118	318	28	54
232	7.17	1.81	0.76	1.25	310	2470	2474	117	321	27	55
233	7.14	1.85	0.77	1.25	305	2480	2464	116	324	27	56
234	7.11	1.89	0.78	1.26	300	2489	2454	116	327	26	57
235	7.09	1.93	0.79	1.27	296	2499	2444	115	330	25	58
236	7.07	1.97	0.80	1.27	291	2508	2433	114	333	25	59
237	7.05	2.00	0.81	1.28	287	2518	2423	114	336	24	60
238	7.03	2.04	0.82	1.28	283	2527	2412	113	339	23	61
239	7.01	2.08	0.83	1.29	278	2536	2401	113	342	22	62
240	6.99	2.12	0.84	1.30	274	2545	2390	113	345	21	63

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec.var.	(v.c.1)	Diff.	Sec.var.	(v.s.2)	Diff.	Sec.var.	(v.c.2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
240	38.51	—0.26	247.33	—1.13	0.12	39.98	—1.00	1.66	158.31	—1.31	0.47	4.95	—0.31	2.46
241	38.25	0.26	246.20	1.14	0.12	38.98	0.98	1.67	157.00	1.31	0.48	4.64	0.30	2.47
242	37.99	0.26	245.06	1.15	0.12	38.00	0.97	1.69	155.69	1.32	0.49	4.34	0.28	2.48
243	37.73	0.27	243.91	1.16	0.13	37.03	0.96	1.70	154.37	1.32	0.50	4.06	0.27	2.49
244	37.46	0.26	242.75	1.18	0.13	36.07	0.95	1.71	153.05	1.32	0.51	3.79	0.25	2.50
245	37.20	—0.27	241.57	—1.18	0.13	35.12	—0.93	1.73	151.73	—1.33	0.53	3.54	—0.24	2.51
246	36.93	0.27	240.39	1.19	0.13	34.19	0.92	1.74	150.40	1.33	0.54	3.30	0.23	2.52
247	36.66	0.27	239.20	1.21	0.13	33.27	0.91	1.75	149.07	1.33	0.55	3.07	0.21	2.53
248	36.39	0.27	237.99	1.22	0.14	32.36	0.90	1.76	147.74	1.33	0.56	2.86	0.20	2.54
249	36.12	0.27	236.77	1.23	0.14	31.46	0.89	1.78	146.41	1.34	0.57	2.66	0.18	2.55
250	35.85	—0.27	235.54	—1.24	0.14	30.57	—0.87	1.79	145.07	—1.34	0.58	2.48	—0.17	2.56
251	35.58	0.27	234.30	1.25	0.14	29.70	0.86	1.80	143.73	1.34	0.59	2.31	0.15	2.57
252	35.31	0.28	233.05	1.25	0.15	28.84	0.85	1.82	142.39	1.34	0.60	2.16	0.14	2.58
253	35.03	0.28	231.80	1.26	0.15	27.99	0.84	1.83	141.05	1.34	0.61	2.02	0.13	2.59
254	34.75	0.28	230.54	1.28	0.16	27.15	0.82	1.85	139.71	1.35	0.62	1.89	0.11	2.60
255	34.47	—0.28	229.26	—1.28	0.16	26.33	—0.81	1.86	138.36	—1.35	0.64	1.78	—0.10	2.61
256	34.19	0.28	227.98	1.30	0.16	25.52	0.79	1.88	137.01	1.35	0.65	1.68	0.08	2.61
257	33.91	0.28	226.68	1.31	0.17	24.73	0.78	1.89	135.66	1.35	0.66	1.60	0.07	2.62
258	33.63	0.29	225.37	1.31	0.17	23.95	0.77	1.91	134.31	1.35	0.67	1.53	0.06	2.63
259	33.34	0.28	224.06	1.32	0.18	23.18	0.75	1.92	132.96	1.35	0.68	1.47	0.04	2.64
260	33.06	—0.28	222.74	—1.33	0.18	22.43	—0.74	1.94	131.61	—1.35	0.69	1.43	—0.03	2.65
261	32.78	0.29	221.41	1.34	0.18	21.69	0.72	1.95	130.26	1.36	0.70	1.40	—0.01	2.66
262	32.49	0.28	220.07	1.35	0.19	20.97	0.71	1.97	128.90	1.35	0.71	1.39	+0.01	2.67
263	32.21	0.29	218.72	1.36	0.19	20.26	0.70	1.98	127.55	1.35	0.73	1.40	0.02	2.67
264	31.92	0.29	217.36	1.37	0.20	19.56	0.68	1.99	126.20	1.35	0.74	1.42	0.03	2.68
265	31.63	—0.29	215.99	—1.37	0.20	18.88	—0.67	2.01	124.85	—1.35	0.75	1.45	+0.05	2.69
266	31.34	0.29	214.62	1.38	0.21	18.21	0.65	2.02	123.50	1.36	0.76	1.50	0.06	2.70
267	31.05	0.29	213.24	1.39	0.21	17.56	0.64	2.03	122.14	1.35	0.77	1.56	0.07	2.71
268	30.76	0.29	211.85	1.40	0.22	16.92	0.63	2.04	120.79	1.35	0.79	1.63	0.09	2.71
269	30.47	0.29	210.45	1.40	0.22	16.29	0.61	2.06	119.44	1.35	0.80	1.72	0.10	2.72
270	30.18	—0.30	209.05	—1.41	0.23	15.68	—0.60	2.07	118.09	—1.35	0.81	1.82	+0.12	2.73
271	29.88	0.29	207.64	1.42	0.24	15.08	0.58	2.08	116.74	1.35	0.82	1.94	0.13	2.74
272	29.59	0.30	206.22	1.43	0.24	14.50	0.56	2.10	115.39	1.35	0.84	2.07	0.14	2.74
273	29.29	0.29	204.79	1.43	0.25	13.94	0.55	2.11	114.04	1.34	0.85	2.21	0.16	2.75
274	29.00	0.29	203.36	1.44	0.25	13.39	0.54	2.13	112.70	1.35	0.87	2.37	0.18	2.75
275	28.71	—0.30	201.92	—1.44	0.26	12.85	—0.52	2.14	111.35	—1.34	0.88	2.55	+0.19	2.76
276	28.41	0.29	200.48	1.45	0.27	12.33	0.50	2.15	110.01	1.34	0.89	2.74	0.21	2.77
277	28.12	0.29	199.03	1.46	0.27	11.83	0.49	2.17	108.67	1.34	0.91	2.95	0.22	2.77
278	27.83	0.29	197.57	1.46	0.28	11.34	0.48	2.18	107.33	1.34	0.92	3.17	0.23	2.78
279	27.54	0.29	196.11	1.47	0.28	10.86	0.46	2.20	105.99	1.33	0.94	3.40	0.25	2.78
280	27.25	—0.30	194.64	—1.47	0.29	10.40	—0.45	2.21	104.66	—1.33	0.95	3.65	+0.26	2.79
281	26.95	0.29	193.17	1.48	0.30	9.95	0.43	2.22	103.33	1.33	0.96	3.91	0.28	2.79
282	26.66	0.30	191.69	1.49	0.31	9.52	0.42	2.23	102.00	1.33	0.98	4.19	0.29	2.80
283	26.36	0.29	190.20	1.49	0.32	9.10	0.40	2.24	100.67	1.33	0.99	4.48	0.30	2.80
284	26.07	0.29	188.71	1.49	0.33	8.70	0.39	2.25	99.34	1.32	1.01	4.78	0.32	2.81
285	25.78	—0.30	187.22	—1.50	0.34	8.31	—0.37	2.27	98.02	—1.32	1.02	5.10	+0.33	2.81
286	25.48	0.29	185.72	1.51	0.34	7.94	0.35	2.28	96.70	1.31	1.03	5.43	0.35	2.82
287	25.19	0.30	184.21	1.51	0.35	7.59	0.34	2.29	95.39	1.31	1.05	5.78	0.36	2.82
288	24.89	0.29	182.70	1.51	0.36	7.25	0.32	2.30	94.08	1.31	1.06	6.14	0.37	2.83
289	24.60	0.29	181.19	1.52	0.37	6.93	0.30	2.31	92.77	1.30	1.08	6.51	0.39	2.83
290	24.31	—0.29	179.67	—1.52	0.38	6.63	—0.29	2.32	91.47	—1.30	1.09	6.90	+0.40	2.84
291	24.02	0.29	178.15	1.52	0.39	6.34	0.27	2.33	90.17	1.30	1.10	7.30	0.42	2.84
292	23.73	0.29	176.63	1.53	0.40	6.07	0.26	2.34	88.87	1.29	1.12	7.72	0.43	2.85
293	23.44	0.29	175.10	1.53	0.41	5.81	0.24	2.36	87.58	1.29	1.13	8.15	3.44	2.85
294	23.15	0.29	173.57	1.53	0.42	5.57	0.23	2.37	86.29	1.28	1.15	8.59	0.46	2.86
295	22.86	—0.29	172.04	—1.54	0.43	5.34	—0.21	2.38	85.01	—1.27	1.16	9.05	+0.47	2.86
296	22.57	0.28	170.50	1.54	0.43	5.13	0.20	2.39	83.74	1.27	1.17	9.52	0.49	2.86
297	22.29	0.29	168.96	1.54	0.44	4.93	0.18	2.40	82.47	1.27	1.19	10.01	0.50	2.87
298	22.00	0.29	167.42	1.54	0.45	4.75	0.16	2.42	81.20	1.27	1.20	10.51	0.51	2.87
299	21.71	0.29	165.88	1.55	0.46	4.59	0.15	2.43	79.93	1.26	1.22	11.02	0.53	2.88
300	21.42		164.33		0.47	4.44		2.44	78.67		1.23	11.55		2.88

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″	″							
240	6.99	2.12	0.84	1.30	274	2545	2390	113	345	21	63
241	6.97	2.16	0.85	1.31	270	2554	2379	113	348	21	64
242	6.96	2.20	0.86	1.31	265	2562	2368	113	350	20	65
243	6.94	2.24	0.87	1.31	261	2571	2357	112	353	19	66
244	6.92	2.28	0.88	1.31	257	2579	2346	112	356	19	67
245	6.90	2.31	0.90	1.32	253	2587	2334	112	359	18	68
246	6.88	2.35	0.91	1.32	249	2595	2322	112	362	18	69
247	6.87	2.38	0.92	1.32	245	2603	2311	112	365	17	70
248	6.85	2.42	0.93	1.32	242	2611	2299	112	368	16	72
249	6.84	2.45	0.94	1.33	238	2618	2287	111	371	16	73
250	6.83	2.48	0.95	1.33	234	2626	2275	111	374	15	74
251	6.82	2.52	0.96	1.33	230	2633	2263	111	377	15	75
252	6.82	2.55	0.97	1.33	227	2641	2251	111	380	14	76
253	6.81	2.58	0.98	1.33	223	2648	2239	111	383	14	77
254	6.80	2.61	0.99	1.34	219	2656	2226	111	386	13	78
255	6.79	2.64	1.00	1.34	215	2663	2214	111	389	13	79
256	6.79	2.67	1.02	1.34	212	2670	2201	111	392	13	80
257	6.78	2.70	1.03	1.34	208	2677	2189	112	395	12	82
258	6.78	2.73	1.04	1.34	204	2684	2176	112	398	12	83
259	6.77	2.76	1.05	1.34	201	2691	2164	112	401	12	84
260	6.77	2.79	1.06	1.34	198	2698	2151	112	404	11	85
261	6.77	2.82	1.07	1.34	195	2704	2138	112	407	11	86
262	6.77	2.85	1.09	1.33	192	2711	2125	113	410	11	87
263	6.77	2.88	1.10	1.33	189	2717	2112	113	413	11	89
264	6.77	2.90	1.11	1.33	186	2723	2098	113	416	11	90
265	6.77	2.93	1.12	1.33	183	2729	2085	114	419	11	91
266	6.77	2.95	1.13	1.32	180	2735	2072	114	422	11	92
267	6.77	2.98	1.14	1.32	177	2740	2058	115	425	11	93
268	6.77	3.00	1.15	1.32	174	2745	2045	115	427	11	94
269	6.77	3.03	1.16	1.32	172	2751	2031	116	430	11	96
270	6.77	3.05	1.17	1.31	169	2756	2018	116	433	11	97
271	6.77	3.07	1.18	1.31	166	2761	2004	117	436	11	98
272	6.77	3.09	1.20	1.31	164	2766	1990	117	439	10	99
273	6.78	3.11	1.21	1.30	161	2771	1977	118	442	10	100
274	6.78	3.13	1.22	1.30	159	2776	1963	119	444	10	102
275	6.78	3.15	1.23	1.29	156	2781	1949	119	447	10	103
276	6.79	3.17	1.24	1.28	154	2785	1935	120	450	10	104
277	6.79	3.19	1.25	1.28	151	2790	1921	121	452	10	105
278	6.80	3.20	1.26	1.27	149	2794	1907	121	455	10	106
279	6.81	3.22	1.27	1.27	146	2798	1892	122	457	10	107
280	6.81	3.23	1.28	1.26	144	2802	1878	123	460	10	108
281	6.82	3.25	1.28	1.25	142	2806	1864	124	463	10	110
282	6.83	3.26	1.29	1.25	139	2809	1849	124	465	10	111
283	6.83	3.27	1.30	1.24	137	2813	1835	125	468	10	112
284	6.84	3.28	1.31	1.23	135	2816	1820	126	471	11	113
285	6.84	3.29	1.32	1.22	133	2819	1805	127	474	11	115
286	6.85	3.30	1.33	1.22	131	2822	1791	127	476	11	116
287	6.85	3.31	1.34	1.21	129	2825	1776	128	479	11	117
288	6.86	3.32	1.34	1.20	127	2828	1761	129	482	11	118
289	6.86	3.32	1.35	1.19	126	2830	1746	130	484	12	120
290	6.87	3.33	1.36	1.18	124	2833	1731	131	487	12	121
291	6.88	3.33	1.37	1.17	123	2835	1716	132	490	12	122
292	6.88	3.34	1.38	1.16	121	2837	1702	133	492	12	123
293	6.89	3.34	1.38	1.15	119	2838	1686	134	495	13	125
294	6.89	3.35	1.39	1.14	118	2840	1671	136	497	13	126
295	6.90	3.35	1.40	1.13	116	2841	1656	137	500	13	127
296	6.90	3.35	1.40	1.12	115	2843	1641	138	503	13	128
297	6.90	3.35	1.41	1.11	114	2844	1626	139	505	14	130
298	6.91	3.35	1.42	1.10	112	2845	1611	141	508	14	131
299	6.91	3.35	1.42	1.09	111	2847	1596	142	510	14	132
300	6.92	3.35	1.43	1.08	110	2848	1581	143	512	15	133

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″		″	″	″	″
300	21.42	−0.29	164.33	−1.55	0.47	4.44	−0.13	2.44	78.67	−1.25	1.23	11.55	+0.54	2.88
301	21.13	0.28	162.78	1.55	0.48	4.31	0.12	2.45	77.42	1.25	1.24	12.09	0.55	2.88
302	20.85	0.28	161.23	1.55	0.49	4.19	0.10	2.46	76.17	1.24	1.26	12.64	0.57	2.88
303	20.57	0.28	159.68	1.56	0.50	4.09	0.08	2.47	74.93	1.23	1.27	13.21	0.58	2.89
304	20.29	0.28	158.12	1.55	0.51	4.01	0.07	2.48	73.70	1.23	1.29	13.79	0.59	2.89
305	20.01	−0.28	156.57	−1.56	0.53	3.94	−0.05	2.49	72.47	−1.22	1.30	14.38	+0.61	2.89
306	19.73	0.28	155.01	1.55	0.54	3.89	0.03	2.50	71.25	1.22	1.31	14.99	0.62	2.89
307	19.45	0.28	153.46	1.56	0.55	3.86	0.01	2.51	70.03	1.21	1.33	15.61	0.63	2.89
308	19.17	0.27	151.90	1.56	0.56	3.85	−0.01	2.52	68.82	1.20	1.34	16.24	0.64	2.90
309	18.90	0.27	150.34	1.55	0.57	3.84	+0.01	2.53	67.62	1.20	1.36	16.88	0.66	2.90
310	18.63	−0.27	148.79	−1.56	0.58	3.85	+0.03	2.54	66.42	−1.19	1.37	17.54	+0.67	2.90
311	18.36	0.27	147.23	1.56	0.59	3.88	0.04	2.55	65.23	1.18	1.39	18.21	0.69	2.90
312	18.09	0.27	145.67	1.55	0.60	3.92	0.06	2.56	64.05	1.18	1.40	18.90	0.70	2.90
313	17.82	0.27	144.12	1.56	0.61	3.98	0.08	2.57	62.87	1.17	1.42	19.60	0.71	2.90
314	17.55	0.27	142.56	1.56	0.62	4.06	0.10	2.58	61.70	1.16	1.43	20.31	0.72	2.90
315	17.28	−0.26	141.00	−1.56	0.64	4.16	+0.11	2.59	60.54	−1.15	1.45	21.03	+0.73	2.91
316	17.02	0.27	139.44	1.56	0.65	4.27	0.13	2.59	59.39	1.15	1.47	21.76	0.75	2.91
317	16.75	0.26	137.88	1.55	0.66	4.40	0.15	2.60	58.24	1.14	1.48	22.51	0.76	2.91
318	16.49	0.26	136.33	1.56	0.67	4.55	0.16	2.61	57.10	1.13	1.50	23.27	0.77	2.91
319	16.23	0.26	134.77	1.55	0.68	4.71	0.18	2.62	55.97	1.12	1.51	24.04	0.78	2.91
320	15.97	−0.26	133.22	−1.55	0.69	4.89	+0.19	2.63	54.85	−1.11	1.53	24.82	+0.80	2.91
321	15.71	0.25	131.67	1.55	0.70	5.08	0.21	2.64	53.74	1.10	1.54	25.62	0.80	2.91
322	15.46	0.25	130.12	1.55	0.71	5.29	0.22	2.65	52.64	1.10	1.56	26.42	0.82	2.91
323	15.21	0.25	128.57	1.55	0.73	5.51	0.24	2.65	51.54	1.09	1.57	27.24	0.83	2.90
324	14.96	0.25	127.02	1.54	0.74	5.75	0.25	2.66	50.45	1.08	1.59	28.07	0.85	2.90
325	14.71	−0.24	125.48	−1.54	0.75	6.00	+0.27	2.67	49.37	−1.07	1.60	28.92	+0.85	2.90
326	14.47	0.24	123.94	1.53	0.76	6.27	0.29	2.68	48.30	1.07	1.61	29.77	0.86	2.90
327	14.23	0.24	122.41	1.53	0.77	6.56	0.31	2.69	47.23	1.06	1.63	30.63	0.88	2.90
328	13.99	0.24	120.88	1.53	0.79	6.87	0.32	2.69	46.17	1.04	1.64	31.51	0.89	2.89
329	13.75	0.24	119.35	1.53	0.80	7.19	0.33	2.70	45.13	1.03	1.66	32.40	0.90	2.89
330	13.51	−0.23	117.82	−1.52	0.81	7.52	+0.35	2.71	44.10	−1.03	1.67	33.30	+0.91	2.89
331	13.28	0.24	116.30	1.52	0.82	7.87	0.37	2.72	43.07	1.01	1.68	34.21	0.92	2.89
332	13.04	0.23	114.78	1.52	0.84	8.24	0.38	2.72	42.06	1.00	1.70	35.13	0.93	2.89
333	12.81	0.23	113.26	1.52	0.85	8.62	0.40	2.73	41.06	1.00	1.71	36.06	0.95	2.88
334	12.58	0.23	111.74	1.51	0.86	9.02	0.42	2.74	40.06	0.99	1.73	37.01	0.96	2.88
335	12.35	−0.22	110.23	−1.51	0.88	9.44	+0.43	2.75	39.07	−0.97	1.74	37.97	+0.97	2.88
336	12.13	0.22	108.72	1.50	0.89	9.87	0.45	2.75	38.10	0.96	1.75	38.94	0.97	2.88
337	11.91	0.22	107.22	1.49	0.90	10.32	0.46	2.76	37.14	0.95	1.77	39.91	0.98	2.88
338	11.69	0.22	105.73	1.49	0.91	10.78	0.48	2.77	36.19	0.95	1.78	40.89	1.00	2.87
339	11.47	0.21	104.24	1.49	0.93	11.26	0.49	2.77	35.24	0.94	1.80	41.89	1.01	2.87
340	11.26	−0.21	102.75	−1.48	0.94	11.75	+0.51	2.78	34.30	−0.92	1.81	42.90	+1.02	2.87
341	11.05	0.21	101.27	1.47	0.95	12.26	0.53	2.78	33.38	0.91	1.82	43.92	1.03	2.87
342	10.84	0.20	99.80	1.47	0.97	12.79	0.54	2.79	32.47	0.90	1.84	44.95	1.04	2.86
343	10.64	0.20	98.33	1.47	0.98	13.33	0.56	2.79	31.57	0.89	1.85	45.99	1.04	2.86
344	10.44	0.20	96.86	1.46	1.00	13.89	0.57	2.80	30.68	0.88	1.87	47.03	1.06	2.86
345	10.24	−0.19	95.40	−1.45	1.01	14.46	+0.59	2.80	29.80	−0.87	1.88	48.09	+1.06	2.85
346	10.05	0.20	93.95	1.45	1.02	15.05	0.60	2.81	28.93	0.85	1.90	49.15	1.07	2.85
347	9.85	0.19	92.50	1.44	1.04	15.65	0.61	2.81	28.08	0.85	1.91	50.22	1.08	2.85
348	9.66	0.19	91.06	1.43	1.05	16.26	0.63	2.82	27.23	0.84	1.93	51.30	1.10	2.85
349	9.47	0.19	89.63	1.42	1.07	16.89	0.65	2.82	26.39	0.82	1.94	52.40	1.10	2.84
350	9.28	−0.18	88.21	−1.42	1.08	17.54	+0.66	2.83	25.57	−0.81	1.96	53.50	+1.11	2.84
351	9.10	0.18	86.79	1.41	1.09	18.20	0.68	2.83	24.76	0.79	1.97	54.61	1.12	2.83
352	8.92	0.18	85.38	1.40	1 11	18.88	0.69	2.84	23.97	0.79	1.99	55.73	1.13	2.83
353	8.74	0.17	83.98	1.40	1.12	19.57	0.71	2.84	23.18	0.78	2.00	56.86	1.13	2.82
354	8.57	0.17	82.58	1.39	1.14	20.28	0.72	2.84	22.40	0.76	2.01	57.99	1.15	2.82
355	8.40	−0.17	81.19	−1.38	1.15	21.00	+0.74	2.85	21.64	−0.75	2.03	59.14	+1.15	2.81
356	8.23	0.16	79.81	1.37	1.16	21.74	0.75	2.85	20.89	0.73	2.04	60.29	1.16	2.80
357	8.07	0.16	78.44	1.37	1.18	22.49	0.76	2.85	20.16	0.73	2.05	61.45	1.17	2.80
358	7.91	0.16	77.07	1.36	1.19	23.25	0.78	2.85	19.43	0.72	2.06	62.62	1.18	2.79
359	7.75	0.15	75.71	1.35	1.21	24.03	0.79	2.86	18.71	0.70	2.08	63.80	1.18	2.79
360	7.60		74.36		1.22	24.82		2.86	18.01		2.09	64.98		2.78

TABLE IX, ARG. 2.—*Continued.*

Arg.	$(v.s.3)$	$(v.c.3)$	$(v.s.4)$	$(v.c.4)$	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	$(\rho.s.3)$	$(\rho.c.3)$
	″	″	″								
300	6.92	3.35	1.43	1.08	110	2848	1581	143	512	15	133
301	6.92	3.35	1.43	1.07	109	2849	1566	144	515	15	134
302	6.93	3.35	1.44	1.06	108	2849	1551	146	517	15	135
303	6.93	3.34	1.44	1.05	107	2850	1535	147	519	16	137
304	6.93	3.34	1.45	1.04	106	2850	1520	149	522	16	138
305	6.94	3.34	1.45	1.03	105	2850	1505	150	524	17	139
306	6.94	3.33	1.46	1.01	104	2850	1490	152	526	17	140
307	6.94	3.33	1.46	1.00	103	2850	1474	153	528	18	141
308	6.94	3.32	1.47	0.99	103	2850	1459	154	531	18	142
309	6.95	3.31	1.47	0.98	102	2849	1444	156	533	19	143
310	6.95	3.31	1.47	0.96	102	2849	1429	157	535	20	144
311	6.95	3.30	1.47	0.95	101	2848	1413	158	538	20	145
312	6.95	3.29	1.48	0.94	101	2847	1398	160	540	21	146
313	6.95	3.28	1.48	0.93	100	2847	1383	162	542	22	147
314	6.94	3.27	1.48	0.92	100	2846	1368	163	545	23	148
315	6.94	3.26	1.48	0.90	99	2845	1352	165	547	24	149
316	6.94	3.25	1.48	0.89	99	2843	1337	167	549	24	150
317	6.94	3.23	1.48	0.88	99	2842	1322	168	551	25	151
318	6.93	3.22	1.48	0.86	98	2840	1307	170	554	26	152
319	6.93	3.21	1.48	0.85	98	2838	1291	171	556	27	153
320	6.92	3.20	1.48	0.84	98	2836	1276	173	558	27	154
321	6.92	3.18	1.48	0.83	98	2834	1261	175	560	28	155
322	6.91	3.17	1.48	0.81	98	2831	1246	176	562	29	156
323	6.91	3.15	1.48	0.80	98	2828	1231	178	564	29	157
324	6.90	3.14	1.48	0.79	99	2825	1216	180	566	30	158
325	6.89	3.12	1.48	0.77	99	2822	1202	181	568	31	159
326	6.88	3.11	1.48	0.76	99	2819	1187	183	570	32	159
327	6.87	3.10	1.47	0.75	100	2815	1172	185	572	33	160
328	6.86	3.08	1.47	0.74	100	2812	1157	186	573	33	161
329	6.85	3.06	1.47	0.72	101	2808	1143	188	575	34	162
330	6.83	3.04	1.46	0.71	101	2804	1128	190	577	35	163
331	6.82	3.03	1.46	0.70	102	2800	1113	192	579	36	164
332	6.81	3.01	1.46	0.69	103	2796	1098	193	581	37	165
333	6.79	2.99	1.45	0.67	103	2792	1084	195	583	38	165
334	6.77	2.97	1.45	0.66	104	2788	1069	197	585	39	166
335	6.75	2.96	1.44	0.65	105	2784	1054	199	587	40	167
336	6.74	2.94	1.44	0.64	106	2779	1039	201	589	41	168
337	6.72	2.92	1.43	0.63	107	2775	1025	203	591	42	169
338	6.70	2.90	1.43	0.61	108	2770	1010	204	592	43	169
339	6.68	2.88	1.42	0.60	109	2765	996	206	594	44	170
340	6.66	2.87	1.42	0.59	110	2760	981	208	596	45	171
341	6.64	2.85	1.41	0.58	111	2755	967	210	598	46	171
342	6.62	2.83	1.40	0.57	112	2750	953	212	599	47	172
343	6.59	2.81	1.40	0.56	114	2744	938	214	601	48	173
344	6.57	2.79	1.39	0.55	115	2739	924	217	603	49	173
345	6.55	2.78	1.38	0.53	116	2733	910	219	605	50	174
346	6.52	2.76	1.37	0.52	117	2727	896	221	606	51	174
347	6.50	2.74	1.36	0.51	119	2721	882	223	608	52	175
348	6.47	2.72	1.36	0.50	121	2715	868	226	610	53	176
349	6.44	2.70	1.35	0.49	122	2709	855	228	611	54	176
350	6.41	2.69	1.34	0.48	124	2703	841	230	613	55	177
351	6.37	2.67	1.33	0.47	126	2696	828	232	614	57	177
352	6.34	2.65	1.32	0.46	128	2689	814	235	616	58	178
353	6.31	2.64	1.31	0.45	130	2682	801	237	617	59	178
354	6.28	2.62	1.30	0.44	132	2675	788	239	619	60	178
355	6.24	2.60	1.29	0.43	134	2668	775	241	620	61	179
356	6.21	2.59	1.28	0.42	136	2660	762	244	621	62	179
357	6.18	2.57	1.26	0.41	139	2653	749	246	623	63	180
358	6.14	2.56	1.25	0.40	141	2645	736	248	624	64	180
359	6.10	2.54	1.24	0.39	144	2637	723	250	626	65	180
360	6.07	2.53	1.23	0.38	146	2629	710	252	627	67	181

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec. var.	(*v.c.*1)	Diff.	Sec. var.	(*v.s.*2)	Diff.	Sec. var.	(*v.c.*2)	Diff.	Sec. var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
360	7.60	−0.15	74.36	−1.34	1.22	24.82	+0.81	2.86	18.01	−0.68	2.09	64.98	+1.19	2.78
361	7.45	0.15	73.02	1.33	1.23	25.63	0.82	2.86	17.33	0.67	2.10	66.17	1.20	2.77
362	7.30	0.15	71.69	1.32	1.25	26.45	0.84	2.87	16.66	0.66	2.12	67.37	1.21	2.77
363	7.15	0.14	70.37	1.32	1.26	27.29	0.85	2.87	16.00	0.65	2.13	68.58	1.21	2.76
364	7.01	0.14	69.05	1.30	1.28	28.14	0.86	2.87	15.35	0.64	2.15	69.79	1.22	2.75
365	6.87	−0.13	67.75	−1.29	1.29	29.00	+0.88	2.88	14.71	−0.62	2.16	71.01	+1.22	2.74
366	6.74	0.13	66.46	1.28	1.31	29.88	0.90	2.88	14.09	0.61	2.17	72.23	1.23	2.74
367	6.61	0.13	65.18	1.28	1.32	30.78	0.91	2.88	13.48	0.60	2.18	73.46	1.24	2.73
368	6.48	0.12	63.90	1.27	1.34	31.69	0.92	2.88	12.88	0.59	2.19	74.70	1.25	2.72
369	6.36	0.12	62.63	1.25	1.35	32.61	0.93	2.89	12.29	0.57	2.21	75.95	1.26	2.71
370	6.24	−0.12	61.38	−1.24	1.37	33.54	+0.95	2.89	11.72	−0.55	2.22	77.21	+1.26	2.71
371	6.12	0.12	60.14	1.23	1.38	34.49	0.96	2.89	11.17	0.54	2.23	78.47	1.27	2.70
372	6.00	0.11	58.91	1.23	1.40	35.45	0.97	2.89	10.63	0.53	2.25	79.74	1.27	2.69
373	5.89	0.11	57.68	1.22	1.41	36.42	0.98	2.89	10.10	0.51	2.26	81.01	1.27	2.68
374	5.78	0.10	56.46	1.20	1.43	37.40	1.00	2.89	9.59	0.50	2.27	82.28	1.28	2.67
375	5.68	−0.11	55.26	−1.19	1.44	38.40	+1.01	2.89	9.09	−0.49	2.29	83.56	+1.29	2.66
376	5.57	0.10	54.07	1.18	1.45	39.41	1.02	2.89	8.60	0.48	2.30	84.85	1.29	2.66
377	5.47	0.09	52.89	1.17	1.47	40.43	1.04	2.89	8.12	0.46	2.31	86.14	1.29	2.65
378	5.38	0.08	51.72	1.15	1.48	41.47	1.05	2.89	7.66	0.45	2.32	87.43	1.30	2.64
379	5.30	0.08	50.57	1.15	1.50	42.52	1.06	2.89	7.21	0.43	2.34	88.73	1.31	2.63
380	5.22	−0.08	49.42	−1.13	1.51	43.58	+1.07	2.89	6.78	−0.42	2.35	90.04	+1.31	2.62
381	5.14	0.07	48.29	1.12	1.52	44.65	1.09	2.89	6.36	0.40	2.36	91.35	1.32	2.61
382	5.07	0.08	47.17	1.11	1.54	45.74	1.10	2.89	5.96	0.39	2.37	92.67	1.32	2.60
383	4.99	0.08	46.06	1.10	1.55	46.84	1.11	2.89	5.57	0.38	2.38	93.99	1.32	2.59
384	4.91	0.07	44.96	1.09	1.57	47.95	1.12	2.89	5.19	0.36	2.39	95.31	1.33	2.58
385	4.84	−0.07	43.87	−1.07	1.58	49.07	+1.13	2.89	4.83	−0.35	2.40	96.64	+1.33	2.57
386	4.77	0.06	42.80	1.06	1.60	50.20	1.14	2.89	4.48	0.33	2.42	97.97	1.33	2.57
387	4.71	0.06	41.74	1.05	1.61	51.34	1.16	2.88	4.15	0.32	2.43	99.30	1.34	2.56
388	4.65	0.05	40.69	1.04	1.63	52.50	1.17	2.88	3.83	0.31	2.44	100.64	1.34	2.55
389	4.60	0.04	39.65	1.02	1.64	53.67	1.18	2.88	3.52	0.29	2.45	101.98	1.35	2.54
390	4.56	−0.04	38.63	−1.01	1.66	54.85	+1.19	2.88	3.23	−0.27	2.46	103.33	+1.35	2.53
391	4.52	0.04	37.62	0.99	1.67	56.04	1.21	2.88	2.96	0.26	2.47	104.68	1.35	2.52
392	4.48	0.03	36.63	0.98	1.69	57.25	1.21	2.88	2.70	0.25	2.48	106.03	1.35	2.51
393	4.45	0.04	35.65	0.98	1.70	58.46	1.22	2.87	2.45	0.23	2.49	107.38	1.36	2.50
394	4.41	0.03	34.67	0.96	1.71	59.68	1.23	2.87	2.22	0.21	2.50	108.74	1.36	2.49
395	4.38	−0.03	33.71	−0.94	1.73	60.91	+1.24	2.87	2.01	−0.20	2.51	110.10	+1.36	2.47
396	4.35	0.02	32.77	0.92	1.74	62.15	1.25	2.87	1.81	0.19	2.52	111.46	1.36	2.46
397	4.33	0.02	31.85	0.91	1.75	63.40	1.27	2.87	1.62	0.17	2.53	112.82	1.36	2.45
398	4.31	0.01	30.94	0.91	1.76	64.67	1.28	2.86	1.45	0.16	2.54	114.18	1.37	2.44
399	4.30	0.01	30.03	0.89	1.78	65.95	1.28	2.86	1.29	0.15	2.55	115.55	1.37	2.43
400	4.29	−0.01	29.14	−0.87	1.79	67.23	+1.29	2.86	1.14	−0.13	2.56	116.92	+1.37	2.42
401	4.28	0.00	28.27	0.85	1.80	68.52	1.30	2.86	1.01	0.11	2.57	118.29	1.37	2.41
402	4.28	0.00	27.42	0.85	1.82	69.82	1.31	2.85	0.90	0.10	2.58	119.66	1.37	2.40
403	4.28	0.00	26.57	0.84	1.83	71.13	1.32	2.85	0.80	0.08	2.59	121.03	1.38	2.39
404	4.28	+0.01	25.73	0.82	1.85	72.45	1.33	2.84	0.72	0.07	2.60	122.41	1.37	2.38
405	4.29	+0.01	24.91	−0.80	1.86	73.78	+1.34	2.84	0.65	−0.05	2.60	123.78	+1.38	2.36
406	4.30	0.02	24.11	0.78	1.88	75.12	1.35	2.84	0.60	0.04	2.61	125.16	1.37	2.35
407	4.32	0.02	23.33	0.77	1.89	76.47	1.36	2.83	0.56	0.03	2.62	126.53	1.38	2.34
408	4.34	0.02	22.56	0.76	1.91	77.83	1.36	2.83	0.53	0.01	2.63	127.91	1.37	2.33
409	4.36	0.03	21.80	0.74	1.92	79.19	1.37	2.82	0.52	0.00	2.64	129.28	1.37	2.32
410	4.39	+0.03	21.06	−0.73	1.94	80.56	+1.38	2.82	0.52	+0.02	2.65	130.65	+1.37	2.31
411	4.42	0.04	20.33	0.71	1.95	81.94	1.39	2.82	0.54	0.03	2.66	132.02	1.38	2.30
412	4.46	0.04	19.62	0.69	1.97	83.33	1.40	2.81	0.57	0.05	2.67	133.40	1.37	2.29
413	4.50	0.04	18.93	0.68	1.98	84.73	1.41	2.81	0.62	0.06	2.67	134.77	1.37	2.27
414	4.54	0.05	18.25	0.67	1.99	86.14	1.41	2.80	0.68	0.08	2.68	136.14	1.37	2.26
415	4.59	+0.05	17.58	−0.65	2.01	87.55	+1.42	2.80	0.76	+0.09	2.69	137.51	+1.37	2.25
416	4.64	0.06	16.93	0.63	2.02	88.97	1.42	2.79	0.85	0.11	2.70	138.88	1.37	2.24
417	4.70	0.06	16.30	0.63	2.03	90.39	1.43	2.79	0.96	0.12	2.71	140.25	1.37	2.22
418	4.76	0.06	15.67	0.61	2.04	91.82	1.44	2.78	1.08	0.14	2.71	141.62	1.37	2.21
419	4.82	0.06	15.06	0.59	2.06	93.26	1.45	2.78	1.22	0.15	2.72	142.99	1.36	2.20
420	4.88		14.47		2.07	94.71		2.77	1.37		2.73	144.35		2.19

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″	″							
360	6.07	2.53	1.23	0.38	146	2629	710	252	627	67	181
361	6.02	2.51	1.22	0.38	148	2621	697	254	628	68	181
362	5.98	2.50	1.21	0.37	151	2613	685	256	629	69	182
363	5.94	2.49	1.20	0.36	153	2605	672	259	631	70	182
364	5.90	2.48	1.19	0.35	156	2596	660	261	632	71	182
365	5.85	2.46	1.17	0.35	159	2588	647	263	633	72	183
366	5.81	2.45	1.16	0.34	162	2579	635	265	634	74	183
367	5.76	2.44	1.15	0.34	164	2570	623	267	635	75	183
368	5.72	2.44	1.14	0.33	167	2562	610	270	636	76	183
369	5.67	2.43	1.12	0.32	170	2553	598	272	638	77	184
370	5.63	2.42	1.11	0.32	173	2544	586	274	639	78	184
371	5.58	2.41	1.10	0.31	176	2535	574	276	640	79	184
372	5.53	2.41	1.08	0.31	180	2526	562	279	641	80	184
373	5.48	2.40	1.07	0.30	183	2516	550	281	642	81	184
374	5.43	2.39	1.05	0.30	186	2507	539	283	643	82	184
375	5.38	2.39	1.04	0.29	190	2498	528	285	645	83	184
376	5.33	2.39	1.03	0.29	194	2488	517	288	646	84	184
377	5.28	2.38	1.01	0.29	197	2479	506	290	647	85	184
378	5 22	2.38	1.00	0.28	201	2469	495	292	648	86	184
379	5.17	2.38	0.98	0.28	205	2460	485	294	649	87	184
380	5.12	2.38	0.97	0.28	208	2450	474	297	650	88	184
381	5.07	2.38	0.96	0.28	212	2440	463	299	651	89	184
382	5.01	2.38	0.94	0.27	216	2429	453	302	652	90	184
383	4.96	2.39	0.93	0.27	220	2419	443	304	653	91	184
384	4.90	2.39	0.91	0.27	224	2408	433	306	654	92	184
385	4.84	2.39	0.90	0.27	228	2397	423	309	655	94	184
386	4.79	2.40	0.88	0.27	232	2386	413	312	656	95	184
387	4.73	2.40	0.86	0.27	236	2374	404	314	657	96	184
388	4.67	2.41	0.85	0.27	240	2363	394	317	657	97	183
389	4.61	2.42	0.84	0.27	245	2351	384	319	658	98	183
390	4.55	2.43	0.82	0.27	249	2340	375	322	659	99	183
391	4 49	2.44	0.81	0.27	253	2328	366	324	660	100	183
392	4.43	2.46	0.79	0.27	258	2316	357	327	661	102	183
393	4.37	2.47	0.78	0.27	262	2305	348	329	662	103	182
394	4.31	2.48	0.76	0.27	266	2293	339	332	662	104	182
395	4.24	2.49	0.75	0.27	271	2281	330	334	663	105	182
396	4.18	2.51	0.73	0.28	276	2269	322	336	664	106	182
397	4.12	2.53	0.72	0.28	280	2257	313	339	665	107	182
398	4.06	2.55	0.70	0.28	285	2245	304	341	666	109	181
399	4.00	2.57	0.69	0.28	290	2232	296	344	667	110	181
400	3.93	2.59	0.67	0.29	294	2220	288	346	667	111	181
401	3.87	2.62	0.66	0.29	299	2207	280	348	668	112	181
402	3.80	2.64	0.65	0.30	304	2195	273	351	668	113	180
403	3.74	2.66	0.63	0.30	309	2182	265	353	669	114	180
404	3.68	2.69	0.62	0.31	314	2170	258	355	669	115	180
405	3.62	2.72	0.60	0.31	320	2157	251	358	669	116	179
406	3.55	2.75	0.59	0.31	325	2144	244	361	669	116	179
407	3.49	2.78	0.57	0.32	331	2131	237	363	670	117	179
408	3.43	2.81	0.56	0.33	337	2119	230	365	670	118	178
409	3.36	2.84	0.55	0.33	342	2106	224	368	670	119	178
410	3.30	2.88	0.53	0.34	348	2093	217	371	670	120	178
411	3.24	2.92	0.52	0.35	354	2080	211	373	670	121	177
412	3.17	2.95	0.50	0.36	359	2066	205	376	670	122	177
413	3.11	2.99	0.49	0.36	365	2053	199	378	671	122	176
414	3.04	3.03	0.48	0.37	371	2039	193	381	671	123	176
415	2.98	3.07	0.46	0.38	377	2026	187	382	671	124	175
416	2.92	3.11	0.45	0.38	383	2012	181	386	671	125	174
417	2.85	3.15	0.44	0.39	389	1998	176	388	671	126	174
418	2.79	3.20	0.42	0.40	395	1985	170	391	671	126	173
419	2.73	3.24	0.41	0.41	401	1971	165	393	672	127	173
420	2.67	3.29	0.40	0.42	407	1957	160	396	672	128	172

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec.var.	(v.c.1)	Diff.	Sec.var.	(v.s.2)	Diff.	Sec.var.	(v.c.2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
420	4.88	+0.07	14.47	−0.57	2.07	94.71	+1.45	2.77	1.37	+0.17	2.73	144.35	+1.37	2.19
421	4.95	0.07	13.90	0.55	2.08	96.16	1.46	2.76	1.54	0.18	2.74	145.72	1.37	2.18
422	5.02	0.08	13.35	0.54	2.10	97.62	1.47	2.76	1.72	0.20	2.74	147.09	1.36	2.16
423	5.10	0.08	12.81	0.53	2.11	99.09	1.47	2.75	1.92	0.21	2.75	148.45	1.35	2.15
424	5.18	0.08	12.28	0.51	2.13	100.56	1.48	2.75	2.13	0.22	2.75	149.80	1.35	2.13
425	5.26	+0.09	11.77	−0.49	2.14	102.04	+1.48	2.74	2.35	+0.24	2.76	151.15	+1.35	2.12
426	5.35	0.09	11.28	0.48	2.15	103.52	1.49	2.73	2.59	0.25	2.77	152.50	1.34	2.11
427	5.44	0.09	10.80	0.46	2.17	105.01	1.49	2.73	2.84	0.27	2.77	153.84	1.35	2.09
428	5.53	0.10	10.34	0.45	2.18	106.50	1.50	2.72	3.11	0.28	2.78	155.19	1.35	2.08
429	5.63	0.10	9.89	0.43	2.20	108.00	1.51	2.72	3.39	0.30	2.78	156.54	1.34	2.06
430	5.73	+0.10	9.46	−0.41	2.21	109.51	+1.51	2.71	3.69	+0.31	2.79	157.88	+1.34	2.05
431	5.83	0.11	9.05	0.39	2.22	111.02	1.51	2.70	4.00	0.33	2.79	159.22	1.33	2.04
432	5.94	0.11	8.66	0.38	2.23	112.53	1.51	2.69	4.33	0.34	2.80	160.55	1.33	2.02
433	6.05	0.12	8.28	0.37	2.24	114.04	1.52	2.68	4.67	0.35	2.80	161.88	1.32	2.01
434	6.17	0.12	7.91	0.35	2.25	115.56	1.53	2.67	5.02	0.37	2.81	163.20	1.32	1.99
435	6.29	+0.12	7.56	−0.33	2.27	117.09	+1.53	2.66	5.39	+0.38	2.81	164.52	+1.32	1.98
436	6.41	0.13	7.23	0.31	2.28	118.62	1.54	2.66	5.77	0.40	2.82	165.84	1.31	1.97
437	6.54	0.13	6.92	0.30	2.29	120.16	1.54	2.65	6.17	0.41	2.82	167.15	1.31	1.95
438	6.67	0.13	6.62	0.28	2.30	121.70	1.54	2.64	6.58	0.42	2.83	168.46	1.30	1.94
439	6.80	0.14	6.34	0.27	2.31	123.24	1.54	2.63	7.00	0.44	2.83	169.76	1.30	1.92
440	6.94	+0.14	6.07	−0.25	2.32	124.78	+1.55	2.62	7.44	+0.45	2.84	171.06	+1.29	1.91
441	7.08	0.14	5.82	0.23	2.33	126.33	1.55	2.61	7.89	0.47	2.84	172.35	1.29	1.90
442	7.22	0.15	5.59	0.21	2.34	127.88	1.55	2.60	8.36	0.48	2.85	173.64	1.29	1.88
443	7.37	0.15	5.38	0.20	2.36	129.43	1.55	2.59	8.84	0.49	2.85	174.93	1.28	1.87
444	7.52	0.15	5.18	0.18	2.37	130.98	1.56	2.58	9.33	0.51	2.86	176.21	1.27	1.85
445	7.67	+0.16	5.00	−0.17	2.38	132.54	+1.56	2.57	9.84	+0.52	2.86	177.48	+1.27	1.84
446	7.83	0.16	4.83	0.15	2.39	134.10	1.56	2.57	10.36	0.53	2.86	178.75	1.26	1.83
447	7.99	0.16	4.68	0.13	2.40	135.66	1.56	2.56	10.89	0.55	2.87	180.01	1.25	1.81
448	8.15	0.16	4.55	0.12	2.42	137.22	1.56	2.55	11.44	0.56	2.87	181.26	1.25	1.80
449	8.31	0.17	4.43	0.10	2.43	138.78	1.57	2.54	12.00	0.57	2.88	182.51	1.24	1.78
450	8.48	+0.17	4.33	−0.08	2.44	140.35	+1.57	2.53	12.57	+0.59	2.88	183.75	+1.24	1.77
451	8.65	0.17	4.25	0.06	2.45	141.92	1.57	2.52	13.16	0.60	2.88	184.99	1.23	1.76
452	8.82	0.18	4.19	0.05	2.46	143.49	1.57	2.51	13.76	0.61	2.88	186.22	1.22	1.74
453	9.00	0.18	4.14	0.04	2.47	145.06	1.57	2.50	14.37	0.62	2.89	187.44	1.22	1.73
454	9.18	0.18	4.10	0.01	2.48	146.63	1.57	2.49	14.99	0.64	2.89	188.66	1.21	1.71
455	9.36	+0.18	4.09	0.00	2.49	148.20	+1.57	2.47	15.63	+0.65	2.89	189.87	+1.20	1.70
456	9.54	0.19	4.09	+0.02	2.50	149.77	1.57	2.46	16.28	0.67	2.89	191.07	1.20	1.69
457	9.73	0.19	4.11	0.03	2.51	151.34	1.57	2.45	16.95	0.68	2.89	192.27	1.19	1.67
458	9.92	0.19	4.14	0.04	2.52	152.91	1.57	2.44	17.63	0.69	2.90	193.46	1.18	1.66
459	10.11	0.19	4.18	0.07	2.53	154.48	1.57	2.43	18.32	0.70	2.90	194.64	1.17	1.64
460	10.30	+0.20	4.25	+0.08	2.54	156.05	+1.57	2.42	19.02	+0.71	2.90	195.81	+1.17	1.63
461	10.50	0.20	4.33	0.10	2.55	157.62	1.57	2.41	19.73	0.72	2.90	196.98	1.16	1.61
462	10.70	0.20	4.43	0.12	2.56	159.19	1.57	2.40	20.45	0.74	2.90	198.14	1.15	1.60
463	10.90	0.20	4.55	0.13	2.57	160.76	1.57	2.39	21.19	0.75	2.90	199.29	1.14	1.58
464	11.10	0.21	4.68	0.15	2.58	162.33	1.56	2.38	21.94	0.77	2.90	200.43	1.13	1.57
465	11.31	+0.21	4.83	+0.16	2.59	163.89	+1.56	2.36	22.71	+0.77	2.91	201.56	+1.12	1.55
466	11.52	0.21	4.99	0.18	2.59	165.45	1.56	2.35	23.48	0.79	2.91	202.68	1.11	1.53
467	11.73	0.21	5.17	0.20	2.60	167.01	1.56	2.34	24.27	0.80	2.91	203.79	1.11	1.52
468	11.94	0.21	5.37	0.22	2.61	168.57	1.56	2.33	25.07	0.81	2.91	204.90	1.10	1.50
469	12.15	0.22	5.59	0.23	2.62	170.13	1.55	2.32	25.88	0.82	2.91	206.00	1.09	1.49
470	12.37	+0.22	5.82	+0.25	2.63	171.68	+1.55	2.31	26.70	+0.83	2.91	207.09	+1.08	1.47
471	12.59	0.22	6.07	0.27	2.64	173.23	1.55	2.30	27.53	0.84	2.91	208.17	1.07	1.46
472	12.81	0.22	6.34	0.28	2.65	174.78	1.55	2.29	28.37	0.85	2.91	209.24	1.06	1.44
473	13.03	0.23	6.62	0.29	2.65	176.33	1.54	2.27	29.22	0.87	2.90	210.30	1.06	1.43
474	13.26	0.23	6.91	0.31	2.66	177.87	1.54	2.26	30.09	0.88	2.90	211.36	1.04	1.41
475	13.49	+0.23	7.22	+0.33	2.67	179.41	+1.54	2.25	30.97	+0.89	2.90	212.40	+1.03	1.40
476	13.72	0.23	7.55	0.35	2.68	180.95	1.53	2.24	31.86	0.90	2.90	213.43	1.02	1.39
477	13.95	0.23	7.90	0.36	2.69	182.48	1.53	2.23	32.76	0.91	2.90	214.45	1.02	1.37
478	14.18	0.23	8.26	0.37	2.69	184.01	1.53	2.21	33.67	0.92	2.89	215.47	1.01	1.36
479	14.41	0.24	8.63	0.39	2.70	185.54	1.52	2.20	34.59	0.93	2.89	216.48	0.99	1.34
480	14.65		9.02		2.71	187.06		2.19	35.52		2.89	217.47		1.33

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	"	"	"	"							
420	2.67	3.29	0.40	0.42	407	1957	160	396	672	128	172
421	2.61	3.34	0.39	0.43	413	1943	155	399	672	129	172
422	2.55	3.39	0.38	0.44	419	1930	150	402	673	129	171
423	2.49	3.44	0.36	0.45	425	1916	146	405	673	130	171
424	2.43	3.50	0.35	0.46	431	1903	141	407	673	131	171
425	2.37	3.55	0.34	0.47	437	1889	136	410	673	132	170
426	2.31	3.61	0.33	0.49	443	1875	132	413	673	133	170
427	2.25	3.66	0.32	0.50	450	1862	128	416	673	133	169
428	2.20	3.72	0.31	0.51	456	1848	124	418	673	134	169
429	2.14	3.78	0.30	0.52	463	1834	120	421	673	135	168
430	2.08	3.84	0.29	0.53	469	1820	117	424	673	136	167
431	2.03	3.90	0.28	0.54	476	1806	114	426	673	136	167
432	1.97	3.96	0.27	0.55	482	1791	111	429	673	137	166
433	1.92	4.03	0.26	0.57	489	1777	108	432	673	138	165
434	1.86	4.10	0.25	0.58	496	1762	105	434	673	139	165
435	1.81	4.16	0.24	0.59	503	1748	102	437	672	139	164
436	1.76	4.23	0.23	0.60	510	1733	100	440	672	140	163
437	1.70	4.30	0.22	0.62	517	1719	98	442	672	141	163
438	1.65	4.37	0.22	0.63	524	1704	96	445	671	142	162
439	1.60	4.45	0.21	0.64	531	1690	94	447	671	142	161
440	1.55	4.52	0.20	0.66	539	1675	92	450	671	143	160
441	1.50	4.60	0.19	0.67	546	1660	91	453	670	144	159
442	1.46	4.67	0.18	0.68	553	1645	90	455	670	144	159
443	1.41	4.75	0.18	0.70	560	1631	89	458	669	145	158
444	1.37	4.83	0.17	0.71	567	1616	88	460	669	145	157
445	1.32	4.91	0.17	0.73	574	1601	87	463	669	146	156
446	1.28	4.99	0.16	0.74	582	1586	87	466	668	146	156
447	1.24	5.07	0.16	0.75	589	1571	86	468	668	147	155
448	1.20	5.15	0.15	0.77	596	1557	86	471	667	147	154
449	1.16	5.24	0.14	0.79	603	1542	86	473	667	147	153
450	1.12	5.32	0.14	0.80	610	1527	86	476	667	148	152
451	1.08	5.40	0.13	0.82	618	1512	87	479	666	148	152
452	1.04	5.49	0.13	0.83	625	1497	87	481	666	149	151
453	1.01	5.57	0.12	0.85	633	1483	88	484	665	149	150
454	0.97	5.66	0.12	0.86	640	1468	88	486	665	149	149
455	0.94	5.75	0.12	0.88	648	1453	89	489	664	150	148
456	0.91	5.84	0.11	0.89	656	1438	90	492	664	150	148
457	0.88	5.94	0.11	0.90	664	1423	92	494	663	150	147
458	0.85	6.03	0.11	0.92	671	1409	93	497	662	151	146
459	0.82	6.13	0.10	0.94	679	1394	95	499	661	151	145
460	0.80	6.23	0.10	0.95	687	1379	96	502	661	151	144
461	0.77	6.32	0.10	0.97	695	1364	98	505	660	151	144
462	0.75	6.42	0.10	0.99	703	1349	100	507	660	152	143
463	0.73	6.51	0.10	1.00	711	1335	102	510	659	152	142
464	0.71	6.61	0.10	1.02	719	1320	105	512	658	152	142
465	0.69	6.71	0.09	1.03	727	1305	108	515	657	152	141
466	0.68	6.81	0.09	1.05	735	1290	110	517	657	153	140
467	0.66	6.91	0.09	1.06	743	1275	113	520	656	153	139
468	0.64	7.01	0.09	1.08	751	1261	116	523	655	153	139
469	0.63	7.11	0.09	1.10	759	1246	119	525	654	154	138
470	0.62	7.21	0.09	1.11	767	1232	123	528	653	154	137
471	0.62	7.31	0.10	1.13	775	1217	127	531	652	154	136
472	0.61	7.42	0.10	1.15	784	1203	131	533	651	155	136
473	0.60	7.52	0.10	1.16	792	1189	135	536	649	155	135
474	0.60	7.62	0.10	1.18	800	1175	139	539	648	155	134
475	0.60	7.72	0.10	1.19	809	1161	143	542	647	156	133
476	0.60	7.83	0.11	1.21	817	1147	148	545	646	156	133
477	0.60	7.94	0.11	1.23	826	1133	153	548	644	156	132
478	0.60	8.04	0.11	1.24	834	1119	158	551	643	156	131
479	0.61	8.15	0.12	1.25	843	1105	163	554	642	156	130
480	0.61	8.26	0.12	1.27	851	1091	168	556	640	156	130

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
480	14.65	+0.24	9.02	+0.41	2.71	187.06	+1.51	2.19	35.52	+0.94	2.89	217.47	+0.99	1.33
481	14.89	0.24	9.43	0.42	2.72	188.57	1.51	2.18	36.46	0.95	2.89	218.46	0.98	1.32
482	15.13	0.24	9.85	0.44	2.72	190.08	1.51	2.16	37.41	0.96	2.89	219.44	0.96	1.30
483	15.37	0.24	10.29	0.45	2.73	191.59	1.50	2.15	38.37	0.98	2.88	220.40	0.95	1.29
484	15.61	0.24	10.74	0.47	2.74	193.09	1.50	2.14	39.35	0.98	2.88	221.35	0.94	1.27
485	15.85	+0.24	11.21	+0.48	2.74	194.59	+1.49	2.12	40.33	+0.99	2.88	222.29	+0.93	1.26
486	16.09	0.24	11.69	0.50	2.75	196.08	1.49	2.11	41.32	1.00	2.88	223.22	0.91	1.25
487	16.33	0.25	12.19	0.52	2.76	197.57	1.48	2.10	42.32	1.01	2.88	224.13	0.91	1.23
488	16.58	0.25	12.71	0.53	2.77	199.05	1.48	2.09	43.33	1.02	2.87	225.04	0.90	1.22
489	16.83	0.25	13.24	0.55	2.77	200.53	1.47	2.07	44.35	1.03	2.87	225.94	0.89	1.20
490	17.08	+0.25	13.79	+0.56	2.78	202.00	+1.46	2.06	45.38	+1.04	2.87	226.83	+0.88	1.19
491	17.33	0.25	14.35	0.58	2.79	203.46	1.46	2.05	46.42	1.04	2.87	227.71	0.87	1.18
492	17.58	0.25	14.93	0.59	2.79	204.92	1.45	2.03	47.46	1.05	2.86	228.58	0.85	1.16
493	17.83	0.26	15.52	0.61	2.80	206.37	1.44	2.02	48.51	1.07	2.86	229.43	0.84	1.15
494	18.09	0.25	16.13	0.62	2.80	207.81	1.43	2.00	49.58	1.07	2.86	230.27	0.83	1.13
495	18.34	+0.26	16.75	+0.64	2.81	209.24	+1.43	1.99	50.65	+1.08	2.85	231.10	+0.82	1.12
496	18.60	0.25	17.39	0.65	2.81	210.67	1.42	1.98	51.73	1.09	2.85	231.92	0.81	1.10
497	18.85	0.26	18.04	0.67	2.82	212.09	1.41	1.96	52.82	1.10	2.85	232.73	0.79	1.09
498	19.11	0.25	18.71	0.68	2.82	213.50	1.40	1.95	53.92	1.11	2.85	233.52	0.78	1.07
499	19.36	0.26	19.39	0.70	2.83	214.90	1.40	1.93	55.03	1.11	2.84	234.30	0.77	1.06
500	19.62	+0.26	20.09	+0.71	2.83	216.30	+1.39	1.92	56.14	+1.12	2.84	235.07	+0.76	1.04
501	19.88	0.26	20.80	0.72	2.83	217.69	1.38	1.91	57.26	1.12	2.83	235.83	0.75	1.03
502	20.14	0.26	21.52	0.74	2.84	219.07	1.37	1.89	58.38	1.14	2.83	236.58	0.73	1.01
503	20.40	0.26	22.26	0.75	2.84	220.44	1.37	1.88	59.52	1.15	2.82	237.31	0.72	1.00
504	20.66	0.26	23.01	0.76	2.84	221.81	1.36	1.86	60.67	1.15	2.82	238.03	0.71	0.99
505	20.92	+0.26	23.77	+0.78	2.85	223.17	+1.35	1.85	61.82	+1.16	2.81	238.74	+0.70	0.97
506	21.18	0.26	24.55	0.79	2.85	224.52	1.33	1.84	62.98	1.16	2.80	239.44	0.68	0.96
507	21.44	0.25	25.34	0.81	2.85	225.85	1.33	1.82	64.14	1.17	2.80	240.12	0.67	0.95
508	21.69	0.26	26.15	0.82	2.85	227.18	1.32	1.81	65.31	1.18	2.79	240.79	0.66	0.94
509	21.95	0.26	26.97	0.84	2.86	228.50	1.31	1.79	66.49	1.19	2.79	241.45	0.65	0.92
510	22.21	+0.26	27.81	+0.85	2.86	229.81	+1.30	1.78	67.68	+1.19	2.78	242.10	+0.63	0.91
511	22.47	0.25	28.66	0.86	2.86	231.11	1.28	1.77	68.87	1.20	2.77	242.73	0.63	0.90
512	22.72	0.26	29.52	0.88	2.87	232.39	1.28	1.75	70.07	1.21	2.77	243.36	0.62	0.88
513	22.98	0.26	30.40	0.88	2.87	233.67	1.27	1.74	71.28	1.21	2.76	243.98	0.60	0.87
514	23.24	0.25	31.28	0.90	2.87	234.94	1.26	1.72	72.49	1.22	2.75	244.58	0.58	0.86
515	23.49	+0.26	32.18	+0.91	2.88	236.20	+1.25	1.71	73.71	+1.22	2.74	245.16	+0.55	0.84
516	23.75	0.25	33.09	0.92	2.88	237.45	1.24	1.69	74.93	1.23	2.74	245.71	0.55	0.83
517	24.00	0.26	34.01	0.94	2.88	238.69	1.23	1.68	76.16	1.23	2.73	246.26	0.54	0.82
518	24.26	0.25	34.95	0.95	2.88	239.92	1.21	1.66	77.39	1.24	2.72	246.80	0.53	0.81
519	24.51	0.25	35.90	0.96	2.89	241.13	1.20	1.65	78.63	1.24	2.72	247.33	0.51	0.79
520	24.76	+0.26	36.86	+0.97	2.89	242.33	+1.19	1.63	79.87	+1.25	2.71	247.84	+0.50	0.78
521	25.02	0.25	37.83	0.99	2.89	243.52	1.18	1.62	81.12	1.25	2.70	248.34	0.49	0.77
522	25.27	0.25	38.82	1.00	2.89	244.70	1.17	1.60	82.37	1.25	2.69	248.83	0.47	0.75
523	25.52	0.25	39.82	1.01	2.89	245.87	1.16	1.59	83.62	1.26	2.68	249.30	0.46	0.74
524	25.77	0.25	40.83	1.02	2.89	247.03	1.15	1.57	84.88	1.26	2.67	249.76	0.45	0.73
525	26.02	+0.25	41.85	+1.03	2.89	248.18	+1.13	1.56	86.14	+1.27	2.66	250.21	+0.43	0.71
526	26.27	0.25	42.88	1.04	2.89	249.31	1.12	1.55	87.41	1.28	2.66	250.64	0.42	0.70
527	26.52	0.24	43.92	1.05	2.89	250.43	1.10	1 53	88.69	1.28	2.65	251.06	0.41	0.69
528	26.76	0.24	44.97	1.07	2.89	251.53	1.10	1.52	89.97	1.28	2.64	251.47	0.39	0.68
529	27.00	0.24	46.04	1.08	2.89	252.63	1.09	1.50	91.25	1.29	2.63	251.86	0.38	0.66
530	27.24	+0.24	47.12	+1.09	2.89	253.72	+1.07	1 49	92.54	+1.29	2.62	252.24	+0.36	0.65
531	27.48	0.24	48.21	1.10	2.89	254.79	1.06	1.48	93.83	1.29	2.61	252.60	0.35	0.64
532	27.72	0.24	49.31	1.11	2.89	255.85	1.05	1.46	95.12	1.30	2.60	252.95	0.34	0.63
533	27.96	0.23	50.42	1.11	2.89	256.90	1.03	1.45	96.42	1.30	2.59	253.29	0.32	0.62
534	28.19	0.23	51.53	1.13	2.89	257.93	1.02	1.43	97.72	1.30	2.58	253.61	0.31	0.61
535	28.42	+0.23	52.66	+1.14	2.88	258.95	+1.01	1.42	99.02	+1.30	2.58	253.92	+0.30	0.60
536	28.65	0.23	53.80	1.15	2.88	259.96	0.99	1.40	100.32	1.31	2.57	254.22	0.28	0.58
537	28.88	0.23	54.95	1.16	2.88	260.95	0.98	1.39	101.63	1.31	2.56	254.50	0.27	0.57
538	29.11	0.22	56.11	1.16	2.88	261.93	0.97	1.37	102 94	1.31	2.55	254.77	0.25	0.56
539	29.33	0.22	57.27	1.18	2.88	262.90	0.95	1.36	104.25	1.31	2.54	255.02	0.24	0.55
540	29.55		58.45		2.88	263.85		1.34	105.56		2.53	255.26		0.54

TABLE IX, Arg. 2.—*Continued.*

Arg.	$(v.s.3)$	$(v.c.3)$	$(v.s.4)$	$(v.c.4)$	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	$(\rho.s.3)$	$(\rho.c.3)$
	″	″	″	″							
480	0.61	8.26	0.11	1.27	851	1091	168	556	640	156	130
481	0.62	8.36	0.12	1.28	860	1076	173	559	639	156	129
482	0.64	8.47	0.13	1.30	868	1063	179	561	637	156	128
483	0.65	8.58	0.13	1.31	877	1049	184	564	636	156	128
484	0.66	8.69	0.14	1.33	885	1035	190	566	634	156	127
485	0.67	8.79	0.14	1.35	894	1021	196	569	633	156	126
486	0.69	8.90	0.15	1.36	902	1007	202	571	632	156	125
487	0.71	9.01	0.15	1.37	911	994	208	574	630	156	124
488	0.73	9.11	0.16	1.39	919	980	214	576	629	156	124
489	0.75	9.22	0.17	1.40	927	966	220	579	627	156	123
490	0.77	9.33	0.17	1.42	936	953	227	581	626	156	122
491	0.80	9.43	0.18	1.43	945	940	234	584	624	156	121
492	0.82	9.54	0.19	1.44	954	926	241	586	623	156	120
493	0.85	9.65	0.19	1.46	963	913	249	589	621	156	120
494	0.88	9.75	0.20	1.47	972	900	256	591	619	156	119
495	0.91	9.86	0.21	1.49	981	887	264	594	618	156	118
496	0.94	9.97	0.22	1.50	990	874	272	596	616	156	117
497	0.98	10.07	0.23	1.51	999	861	280	599	615	156	117
498	1.02	10.18	0.23	1.53	1008	848	288	601	613	155	116
499	1.06	10.29	0.24	1.54	1017	835	296	604	611	155	115
500	1.10	10.39	0.25	1.55	1026	822	304	606	610	155	115
501	1.14	10.50	0.26	1.57	1035	810	312	608	608	155	114
502	1.19	10.60	0.27	1.58	1044	797	321	611	606	155	114
503	1.24	10.70	0.28	1.59	1053	785	329	613	604	154	113
504	1.29	10.81	0.29	1.60	1063	772	338	615	602	154	113
505	1.34	10.91	0.30	1.62	1072	760	346	617	600	154	112
506	1.39	11.01	0.31	1.63	1081	748	355	620	598	154	112
507	1.44	11.12	0.32	1.64	1090	736	364	622	596	154	111
508	1.50	11.22	0.33	1.65	1099	724	373	624	594	154	110
509	1.56	11.32	0.34	1.66	1108	712	382	626	592	153	110
510	1.62	11.42	0.35	1.67	1117	700	391	628	590	153	109
511	1.68	11.52	0.37	1.68	1126	688	401	631	588	153	109
512	1.74	11.62	0.38	1.69	1135	677	410	633	585	153	108
513	1.80	11.71	0.39	1.70	1144	666	420	635	583	153	108
514	1.87	11.81	0.40	1.71	1153	654	430	637	581	153	107
515	1.94	11.91	0.41	1.72	1162	643	440	639	578	153	106
516	2.01	12.00	0.42	1.73	1171	632	450	642	576	153	106
517	2.08	12.09	0.43	1.74	1180	621	460	644	573	152	106
518	2.16	12.19	0.45	1.75	1189	610	471	646	571	152	105
519	2.23	12.28	0.46	1.76	1197	599	481	648	569	152	105
520	2.31	12.37	0.47	1.77	1206	588	492	650	566	152	104
521	2.39	12.46	0.49	1.78	1215	577	503	652	564	152	104
522	2.47	12.55	0.50	1.79	1224	567	514	654	561	151	103
523	2.55	12.63	0.51	1.79	1233	557	525	656	558	151	103
524	2.63	12.72	0.52	1.80	1242	546	536	658	556	151	103
525	2.71	12.81	0.54	1.81	1250	536	548	660	553	151	102
526	2.80	12.89	0.55	1.82	1259	526	559	662	550	151	102
527	2.88	12.98	0.57	1.83	1267	517	571	664	548	150	101
528	2.97	13.06	0.58	1.83	1276	507	583	666	545	150	101
529	3.06	13.14	0.60	1.84	1284	498	595	668	542	150	100
530	3.15	13.22	0.61	1.85	1293	488	607	670	540	150	100
531	3.24	13.30	0.62	1.85	1301	479	618	672	537	150	100
532	3.34	13.37	0.64	1.86	1310	470	630	673	535	149	99
533	3.43	13.45	0.65	1.86	1318	461	642	675	532	149	99
534	3.53	13.52	0.67	1.87	1327	452	654	677	529	149	98
535	3.63	13.59	0.68	1.87	1335	443	666	678	527	149	98
536	3.73	13.66	0.69	1.88	1343	434	678	680	524	148	98
537	3.83	13.73	0.71	1.88	1352	426	690	682	521	148	98
538	3.93	13.80	0.72	1.89	1360	417	702	683	519	148	97
539	4.03	13.87	0.74	1.89	1368	409	714	685	516	148	97
540	4.13	13.93	0.75	1.90	1376	401	726	686	513	147	97

TABLE IX, Arg. 2.—*Concluded.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
540	29.55	+0.22	58.45	+1.18	2.88	263.85	+0.94	1.34	105.56	+1.32	2.53	255.26	+0.23	0.54
541	29.77	0.22	59.63	1.19	2.88	264.79	0.92	1.33	106.88	1.31	2.52	255.49	0.21	0.53
542	29.99	0.22	60.82	1.20	2.88	265.71	0.91	1.32	108.19	1.32	2.51	255.70	0.20	0.52
543	30.21	0.22	62.02	1.22	2.87	266.62	0.90	1.30	109.51	1.32	2.50	255.90	0.18	0.51
544	30.43	0.21	63.24	1.22	2.87	267.52	0.88	1.29	110.83	1.32	2.49	256.08	0.17	0.50
545	30.64	+0.21	64.46	+1.23	2.87	268.40	+0.87	1.27	112.15	+1.32	2.47	256.25	+0.16	0.49
546	30.85	0.21	65.69	1.24	2.87	269.27	0.85	1.26	113.47	1.33	2.46	256.41	0.14	0.48
547	31.06	0.21	66.93	1.25	2.87	270.12	0.84	1.25	114.80	1.32	2.45	256.55	0.13	0.47
548	31.27	0.20	68.18	1.25	2.86	270.96	0.83	1.23	116.12	1.33	2.44	256.68	0.11	0.46
549	31.47	0.20	69.43	1.26	2.86	271.79	0.81	1.22	117.45	1.32	2.43	256.79	0.10	0.45
550	31.67	+0.20	70.69	+1.27	2.86	272.60	+0.80	1.21	118.77	+1.32	2.42	256.89	+0.09	0.44
551	31.87	0.19	71.96	1.27	2.86	273.40	0.78	1.20	120.09	1.33	2.41	256.98	0.07	0.43
552	32.06	0.19	73.23	1.28	2.85	274.18	0.77	1.18	121.42	1.32	2.40	257.05	0.06	0.42
553	32.25	0.19	74.51	1.29	2.85	274.95	0.75	1.17	122.74	1.33	2.39	257.11	0.04	0.41
554	32.44	0.19	75.80	1.30	2.84	275.70	0.74	1.15	124.07	1.32	2.38	257.15	0.03	0.40
555	32.63	+0.18	77.10	+1.30	2.84	276.44	+0.72	1.14	125.39	+1.32	2.36	257.18	+0.02	0.40
556	32.81	0.18	78.40	1.31	2.84	277.16	0.70	1.12	126.71	1.32	2.35	257.20	0.00	0.39
557	32.99	0.18	79.71	1.32	2.83	277.86	0.69	1.11	128.03	1.32	2.34	257.20	−0.01	0.38
558	33.17	0.18	81.03	1.32	2.83	278.55	0.68	1.09	129.35	1.32	2.33	257.19	0.03	0.37
559	33.35	0.17	82.35	1.33	2.82	279.23	0.66	1.08	130.67	1.34	2.32	257.16	0.04	0.36
560	33.52	+0.17	83.68	+1.33	2.82	279.89	+0.65	1.06	132.01	+1.32	2.31	257.12	−0.05	0.35
561	33.69	0.17	85.01	1.34	2.82	280.54	0.63	1.05	133.33	1.32	2.30	257.07	0.07	0.34
562	33.86	0.17	86.35	1.34	2.81	281.17	0.61	1.03	134.65	1.32	2.29	257.00	0.08	0.33
563	34.03	0.16	87.69	1.35	2.81	281.78	0.60	1.02	135.97	1.31	2.27	256.92	0.10	0.33
564	34.19	0.16	89.04	1.36	2.80	282.38	0.59	1.01	137.28	1.32	2.26	256.82	0.11	0.32
565	34.35	+0.16	90.40	+1.36	2.80	282.97	+0.57	0.99	138.60	+1.31	2.25	256.71	−0.12	0.31
566	34.51	0.16	91.76	1.37	2.79	283.54	0.55	0.98	139.91	1.31	2.24	256.59	0.14	0.30
567	34.67	0.15	93.13	1.37	2.79	284.09	0.54	0.97	141.22	1.32	2.23	256.45	0.15	0.29
568	34.82	0.15	94.50	1.38	2.78	284.63	0.53	0.96	142.54	1.31	2.21	256.30	0.16	0.29
569	34.97	0.15	95.88	1.38	2.78	285.16	0.51	0.94	143.85	1.31	2.20	256.14	0.18	0.28
570	35.12	+0.15	97.26	+1.38	2.77	285.67	+0.49	0.93	145.16	+1.30	2.19	255.96	−0.19	0.27
571	35.27	0.14	98.64	1.39	2.76	286.16	0.48	0.92	146.46	1.30	2.18	255.77	0.20	0.26
572	35.41	0.14	100.03	1.39	2.76	286.64	0.46	0.90	147.76	1.29	2.16	255.57	0.21	0.26
573	35.55	0.14	101.42	1.40	2.75	287.10	0.45	0.89	149.05	1.30	2.15	255.36	0.23	0.25
574	35.69	0.13	102.82	1.40	2.75	287.55	0.43	0.87	150.35	1.29	2.13	255.13	0.24	0.25
575	35.82	+0.13	104.22	+1.40	2.74	287.98	+0.42	0.86	151.64	+1.29	2.12	254.89	−0.26	0.24
576	35.95	0.13	105.62	1.41	2.73	288.40	0.40	0.85	152.93	1.29	2.11	254.63	0.27	0.23
577	36.08	0.13	107.03	1.41	2.73	288.80	0.38	0.83	154.22	1.28	2.09	254.36	0.28	0.23
578	36.21	0.12	108.44	1.41	2.72	289.18	0.37	0.82	155.50	1.27	2.08	254.08	0.29	0.22
579	36.33	0.12	109.85	1.41	2.72	289.55	0.36	0.80	156.77	1.28	2.06	253.79	0.31	0.22
580	36.45	+0.12	111.26	+1.42	2.71	289.91	+0.34	0.79	158.05	+1.27	2.05	253.48	−0.32	0.21
581	36.57	0.12	112.68	1.42	2.70	290.25	0.32	0.78	159.32	1.27	2.04	253.16	0.33	0.21
582	36.69	0.12	114.10	1.42	2.69	290.57	0.31	0.77	160.59	1.26	2.02	252.83	0.35	0.20
583	36.81	0.12	115.52	1.43	2.68	290.88	0.30	0.76	161.85	1.26	2.01	252.48	0.36	0.20
584	36.93	0.11	116.95	1.43	2.67	291.18	0.28	0.75	163.11	1.26	1.99	252.12	0.37	0.19
585	37.04	+0.11	118.38	+1.43	2.67	291.46	+0.26	0.73	164.37	+1.25	1.98	251.75	−0.39	0.19
586	37.15	0.11	119.81	1.43	2.66	291.72	0.24	0.72	165.62	1.25	1.97	251.36	0.40	0.18
587	37.26	0.11	121.24	1.43	2.65	291.96	0.23	0.71	166.87	1.24	1.95	250.96	0.41	0.18
588	37.37	0.11	122.67	1.44	2.64	292.19	0.22	0.70	168.11	1.24	1.94	250.55	0.42	0.17
589	37.48	0.10	124.11	1.43	2.63	292.41	0.20	0.69	169.35	1.23	1.92	250.13	0.44	0.17
590	37.58	+0.10	125.54	+1.44	2.62	292.61	+0.18	0.68	170.58	+1.23	1.91	249.69	−0.45	0.16
591	37.68	0.10	126.98	1.43	2.61	292.79	0.17	0.67	171.81	1.22	1.90	249.24	0.46	0.16
592	37.78	0.10	128.41	1.44	2.60	292.96	0.16	0.66	173.03	1.22	1.88	248.78	0.47	0.15
593	37.88	0.10	129.85	1.44	2.59	293.12	0.14	0.64	174.25	1.21	1.87	248.31	0.48	0.15
594	37.98	0.10	131.29	1.44	2.58	293.26	0.12	0.63	175.46	1.21	1.85	247.83	0.50	0.14
595	38.08	+0.10	132.73	+1.44	2.58	293.38	+0.11	0.62	176.67	+1.20	1.84	247.33	−0.51	0.14
596	38.18	0.09	134.17	1.44	2.57	293.49	0.09	0.61	177.87	1.20	1.83	246.82	0.52	0.14
597	38.27	0.09	135.61	1.44	2.56	293.58	0.08	0.60	179.07	1.19	1.81	246.30	0.53	0.13
598	38.36	0.09	137.05	1.45	2.55	293.66	0.07	0.58	180.26	1.18	1.80	245.77	0.54	0.13
599	38.45	0.09	138.50	1.44	2.54	293.73	0.05	0.57	181.44	1.18	1.78	245.23	0.56	0.12
600	38.54		139.94		2.53	293.78		0.56	182.62		1.77	244.67		0.12

TABLE IX, Arg. 2.—*Concluded.*

Arg.	($v.s.3$)	($v.c.3$)	($v.s.4$)	($v.c.4$)	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)	($\rho.s.2$)	($\rho.c.2$)	($\rho.s.3$)	($\rho.c.3$)
	"	"	"	"							
540	4.13	13.93	0.75	1.90	1376	401	726	686	513	147	97
541	4.23	13.99	0.76	1.90	1384	393	738	688	510	147	97
542	4.34	14.05	0.78	1.90	1392	386	751	689	507	147	96
543	4.45	14.11	0.79	1.90	1400	378	763	691	503	146	96
544	4.56	14.17	0.81	1.91	1408	371	776	692	500	146	95
545	4.67	14.23	0.83	1.91	1416	363	788	693	497	146	95
546	4.79	14.28	0.84	1.91	1424	356	801	695	494	146	95
547	4.90	14.34	0.85	1.91	1431	349	813	696	491	146	94
548	5.01	14.39	0.87	1.92	1439	343	826	698	487	145	94
549	5.12	14.44	0.89	1.92	1446	336	838	699	484	145	94
550	5.24	14.49	0.90	1.92	1454	329	851	700	481	145	93
551	5.35	14.54	0.92	1.92	1461	323	864	702	478	145	93
552	5.47	14.58	0.93	1.92	1468	316	877	703	475	145	93
553	5.58	14.63	0.95	1.92	1475	310	890	704	471	144	93
554	5.70	14.67	0.96	1.92	1482	304	903	705	468	144	92
555	5.82	14.71	0.98	1.92	1489	298	916	706	465	144	92
556	5.94	14.75	0.99	1.92	1496	292	929	708	462	144	92
557	6.06	14.79	1.01	1.92	1502	287	943	709	458	144	92
558	6.18	14.82	1.02	1.92	1509	281	956	710	455	143	92
559	6.30	14.85	1.04	1.92	1515	276	970	711	452	143	91
560	6.42	14.88	1.05	1.92	1522	270	983	712	449	143	91
561	6.54	14.91	1.07	1.92	1528	265	996	713	445	143	91
562	6.67	14.93	1.08	1.92	1534	260	1010	714	442	143	91
563	6.79	14.96	1.10	1.91	1541	255	1023	715	438	142	91
564	6.91	14.99	1.11	1.91	1547	250	1036	716	435	142	90
565	7.04	15.01	1.13	1.91	1553	246	1049	716	431	142	90
566	7.16	15.03	1.14	1.91	1559	242	1063	717	428	142	90
567	7.29	15.05	1.16	1.90	1564	238	1076	717	424	142	90
568	7.41	15.06	1.17	1.90	1570	234	1089	717	421	141	90
569	7.54	15.08	1.18	1.90	1575	230	1102	718	417	141	89
570	7.66	15.09	1.20	1.89	1581	226	1116	718	413	141	89
571	7.79	15.09	1.21	1.89	1586	223	1129	718	410	141	89
572	7.91	15.10	1.23	1.89	1591	219	1142	718	406	141	89
573	8.04	15.10	1.24	1.88	1596	216	1155	719	403	140	89
574	8.16	15.11	1.26	1.88	1601	213	1169	719	399	140	89
575	8.29	15.11	1.27	1.87	1605	210	1182	719	395	140	88
576	8.42	15.12	1.29	1.86	1610	207	1195	719	392	140	88
577	8.54	15.12	1 30	1.86	1614	205	1208	719	388	140	88
578	8.67	15.11	1.32	1.85	1618	202	1221	719	385	140	88
579	8.80	15.10	1.33	1.84	1622	200	1234	719	381	139	88
580	8.93	15.09	1.34	1.84	1626	198	1248	719	378	139	88
581	9.05	15.08	1.35	1.83	1630	196	1261	719	374	139	87
582	9.18	15.07	1.36	1.83	1634	194	1274	719	371	139	87
583	9.31	15.05	1.38	1.82	1637	192	1288	719	367	139	87
584	9.43	15.04	1.39	1.81	1641	191	1301	719	364	139	87
585	9.56	15.02	1.40	1.81	1645	189	1315	718	360	139	87
586	9.68	.15.00	1.42	1.80	1648	188	1328	718	357	139	87
587	9.81	14.98	1.43	1.79	1651	186	1342	718	353	139	87
588	9.93	14.96	1.44	1.79	1654	185	1356	717	350	138	86
589	10.06	14.93	1.45	1.78	1657	184	1369	717	346	138	86
590	10.18	14.91	1.46	1.77	1660	183	1382	717	343	138	86
591	10.31	14.88	1.48	1.76	1662	182	1396	716	339	138	86
592	10.43	14.85	1.49	1.75	1665	182	1409	716	335	138	86
593	10.55	14.81	1.50	1.74	1667	181	1422	716	332	138	86
594	10.68	14.78	1.51	1.73	1669	181	1436	715	328	138	85
595	10.80	14.74	1.52	1.72	1671	181	1448	715	324	138	85
596	10.92	14.71	1.53	1.72	1673	181	1461	714	321	138	85
597	11.04	14.67	1.55	1.71	1674	181	1474	714	317	137	85
598	11.16	14.63	1.56	1.70	1676	181	1487	713	313	137	85
599	11.28	14.58	1.57	1.69	1677	182	1500	712	310	137	85
600	11.40	14.53	1.58	1.68	1678	182	1513	711	306	137	85

TABLE X, Arg. 3.—Action of Neptune.

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	92.96	—0.29	0.87	+0.11	34.22	+0.55	7.42	—0.06	3.95	—0.13	0.80	1.15	—0.85	—1.13
1	92.67	0.29	0.98	0.12	34.77	0.54	7.36	0.06	3.82	0.12	0.81	1.16	0.86	1.14
2	92.38	0.31	1.10	0.13	35.31	0.54	7.30	0.07	3.70	0.12	0.83	1.18	0.87	1.15
3	92.07	0.30	1.23	0.15	35.85	0.54	7.23	0.07	3.58	0.11	0.84	1.20	0.89	1.16
4	91.77	0.31	1.38	0.15	36.39	0.53	7.16	0.07	3.47	0.11	0.86	1.22	0.91	1.17
5	91.46	—0.31	1.53	+0.17	36.92	+0.52	7.09	—0.08	3.36	—0.11	0.88	1.23	—0.93	—1.18
6	91.15	0.31	1.70	0.18	37.44	0.52	7.01	0.09	3.25	0.10	0.90	1.24	0.95	1.18
7	90.84	0.32	1.88	0.20	37.96	0.52	6.92	0.10	3.15	0.10	0.91	1.25	0.97	1.19
8	90.52	0.33	2.08	0.21	38.48	0.52	6.82	0.10	3.05	0.10	0.93	1.26	1.00	1.19
9	90.19	0.33	2.29	0.22	39.00	0.51	6.72	0.11	2.95	0.09	0.95	1.27	1.02	1.20
10	89.86	—0.33	2.51	+0.24	39.51	+0.51	6.61	—0.10	2.86	—0.09	0.97	1.28	—1.04	—1.20
11	89.53	0.35	2.75	0.24	40.02	0.50	6.51	0.11	2.77	0.08	1.00	1.28	1.06	1.19
12	89.18	0.35	2.99	0.27	40.52	0.49	6.40	0.12	2.69	0.08	1.02	1.29	1.08	1.19
13	88.83	0.36	3.26	0.28	41.01	0.49	6.28	0.12	2.61	0.07	1.05	1.29	1.09	1.18
14	88.47	0.38	3.54	0.29	41.50	0.49	6.16	0.12	2.54	0.07	1.07	1.29	1.11	1.18
15	88.09	—0.38	3.83	+0.30	41.99	+0.48	6.04	—0.13	2.47	—0.06	1.09	1.29	—1.12	—1.17
16	87.71	0.40	4.13	0.32	42.47	0.47	5.91	0.13	2.41	0.05	1.12	1.29	1.13	1.15
17	87.31	0.41	4.45	0.33	42.94	0.46	5.78	0.14	2.36	0.05	1.14	1.28	1.15	1.13
18	86.90	0.41	4.78	0.35	43.40	0.46	5.64	0.14	2.31	0.04	1.17	1.27	1.16	1.12
19	86.49	0.43	5.13	0.36	43.86	0.45	5.50	0.14	2.27	0.04	1.19	1.26	1.17	1.10
20	86.06	—0.44	5.49	+0.38	44.31	+0.43	5.36	—0.14	2.23	—0.03	1.21	1.25	—1.18	—1.08
21	85.62	0.46	5.87	0.39	44.74	0.42	5.22	0.14	2.20	0.02	1.23	1.23	1.19	1.06
22	85.16	0.47	6.26	0.41	45.16	0.42	5.08	0.15	2.18	—0.02	1.25	1.22	1.19	1.03
23	84.69	0.47	6.67	0.42	45.58	0.40	4.93	0.15	2.16	0.00	1.27	1.20	1.20	1.00
24	84.22	0.50	7.09	0.44	45.98	0.40	4.78	0.15	2.16	0.00	1.29	1.19	1.20	0.99
25	83.72	—0.50	7.53	+0.45	46.38	+0.39	4.63	—0.15	2.16	0.00	1.31	1.16	—1.20	—0.97
26	83.22	0.52	7.98	0.47	46.77	0.37	4.48	0.15	2.16	+0.01	1.33	1.14	1.19	0.95
27	82.70	0.53	8.45	0.49	47.14	0.35	4.33	0.15	2.17	0.02	1.35	1.12	1.19	0.93
28	82.17	0.55	8.94	0.50	47.49	0.34	4.18	0.16	2.19	0.03	1.36	1.10	1.18	0.91
29	81.62	0.57	9.44	0.51	47.83	0.33	4.02	0.15	2.22	0.04	1.37	1.07	1.17	0.90
30	81.05	—0.57	9.95	+0.53	48.16	+0.31	3.87	—0.15	2.26	+0.04	1.38	1.05	—1.17	—0.89
31	80.48	0.59	10.48	0.54	48.47	0.29	3.72	0.14	2.30	0.05	1.38	1.02	1.15	0.88
32	79.89	0.60	11.02	0.56	48.76	0.28	3.58	0.15	2.35	0.06	1.39	1.00	1.13	0.86
33	79.29	0.61	11.58	0.57	49.04	0.26	3.43	0.15	2.41	0.07	1.40	0.97	1.11	0.85
34	78.68	0.63	12.15	0.58	49.30	0.25	3.28	0.14	2.48	0.07	1.40	0.94	1.09	0.84
35	78.05	—0.64	12.73	+0.60	49.55	+0.22	3.14	—0.13	2.55	+0.09	1.40	0.91	—1.09	—0.83
36	77.41	0.65	13.33	0.61	49.77	0.21	3.01	0.14	2.64	0.09	1.40	0.88	1.07	0.83
37	76.76	0.66	13.94	0.63	49.98	0.18	2 87	0.13	2.73	0.10	1.40	0.85	1.05	0.82
38	76.10	0.68	14.57	0.64	50.16	0.16	2.74	0.13	2.83	0.11	1.39	0.82	1.03	0.82
39	75.42	0.69	15.21	0.65	50.32	0.14	2.61	0.12	2.94	0.11	1.38	0.79	0.99	0.81
40	74.73	—0.69	15.86	+0.66	50.46	+0.12	2.49	—0.11	3.05	+0.12	1.37	0.76	—0.99	—0.81
41	74.04	0.71	16.52	0.67	50.58	0.10	2.38	0.11	3.17	0.13	1.35	0.73	0.97	0.81
42	73.33	0.72	17.19	0.68	50.68	0.08	2.27	0.10	3.30	0.14	1.34	0.70	0.95	0.82
43	72.61	0.73	17.87	0.69	50.76	0.05	2.17	0.10	3.44	0.14	1.32	0.67	0.93	0.82
44	71.88	0.74	18.56	0.70	50.81	+0.02	2.07	0.10	3.58	0.14	1.30	0.65	0.91	0.83
45	71.14	—0.75	19.26	+0.71	50.83	0.00	1.97	—0.09	3.72	+0.15	1.28	0.62	—0.91	—0.83
46	70.39	0.76	19.97	0.71	50.83	—0.02	1.88	0.07	3.87	0.16	1.25	0.60	0.91	0.84
47	69.63	0.76	20.68	0.72	50.81	0.06	1.81	0.06	4.03	0.16	1.23	0.58	0.87	0.86
48	68.87	0.77	21.40	0.72	50.75	0.08	1.75	0.06	4.19	0.17	1.20	0.56	0.86	0.87
49	68.10	0.78	22.12	0.73	50.67	0.10	1.69	0.05	4.36	0.17	1.17	0.54	0.85	0.88
50	67.32	—0.78	22.85	+0.73	50.57	—0.13	1.64	—0.03	4.53	+0.18	1.14	0.52	—0.84	—0.90
51	66.54	0.79	23.58	0.74	50.44	0.15	1.61	0.03	4.71	0.18	1.10	0.51	0.84	0.92
52	65.75	0.79	24.32	0.74	50.29	0.18	1.58	0.02	4.89	0.18	1.07	0.49	0.83	0.94
53	64.96	0.80	25.06	0.73	50.11	0.21	1.56	0.01	5.07	0.18	1.03	0.48	0.82	0.96
54	64.16	0.81	25.79	0.74	49.90	0.24	1.55	0.00	5.25	0.18	1.00	0.47	0.81	0.98
55	63.35	—0.81	26.53	+0.74	49.66	—0.27	1.55	+0.02	5.43	+0.19	0.96	0.47	—0.81	—1.00
56	62.54	0.81	27.27	0.72	49.39	0.28	1.57	0.02	5.62	0.19	0.92	0.47	0.81	1.01
57	61.73	0.81	27.99	0.73	49 11	0.32	1.59	0.03	5.81	0.18	0.89	0.47	0.82	1.03
58	60.92	0.81	28.72	0.73	48.79	0.35	1.62	0.04	5.99	0.18	0.85	0.48	0.82	1.05
59	60.11	0.81	29.45	0.72	48.44	0.37	1.66	0.06	6.17	0.19	0.81	0.48	0.83	1.07
60	59.30		30.17		48.07		1.72		6.36		0.77	0 49	—0.83	—1.09

TABLE X, ARG. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
60	59.30	−0.81	30.17	+0.71	48.07	−0.41	1.72	+0.06	6.36	+0.18	0.77	0.49	−0.83	−1.09
61	58.49	0.82	30.88	0.71	47.66	0.42	1.78	0.08	6.54	0.17	0.73	0.50	0.84	1.10
62	57.67	0.81	31.59	0.70	47.24	0.45	1.86	0.09	6.71	0.18	0.69	0.52	0.85	1.11
63	56.86	0.81	32.29	0.68	46.79	0.48	1.95	0.10	6.89	0.17	0.66	0.54	0.87	1.13
64	56.05	0.81	32.97	0.67	46.31	0.50	2.05	0.11	7.06	0.17	0.62	0.56	0.88	1.14
65	55.24	−0.80	33.64	+0.66	45.81	−0.53	2.16	+0.12	7.23	+0.16	0.59	0.59	−0.89	−1.15
66	54.44	0.80	34.30	0.65	45.28	0.55	2.28	0.13	7.39	0.15	0.55	0.62	0.91	1.16
67	53.64	0.80	34.95	0.63	44.73	0.57	2.41	0.14	7.54	0.15	0.53	0.65	0.93	1.16
68	52.84	0.79	35.58	0.63	44.16	0.60	2.55	0.15	7.69	0.14	0.50	0.68	0.95	1.17
69	52.05	0.79	36.21	0.61	43.56	0.63	2.70	0.15	7.83	0.13	0.47	0.71	0.97	1.18
70	51.26	−0.79	36.82	+0.59	42.93	−0.65	2.85	+0.17	7.96	+0.12	0.45	0.75	−0.99	−1.19
71	50.47	0.77	37.41	0.57	42.28	0.66	3.02	0.17	8.08	0.12	0.43	0.78	1.00	1.19
72	49.70	0.77	37.98	0.55	41.62	0.68	3.19	0.18	8.20	0.11	0.41	0.82	1.02	1.19
73	48.93	0.76	38.53	0.53	40.94	0.71	3.37	0.19	8.31	0.09	0.39	0.87	1.04	1.18
74	48.17	0.74	39.06	0.50	40.23	0.72	3.56	0.19	8.40	0.09	0.38	0.91	1.06	1.18
75	47.43	−0.74	39.56	+0.49	39.51	−0.74	3.75	+0.21	8.49	+0.08	0.37	0.96	−1.08	−1.18
76	46.69	0.73	40.05	0.47	38.77	0.76	3.96	0.20	8.57	0.07	0.36	1.00	1.10	1.17
77	45.96	0.72	40.52	0.44	38.01	0.78	4.16	0.21	8.64	0.05	0.36	1.05	1.11	1.16
78	45.24	0.72	40.96	0.42	37.23	0.79	4.37	0.22	8.69	0.04	0.36	1.10	1.12	1.15
79	44.52	0.70	41.38	0.40	36.44	0.81	4.59	0.21	8.73	0.03	0.36	1.14	1.13	1.13
80	43.82	−0.68	41.78	+0.38	35.63	−0.82	4.80	+0.22	8.76	+0.01	0.37	1.18	−1.14	−1.12
81	43.14	0.68	42.16	0.35	34.81	0.83	5.02	0.22	8.77	+0.01	0.38	1.23	1.15	1.10
82	42.46	0.66	42.51	0.32	33.98	0.85	5.24	0.23	8.78	−0.01	0.40	1.28	1.16	1.08
83	41.80	0.65	42.83	0.29	33.13	0.85	5.47	0.22	8.77	0.02	0.41	1.32	1.17	1.06
84	41.15	0.63	43.12	0.26	32.28	0.86	5.69	0.22	8.75	0.03	0.43	1.36	1.17	1.04
85	40.52	−0.62	43.38	+0.23	31.42	−0.87	5.91	+0.22	8.72	−0.04	0.46	1.41	−1.18	−1.02
86	39.90	0.60	43.61	0.21	30.55	0.87	6.13	0.22	8.68	0.06	0.49	1.45	1.18	1.01
87	39.30	0.59	43.82	0.18	29.68	0.88	6.35	0.22	8.62	0.07	0.52	1.49	1.18	0.99
88	38.71	0.57	44.00	0.16	28.80	0.89	6.57	0.22	8.55	0.08	0.55	1.52	1.17	0.97
89	38.14	0.56	44.16	0.12	27.91	0.90	6.79	0.21	8.47	0.09	0.59	1.56	1 16	0.95
90	37.58	−0.54	44.28	+0.10	27.01	−0.89	7.00	+0.21	8.38	−0.11	0.62	1.59	−1.16	−0.94
91	37.04	0.52	44.38	0.06	26.12	0.89	7.21	0.20	8.27	0.12	0.66	1.61	1.15	0.92
92	36.52	0.50	44.44	+0.04	25.23	0.90	7.41	0.19	8.15	0.13	0.71	1.64	1.14	0.91
93	36.02	0.49	44.48	0.00	24.33	0.90	7.60	0.19	8.02	0.13	0.76	1.66	1.13	0.90
94	35.53	0.46	44.48	−0.02	23.43	0.89	7.79	0.18	7.89	0.15	0.80	1.68	1.12	0.89
95	35.07	−0.45	44.46	−0.06	22.54	−0.89	7.97	+0.17	7.74	−0.16	0.85	1.70	−1.11	−0.88
96	34.62	0.42	44.40	0.08	21.65	0.88	8.14	0.16	7.58	0.17	0.90	1.71	1.10	0.87
97	34.20	0.41	44.32	0.12	20.77	0.87	8.30	0.15	7.41	0.17	0.95	1.72	1.08	0.86
98	33.79	0.38	44.20	0.13	19.90	0.87	8.45	0.15	7.24	0.19	1.00	1.72	1.06	0.85
99	33.41	0.37	44.07	0.17	19.03	0.87	8.60	0.13	7.05	0.20	1.05	1.73	1.04	0.84
100	33.04	−0.34	43.90	−0.20	18.16	−0.85	8.73	+0.13	6.85	−0.20	1.10	1.72	−1.03	−0.83
101	32.70	0.32	43.70	0.23	17.31	0.85	8.86	0.11	6.65	0.21	1.15	1.72	1.02	0.83
102	32.38	0.30	43.47	0.26	16.46	0.84	8.97	0.09	6.44	0.21	1.21	1.70	1.00	0.83
103	32.08	0.28	43.21	0.29	15.62	0.82	9.06	0.09	6.23	0.22	1.26	1.69	0.98	0.83
104	31.80	0.26	42.92	0.31	14.80	0.81	9.15	0.08	6.01	0.22	1.31	1.67	0.96	0.83
105	31.54	−0.23	42.61	−0.34	13.99	−0.79	9.23	+0.07	5.79	−0.22	1.35	1.65	−0.95	−0.83
106	31.31	0.20	42.27	0.38	13.20	0.78	9.30	0.05	5.57	0.23	1.40	1.63	0.94	0.84
107	31.11	0.19	41.89	0.40	12.42	0.76	9.35	0.04	5.34	0.24	1.44	1.60	0.92	0.85
108	30.92	0.16	41.49	0.41	11.66	0.74	9.39	0.03	5.10	0.23	1.48	1.56	0.90	0.86
109	30.76	0.14	41.08	0.44	10.92	0.73	9.42	0.01	4.87	0.23	1.53	1.53	0.89	0.87
110	30.62	−0.11	40.64	−0.47	10.19	−0.70	9.43	+0.01	4.64	−0.24	1.56	1.49	−0.88	−0.88
111	30.51	0.09	40.17	0.49	9.49	0.69	9.44	−0.02	4.40	0.23	1.59	1.45	0.87	0.89
112	30.42	0.07	39.68	0.52	8.80	0.66	9.42	0.02	4.17	0.23	1.62	1.41	0.86	0.91
113	30.35	0.05	39.16	0.55	8.14	0.64	9.40	0.04	3.94	0.23	1.65	1.37	0.86	0.93
114	30.30	−0.02	38.61	0.56	7.50	0.62	9.36	0.05	3.71	0.23	1.67	1.32	0.85	0.95
115	30.28	+0.01	38.05	−0.58	6.88	−0.60	9.31	−0.07	3.48	−0.22	1.70	1.27	−0.84	−0.96
116	30.29	0.03	37.47	0.61	6.28	0.57	9.24	0.07	3.26	0.22	1.72	1.22	0.84	0.97
117	30.32	0.05	36.86	0.62	5.71	0.54	9.17	0.09	3.04	0.22	1.73	1.17	0.84	0.99
118	30.37	0.09	36.24	0.64	5.17	0.52	9.08	0.09	2.82	0.21	1.74	1.12	0.84	1.01
119	30.46	0.10	35.60	0.65	4.65	0.49	8.99	0.11	2.61	0.20	1.74	1.07	0.84	1.02
120	30.56		34.95		4.16		8.88		2.41		1.74	1.02	−0.84	−1.04

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
120	30.56	+0.14	34.95	−0.68	4.16	−0.47	8.88	−0.12	2.41	−0.20	1.74	1.02	−0.84	−1.04
121	30.70	0.15	34.27	0.69	3.69	0.43	8.76	0.13	2.21	0.19	1.74	0.97	0.84	1.06
122	30.85	0.18	33.58	0.70	3.26	0.41	8.63	0.14	2.02	0.18	1.73	0.91	0.85	1.08
123	31.03	0.20	32.88	0.73	2.85	0.38	8.49	0.15	1.84	0.17	1.72	0.86	0.86	1.09
124	31.23	0.23	32.15	0.73	2.47	0.34	8.34	0.16	1.67	0.16	1.71	0.81	0.87	1.10
125	31.46	+0.25	31.42	−0.75	2.13	−0.32	8.18	−0.16	1.51	−0.15	1.70	0.77	−0.88	−1.11
126	31.71	0.27	30.67	0.75	1.81	0.29	8.02	0.17	1.36	0.15	1.67	0.72	0.90	1.12
127	31.98	0.30	29.92	0.76	1.52	0.26	7.85	0.18	1.21	0.13	1.65	0.68	0.92	1.13
128	32.28	0.33	29.16	0.77	1.26	0.23	7.67	0.19	1.08	0.12	1.62	0.63	0.93	1.13
129	32.61	0.35	28.39	0.78	1.03	0.20	7.48	0.19	0.96	0.12	1.59	0.59	0.94	1.14
130	32.96	+0.38	27.60	−0.79	0.83	−0.16	7.29	−0.20	0.84	−0.10	1.56	0.55	−0.95	−1.15
131	33.34	0.40	26.82	0.79	0.67	0.13	7.09	0.21	0.74	0.09	1.53	0.51	0.96	1.15
132	33.74	0.42	26.03	0.79	0.54	0.10	6.88	0.21	0.65	0.08	1.49	0.48	0.98	1.15
133	34.16	0.44	25.24	0.80	0.44	0.07	6.67	0.21	0.57	0.06	1.45	0.45	0.99	1.15
134	34.60	0.46	24.44	0.79	0.37	0.04	6.46	0.21	0.51	0.05	1.41	0.42	1.01	1.16
135	35.06	+0.48	23.65	−0.79	0.33	−0.01	6.25	−0.22	0.46	−0.05	1.37	0.40	−1.03	−1.16
136	35.54	0.51	22.86	0.80	0.32	+0.03	6.03	0.22	0.41	0.03	1.32	0.38	1.05	1.15
137	36.05	0.53	22.06	0.79	0.35	0.05	5.81	0.22	0.38	−0.02	1.28	0.37	1.06	1.14
138	36.58	0.55	21.27	0.79	0.40	0.09	5.59	0.21	0.36	0.00	1.23	0.35	1.07	1.13
139	37.13	0.57	20.48	0.79	0.49	0.12	5.38	0.22	0.36	0.00	1.18	0.34	1.08	1.12
140	37.70	+0.60	19.69	−0.79	0.61	+0.15	5.16	−0.21	0.36	+0.02	1.13	0.34	−1.09	−1.11
141	38.30	0.62	18.90	0.77	0.76	0.18	4.95	0.22	0.38	0.03	1.09	0.33	1.09	1.09
142	38.92	0.63	18.13	0.77	0.94	0.21	4.73	0.21	0.41	0.04	1.04	0.34	1.10	1.07
143	39.55	0.66	17.36	0.76	1.15	0.24	4.52	0.21	0.45	0.05	1.00	0.34	1.11	1.06
144	40.21	0.67	16.60	0.75	1.39	0.27	4.31	0.21	0.50	0.07	0.95	0.35	1.12	1.05
145	40.88	+0.68	15.85	−0.73	1.66	+0.29	4.10	−0.20	0.57	+0.08	0.91	0.36	−1.13	−1.04
146	41.56	0.71	15.12	0.73	1.95	0.33	3.90	0.19	0.65	0.08	0.86	0.38	1.13	1.03
147	42.27	0.73	14.39	0.72	2.28	0.35	3.71	0.19	0.73	0.09	0.82	0.40	1.13	1.02
148	43.00	0.75	13.67	0.70	2.63	0.38	3.52	0.19	0.82	0.11	0.78	0.41	1.13	1.01
149	43.75	0.76	12.97	0.69	3.01	0.41	3.33	0.18	0.93	0.11	0.74	0.44	1.14	0.99
150	44.51	+0.79	12.28	−0.68	3.42	+0.44	3.15	−0.17	1.04	+0.12	0.71	0.46	−1.14	−0.98
151	45.30	0.79	11.60	0.66	3.86	0.46	2.98	0.16	1.16	0.13	0.67	0.50	1.14	0.96
152	46.09	0.81	10.94	0.64	4.32	0.48	2.82	0.16	1.29	0.14	0.64	0.53	1.13	0.95
153	46.90	0.83	10.30	0.62	4.80	0.49	2.66	0.15	1.43	0.15	0.62	0.56	1.13	0.94
154	47.73	0.84	9.68	0.60	5.29	0.53	2.51	0.14	1.58	0.16	0.59	0.59	1.12	0.93
155	48.57	+0.85	9.08	−0.59	5.82	+0.54	2.37	−0.12	1.74	+0.16	0.57	0.63	−1.11	−0.92
156	49.42	0.87	8.49	0.57	6.36	0.56	2.25	0.12	1.90	0.17	0.55	0.66	1.10	0.91
157	50.29	0.88	7.92	0.55	6.92	0.59	2.13	0.12	2.07	0.17	0.53	0.70	1.09	0.90
158	51.17	0.90	7.37	0.53	7.51	0.61	2.01	0.10	2.24	0.18	0.52	0.74	1.08	0.89
159	52.07	0.90	6.84	0.50	8.12	0.62	1.91	0.09	2.42	0.18	0.51	0.78	1.06	0.88
160	52.97	+0.92	6.34	−0.49	8.74	+0.65	1.82	−0.08	2.60	+0.18	0.50	0.82	−1.05	−0.87
161	53.89	0.93	5.85	0.46	9.39	0.66	1.74	0.08	2.78	0.19	0.50	0.86	1.04	0.87
162	54.82	0.93	5.39	0.44	10.05	0.67	1.66	0.06	2.97	0.19	0.50	0.90	1.03	0.87
163	55.75	0.95	4.95	0.41	10.72	0.68	1.60	0.05	3.16	0.19	0.50	0.94	1.02	0.86
164	56.70	0.96	4.54	0.40	11.40	0.70	1.55	0.04	3.35	0.20	0.51	0.97	1.00	0.86
165	57.66	+0.97	4.14	−0.37	12.10	+0.70	1.51	−0.03	3.55	+0.20	0.52	1.01	−0.99	−0.86
166	58.63	0.97	3.77	0.35	12.80	0.72	1.48	0.02	3.75	0.19	0.53	1.05	0.97	0.87
167	59.60	0.98	3.42	0.32	13.52	0.73	1.46	−0.01	3.94	0.20	0.54	1.08	0.96	0.87
168	60.58	0.99	3.10	0.30	14.25	0.75	1.45	0.00	4.14	0.20	0.56	1.11	0.95	0.88
169	61.57	0.99	2.80	0.28	15.00	0.74	1.45	+0.01	4.34	0.19	0.57	1.15	0.94	0.88
170	62.56	+1.00	2.52	−0.25	15.74	+0.75	1.46	+0.02	4.53	+0.19	0.59	1.18	−0.93	−0.89
171	63.56	1.00	2.27	0.22	16.49	0.76	1.48	0.03	4.72	0.19	0.62	1.21	0.92	0.91
172	64.56	1.01	2.05	0.21	17.25	0.77	1.51	0.04	4.91	0.18	0.65	1.23	0.91	0.92
173	65.57	1.01	1.84	0.17	18.02	0.76	1.55	0.05	5.09	0.19	0.67	1.25	0.90	0.93
174	66.58	1.02	1.67	0.16	18.78	0.77	1.60	0.06	5.28	0.18	0.70	1.27	0.89	0.94
175	67.60	+1.01	1.51	−0.13	19.55	+0.78	1.66	+0.07	5.46	+0.18	0.72	1.29	−0.89	−0.95
176	68.61	1.02	1.38	0.11	20.33	0.76	1.73	0.08	5.64	0.17	0.75	1.30	0.88	0.96
177	69.63	1.02	1.27	0.08	21.09	0.77	1.81	0.08	5.81	0.16	0.78	1.31	0.88	0.97
178	70.65	1.02	1.19	0.06	21.86	0.77	1.89	0.09	5.97	0.15	0.81	1.32	0.88	0.98
179	71.67	1.02	1.13	0.05	22.63	0.77	1.98	0.10	6.12	0.15	0.84	1.33	0.88	0.99
180	72.69		1.08		23.40		2.08		6.27		0.88	1.34	−0.88	−1.00

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
180	72.69		1.08		23.40		2.08		6.27		0.88	1.34	−0.88	−1.00
181	73.71	+1.02	1.07	−0.01	24.17	+0.77	2.20	+0.12	6.42	+0.15	0.91	1.34	0.88	1.02
182	74.73	1.02	1.08	+0.01	24.92	0.75	2.31	0.11	6.55	0.13	0.94	1.34	0.88	1.03
183	75.75	1.02	1.11	0.03	25.68	0.76	2.43	0.12	6.68	0.13	0.97	1.34	0.89	1.04
184	76.76	1.01	1.16	0.05	26.43	0.75	2.56	0.13	6.81	0.13	1.00	1.33	0.89	1.05
185	77.77	1.01	1.24	0.08	27.17	0.74	2.69	0.13	1.93	0.12	1.03	1.33	−0.90	−1.06
186	78.78	+1.01	1.34	+0.10	27.90	+0.73	2.83	+0.14	7.04	+0.11	1.06	1.32	0.92	1.06
187	79.78	1.00	1.45	0.11	28.63	0.73	2.96	0.13	7.13	0.09	1.09	1.31	0.93	1.07
188	80.77	0.99	1.59	0.14	29.34	0.71	3.10	0.14	7.22	0.09	1.11	1.29	0.94	1.08
189	81.76	0.99	1.74	0.15	30.05	0.71	3.25	0.15	7.30	0.08	1.13	1.28	0.95	1.09
190	82.75	0.99	1.91	0.17	30.75	0.70	3.40	0.15	7.37	0.07	1.16	1.26	−0.96	−1.10
191	83.74	+0.99	2.10	+0.19	31.43	+0.68	3.55	+0.15	7.43	+0.06	1.18	1.24	0.97	1.10
192	84.71	0.97	2.32	0.22	32.10	0.67	3.71	0.16	7.48	0.05	1.20	1.22	0.98	1.10
193	85.65	0.95	2.55	0.23	32.76	0.66	3.86	0.15	7.53	0.05	1.21	1.19	0.99	1.10
194	86.61	0.95	2.79	0.24	33.41	0.65	4.02	0.16	7.57	0.04	1.23	1.17	1.00	1.10
195	87.56	0.95	3.05	0.26	34.05	0.64	4.17	0.15	7.60	0.03	1.24	1.15	−1.01	−1.11
196	88.50	+0.94	3.33	+0.28	34.67	+0.62	4.33	+0.16	7.62	+0.02	1.25	1.12	1.01	1.11
197	89.42	0.92	3.62	0.29	35.27	0.60	4.49	0.16	7.63	+0.01	1.26	1.09	1.02	1.10
198	90.34	0.92	3.93	0.31	35.86	0.59	4.64	0.15	7.63	0.00	1.27	1.07	1.03	1.10
199	91.24	0.90	4.25	0.32	36.43	0.57	4.79	0.15	7.62	−0.01	1.27	1.04	1.04	1.10
200	92.14	0.90	4.58	0.33	36.99	0.56	4.94	0.15	7.60	0.02	1.28	1.02	−1.05	−1.09
201	93.02	+0.88	4.92	+0.34	37.53	+0.54	5.09	+0.15	7.57	−0.03	1.28	1.00	1.05	1.08
202	93.89	0.87	5.28	0.36	38.05	0.52	5.23	0.14	7.54	0.03	1.28	0.97	1.06	1.07
203	94.74	0.85	5.65	0.37	38.56	0.51	5.37	0.14	7.50	0.04	1.27	0.94	1.07	1.06
204	95.59	0.85	6.03	0.38	39.04	0.48	5.51	0.14	7.46	0.04	1.26	0.92	1.08	1.05
205	96.41	0.82	6.42	0.39	39.51	0.47	5.64	0.13	7.41	0.05	1.25	0.89	−1.09	−1.04
206	97.23	+0.82	6.81	+0.39	39.96	+0.45	5.77	+0.13	7.35	−0.06	1.24	0.87	1.09	1.04
207	98.02	0.79	7.22	0.41	40.40	0.44	5.89	0.12	7.28	0.07	1.23	0.85	1.09	1.03
208	98.81	0.79	7.64	0.42	40.81	0.41	6.01	0.12	7.21	0.07	1.22	0.83	1.09	1.02
209	99.57	0.76	8 06	0.42	41.21	0.40	6.12	0.11	7.13	0.08	1.21	0.81	1.09	1.01
210	100.32	0.75	8.49	0.43	41.58	0.37	6.22	0.10	7.05	0.08	1.19	0.79	−1.10	−1.00
211	101.06	+0.74	8.92	+0.43	41.94	+0.36	6.32	+0.10	6.96	−0.09	1.17	0 77	1.10	0.99
212	101.78	0.72	9.36	0.44	42.28	0.34	6.41	0.09	6.86	0.10	1.15	0.76	1.09	0.98
213	102.48	0.70	9.80	0.44	42.59	0.31	6.50	0.09	6.76	0.10	1.13	0.75	1.09	0.97
214	103.17	0.69	10.24	0.44	42.89	0.30	6.58	0.08	6.66	0.10	1.11	0.73	1.09	0.97
215	103.84	0.67	10.69	0.45	43.18	0.29	6.65	0.07	6.56	0.10	1.09	0.72	−1.08	−0.96
216	104.49	+0.65	11.14	+0.45	43.44	+0.26	6.71	+0.06	6.45	−0.11	1.06	0.71	1.08	0.96
217	105.12	0.63	11.59	0.45	43.68	0.24	6.77	0.06	6.33	0.12	1.04	0.70	1.07	0.95
218	105.73	0.61	12.04	0.45	43.90	0.22	6.82	0.05	6.22	0.11	1.02	0.70	1.06	0 94
219	106.33	0.60	12.50	0.46	44.11	0.21	6.87	0.05	6.11	0.11	1.00	0.70	1.05	0.93
220	106.90	0.57	12.95	0.45	44.29	0.18	6.91	0.04	5.99	0.12	0.98	0.70	−1.04	−0.92
221	107.45	+0.55	13.41	+0.46	44.45	+0.16	6.94	+0.03	5.87	−0.12	0.95	0.70	1.04	0.92
222	107.99	0.54	13.86	0.45	44.60	0.15	6.96	0.02	5.76	0.11	0.92	0.70	1.03	0.92
223	108.50	0.51	14.30	0.44	44.73	0.13	6.98	0.02	5.65	0.11	0.90	0.70	1.02	0.92
224	109.00	0.50	14.75	0.45	44.84	0.11	6.99	+0.01	5.53	0.12	0.88	0.71	1.01	0.92
225	109.48	0.48	15.19	0.44	44.94	0.10	6.99	0.00	5.42	0.11	0.86	0.72	−1.00	−0.92
226	109.94	+0.46	15.63	+0.44	45.02	+0.08	6.98	−0.01	5.31	−0.11	0.84	0.72	1.00	0.92
227	110.37	0.43	16.07	0.44	45.08	0.06	6.97	0.01	5.20	0.11	0.81	0.73	1.00	0.92
228	110.78	0.41	16.50	0.43	45.12	0.04	6.96	0.01	5.09	0.11	0.79	0.74	0.99	0.92
229	111.18	0.40	16.93	0.43	45.14	0.02	6.94	0.02	4.98	0.11	0.77	0.76	0.98	0.92
230	111.55	0.37	17.35	0.42	45.15	+0.01	6.91	0.03	4.88	0.10	0.76	0.78	−0.97	−0.93
231	111.89	+0.34	17.76	+0.41	45.15	0.00	6.88	−0.03	4.78	−0.10	0.74	0.79	0.97	0.93
232	112.23	0.34	18.17	0.41	45.13	−0.02	6.85	0.03	4.68	0.10	0.72	0.81	0.96	0.94
233	112.54	0.31	18.57	0.40	45.09	0.04	6.80	0.05	4.58	0.10	0.71	0.83	0.95	0.95
234	112.83	0.29	18.96	0.39	45.04	0.05	6.75	0.05	4.49	0.09	0.69	0.85	0.94	0.96
235	113.10	0.27	19.34	0.38	44.98	0.06	6.70	0.05	4.40	0.09	0.68	0.87	−0.93	−0.97
236	113.35	+0.25	19.72	+0.38	44.91	−0.07	6.64	−0.06	4.31	−0.09	0.67	0.89	0.93	0.97
237	113.57	0.22	20.09	0.37	44.82	0.09	6.58	0.06	4.23	0.08	0.66	0.91	0.93	0.98
238	113.77	0.20	20.45	0.36	44.71	0.11	6.51	0.07	4.16	0.07	0.65	0.93	0.93	0.99
239	113.95	0.18	20.80	0.35	44.59	0.12	6.45	0.06	4.09	0.07	0.65	0.96	0.93	1.00
240	114.11	0.16	21.15	0.35	44.47	0.12	6.38	0.07	4.02	0.07	0.65	0.98	−0.93	−1.01

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
240	114.11		21.15		44.47		6.38		4.02		0.65	0.98	—0.93	—1.01
241	114.24	+0.13	21.48	+0.33	44.33	—0.14	6.30	—0.08	3.96	—0.06	0.65	1.00	0.93	1.01
242	114.36	0.12	21.80	0.32	44.18	0.15	6.23	0.07	3.90	0.06	0.65	1.03	0.93	1.02
243	114.46	0.10	22.12	0.32	44.02	0.16	6.15	0.08	3.85	0.05	0.65	1.05	0.94	1.03
244	114.54	0.08	22.43	0.31	43.86	0.16	6.07	0.08	3.80	0.05	0.65	1.07	0.95	1.04
245	114.60	0.06	22.73	0.30	43.68	0.18	5.99	0.08	3.75	0.05	0 65	1.10	—0.95	—1.04
246	114.63	+0.03	23.01	+0.28	43.49	—0.19	5.91	—0.08	3.71	—0.04	0.65	1.12	0.96	1.04
247	114.65	+0.02	23.29	0.28	43.30	0.19	5.83	0.08	3.68	0.03	0.66	1.14	0.97	1.04
248	114.64	—0.01	23.56	0.27	43.10	0.20	5.74	0.09	3.65	0.03	0.66	1.16	0.97	1.05
249	114.61	0.03	23.82	0.26	42.88	0.22	5.66	0.08	3.62	0.03	0.67	1.19	0.98	1.06
250	114.55	0.06	24.06	0.24	42.67	0.21	5.58	0.08	3.60	0.02	0.69	1.21	—0.98	—1.06
251	114.48	—0.07	24.30	+0.24	42.45	—0.22	5.49	—0.09	3.59	—0.01	0.70	1.23	0.98	1.06
252	114.39	0.09	24.53	0.23	42.22	0.23	5.41	0.08	3.58	0.01	0.72	1.24	0.99	1.06
253	114.29	0.10	24.75	0.22	41.98	0.24	5.33	0.08	3.57	—0.01	0.73	1.26	1.00	1.07
254	114.16	0.13	24.95	0.20	41.74	0.24	5.25	0.08	3.57	0.00	0.75	1.28	1.01	1.07
255	114.01	0.15	25.15	0.20	41.49	0.25	5.16	0.09	3.57	0.00	0.77	1.30	—1.01	—1.07
256	113.84	—0.17	25.34	+0.19	41.24	—0.25	5.08	—0.08	3.57	0.00	0.78	1.31	1.01	1.07
257	113.66	0.18	25.52	0.18	40.98	0.26	5.01	0.07	3.57	0.00	0.80	1.32	1.01	1.07
258	113.45	0.21	25.70	0.18	40.73	0.25	4.93	0.08	3.58	+0.01	0.82	1.34	1.02	1.06
259	113.22	0.23	25.86	0.16	40.47	0.26	4.85	0.08	3.60	0.02	0.84	1.35	1.03	1.05
260	112.98	0.24	26.02	0.16	40.21	0.26	4.78	0.07	3.62	0.02	0.87	1.36	—1.03	—1.05
261	112.71	—0.27	26.16	+0.14	39.95	—0.26	4.71	—0.07	3.64	+0.02	0.89	1.37	1.03	1.04
262	112.43	0.28	26.30	0.14	39.68	0.27	4.64	0.07	3.66	0.02	0.91	1.38	1.03	1.03
263	112.14	0.29	26.43	0.13	39.41	0.27	4.57	0.07	3.69	0.03	0.93	1.38	1.04	1.02
264	111.84	0.30	26.55	0.12	39.13	0.28	4.51	0.06	3.72	0.03	0.96	1.39	1.05	1.01
265	111.51	0.33	26.66	0.11	38.86	0.27	4.45	0.06	3.75	0.03	0.98	1.39	—1.05	—1.01
266	111.16	—0.35	26.77	+0.11	38.59	—0.27	4.39	—0.06	3.79	+0.04	1.00	1.39	1.05	1.01
267	110.80	0.36	26.87	0.10	38.31	0.28	4.33	0.06	3.83	0.04	1.02	1.39	1.05	1.01
268	110.43	0.37	26.96	0.09	38.03	0.28	4.28	0.05	3.86	0.03	1.05	1.38	1.05	1.00
269	110.04	0.39	27.04	0.08	37.76	0.27	4.22	0.06	3.90	0.04	1.07	1.38	1.05	0.99
270	109.63	0.41	27.12	0.08	37.49	0.27	4.17	0.05	3.94	0.04	1.09	1.38	—1.05	—0.99
271	109.21	—0.42	27.19	+0.07	37.21	—0.28	4.12	—0.05	3.99	+0.05	1.12	1.37	1.05	0.99
272	108.77	0.44	27.26	0.07	36.93	0.28	4.07	0.05	4.03	0.04	1.14	1.37	1.05	0.98
273	108.32	0.45	27.32	0.06	36.65	0.28	4.02	0.05	4.08	0.05	1.16	1.36	1.04	0.98
274	107.86	0.46	27.37	0.05	36.37	0.28	3.98	0.04	4.12	0.04	1.18	1.35	1.04	0.97
275	107.38	0.48	27.42	0.05	36.10	0.27	3.95	0.03	4.17	0.05	1.20	1.34	—1.04	—0.97
276	106.90	—0.48	27.46	+0.04	35.82	—0.28	3.92	—0.03	4.22	+0.05	1.22	1.32	1.03	0.97
277	106.40	0.50	27.50	0.04	35.54	0.28	3.89	0.03	4.27	0.05	1.24	1.31	1.02	0.97
278	105.89	0.51	27.53	0.03	35.27	0.27	3.86	0.03	4.32	0.05	1.26	1.30	1.01	0.97
279	105.37	0.52	27.56	0.03	35.00	0.27	3.83	0.03	4.37	0.05	1.28	1.28	1.00	0.96
280	104.84	0.53	27.59	0.03	34.72	0.28	3.80	0.03	4.42	0.05	1.29	1.26	—1.00	—0.96
281	104.30	—0.54	27.61	+0.02	34.44	—0.28	3.77	—0.03	4.47	+0.05	1.31	1.24	1.00	0.96
282	103.75	0.55	27.63	0.02	34.17	0.27	3.75	0.02	4.53	0.06	1.32	1.22	1.00	0.96
283	103.19	0.56	27.64	0.01	33.90	0.27	3.74	0.01	4.58	0.05	1.34	1.20	0.99	0.96
284	102.62	0.57	27.65	+0.01	33.63	0.27	3.72	0.02	4.64	0.06	1.35	1.18	0.98	0.95
285	102.04	0.58	27.65	0.00	33.36	0.27	3.71	0.01	4.69	0.05	1.36	1.16	—0.98	—0.95
286	101.46	—0.58	27.65	0.00	33.09	—0.27	3.70	—0.01	4.74	+0.05	1.37	1.14	0.98	0.95
287	100.87	0.59	27.65	0.00	32.82	0.27	3.69	0.01	4.79	0.05	1.38	1.12	0.98	0.96
288	100.27	0.60	27.64	—0.01	32.55	0.27	3.68	0.01	4.85	0.06	1.38	1.09	0.97	0.97
289	99.67	0.60	27.63	0.01	32.29	0.26	3.67	—0.01	4.90	0.05	1.39	1.07	0.96	0.97
290	99.06	0.61	27.62	0.01	32.02	0.27	3.67	0.00	4.95	0.05	1.39	1.05	—0.96	—0.98
291	98.45	—0.61	27.60	—0.02	31.76	—0.26	3.67	0.00	5.00	+0.05	1.40	1.02	0.96	0.98
292	97.83	0.62	27.58	0.02	31.50	0.26	3.67	0.00	5.06	0.06	1.40	1.00	0.96	0.99
293	97.21	0.62	27.56	0.02	31.23	0.27	3.67	0.00	5.11	0.05	1.40	0.98	0.95	1.00
294	96.58	0.63	27.54	0.02	30.97	0.26	3.68	+0.01	5.16	0.05	1.40	0.95	0.95	1.01
295	95.95	0.63	27.52	0.02	30.71	0.26	3.69	0.01	5.21	0.05	1.39	0.93	—0.95	—1.02
296	95.31	—0.64	27.48	—0.04	30.45	—0.26	3.70	+0.01	5.26	+0.05	1.39	0.91	0.95	1.02
297	94.67	0.64	27.45	0.03	30.19	0.26	3.71	0.01	5.32	0.06	1.38	0.88	0.96	1.02
298	94.04	0.63	27.42	0.03	29.93	0.26	3.72	0.01	5.37	0.05	1.38	0.86	0.96	1.03
299	93.40	0.64	27.38	0.04	29.67	0.26	3.74	0.02	5.42	0.05	1.37	0.84	0.97	1.03
300	92.77	0.63	27.34	0.04	29.40	0.27	3.76	0.02	5.47	0.05	1.35	0.82	—0.97	—1.03

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
300	92.77	—0.64	27.34	—0.04	29.40	—0.24	3.76	+0.01	5.47	+0.04	1.35	0.82	—0.97	—1.03
301	92.13	0.63	27.30	0.04	29.16	0.27	3.77	0.03	5.51	0.05	1.34	0.80	0.97	1.03
302	91.50	0.63	27.26	0.05	28.89	0.26	3.80	0.02	5.56	0.05	1.33	0.78	0.98	1.03
303	90.87	0.63	27.21	0.05	28.63	0.26	3.82	0.03	5.61	0.05	1.31	0.76	0.98	1.04
304	90.24	0.64	27.16	0.06	28.37	0.26	3.85	0.02	5.66	0.05	1.30	0.74	0.99	1.04
305	89.60	—0.63	27.10	—0.05	28.11	—0.25	3.87	+0.04	5.71	+0.04	1.28	0.72	—0.99	—1.04
306	88.97	0.62	27.05	0.06	27.86	0.26	3.91	0.03	5.75	0.05	1.27	0.71	1.00	1.04
307	88.35	0.62	26.99	0.06	27.60	0.25	3.94	0.03	5.80	0.04	1.25	0.69	1.00	1.04
308	87.72	0.62	26.93	0.07	27.35	0.25	3.97	0.04	5.84	0.04	1.23	0.68	1.01	1.04
309	87.10	0.61	26.86	0.07	27.10	0.25	4.01	0.04	5.88	0.04	1.21	0.66	1.01	1.04
310	86.49	—0.60	26.79	—0.07	26.85	—0.26	4.05	+0.04	5.92	+0.04	1.19	0.65	—1.02	—1.04
311	85.89	0.60	26.72	0.07	26.59	0.25	4.09	0.05	5.96	0.04	1.17	0.64	1.02	1.04
312	85.29	0.60	26.65	0.07	26.34	0.25	4.14	0.04	6.00	0.03	1.15	0.63	1.03	1.03
313	84.69	0.60	26.58	0.09	26.09	0.25	4.18	0.04	6.03	0.04	1.12	0.62	1.03	1.03
314	84.09	0.58	26.49	0.09	25.84	0.25	4.22	0.05	6.07	0.03	1.10	0.62	1.04	1.03
315	83.51	—0.58	26.40	—0.09	25.59	—0.25	4.27	+0.05	6.10	+0.04	1.08	0.61	—1.04	—1.03
316	82.93	0.56	26.31	0.09	25.34	0.24	4.32	0.05	6.14	0.03	1.05	0.61	1.04	1.02
317	82.37	0.56	26.22	0.10	25.10	0.25	4.37	0.06	6.17	0.02	1.03	0.61	1.04	1.02
318	81.81	0.54	26.12	0.10	24.85	0.25	4.43	0.05	6.19	0.03	1.01	0.61	1.04	1.01
319	81.27	0.54	26.02	0.11	24.60	0.25	4.48	0.06	6.22	0.03	0.98	0.61	1.04	1.00
320	80.73	—0.53	25.91	—0.11	24.35	—0.24	4.54	+0.06	6.25	+0.02	0.96	0.61	—1.04	—1.00
321	80.20	0.52	25.80	0.12	24.11	0.24	4.60	0.06	6.27	0.02	0.94	0.61	1.04	0.99
322	79.68	0.51	25.68	0.12	23.87	0.24	4.66	0.06	6.29	0.01	0.91	0.62	1.03	0.99
323	79.17	0.49	25.56	0.12	23.63	0.24	4.72	0.06	6.30	0.02	0 89	0.62	1.03	0.98
324	78.68	0.49	25.44	0.13	23.39	0.23	4.78	0.07	6.32	0.02	0.87	0.63	1.03	0.98
325	78.19	—0.47	25.31	—0.14	23.16	—0.24	4.85	+0.07	6.34	+0.01	0.85	0.64	—1.03	—0.97
326	77.72	0.46	25.17	0.15	22.92	0.23	4.92	0.07	6.35	0.00	0.83	0.65	1.03	0.97
327	77.26	0.44	25.02	0.15	22.69	0.23	4.99	0.07	6.35	0.00	0.81	0.66	1.02	0.96
328	76.82	0.43	24.87	0.15	22.46	0.23	5.06	0.07	6.35	0.00	0.79	0.67	1.02	0.96
329	76.39	0.42	24.72	0.16	22.23	0.23	5.13	0.07	6.35	0.00	0.77	0.69	1.01	0.95
330	75.97	—0.40	24.56	—0.17	22.00	—0.22	5.20	+0.07	6.35	0.00	0.75	0.70	—1.01	—0.95
331	75.57	0.39	24.39	0.18	21.78	0.21	5.27	0.07	6.35	—0.01	0.73	0.72	1.00	0.95
332	75.18	0.37	24.21	0.18	21.57	0.22	5.34	0.07	6.34	0.01	0.72	0.74	1.00	0.95
333	74.81	0.36	24.03	0.19	21.35	0.21	5.41	0.07	6.33	0.01	0.70	0.76	0.99	0.95
334	74.45	0.34	23.84	0.19	21.14	0.20	5.48	0.08	6.32	0.02	0.69	0.78	0.99	0.95
335	74.11	—0.32	23.65	—0.20	20.94	—0.21	5.56	+0.07	6.30	—0.02	0.68	0.80	—0.99	—0.95
336	73.79	0.31	23.45	0.21	20.73	0.20	5.63	0.07	6.28	0.03	0.67	0.82	0.98	0.95
337	73.48	0.30	23.24	0.22	20.53	0.19	5.70	0.08	6.25	0.03	0.66	0.84	0.97	0.95
338	73.18	0.27	23.02	0.22	20.34	0.19	5.78	0.07	6.22	0.03	0.65	0.86	0.96	0.95
339	72.91	0.25	22.80	0.23	20.15	0.19	5.85	0.07	6.19	0.04	0.64	0.88	0.95	0.95
340	72.66	—0.24	22.57	—0.24	19.96	—0.17	5.92	+0.06	6.15	—0.04	0.64	0.91	—0.95	—0.95
341	72.42	0.22	22.33	0.24	19.79	0.17	5.98	0.07	6.11	0.04	0.63	0.93	0.95	0.96
342	72.20	0.20	22.09	0.25	19.62	0.17	6.05	0.07	6.07	0.05	0.63	0.95	0.94	0.96
343	72.00	0 18	21.84	0.26	19.45	0.15	6.12	0.07	6.02	0.05	0.63	0.98	0.94	0.96
344	71.82	0.16	21.58	0.27	19.30	0.15	6.19	0.06	5.97	0.06	0.63	1.00	0.93	0.97
345	71.66	—0.14	21.31	—0.27	19.15	—0.15	6 25	+0.06	5.91	—0.05	0.63	1.02	—0.93	—0.97
346	71.52	0.13	21.04	0.29	19.00	0.13	6.31	0.06	5.86	0.06	0.64	1.05	0.93	0.98
347	71.39	0.10	20.75	0.29	18.87	0.13	6.37	0.06	5.80	0.07	0.64	1.07	0.94	0.99
348	71.29	0.08	20.46	0.30	18.74	0.12	6.43	0.05	5.73	0.07	0.65	1.09	0.94	1.00
349	71.21	0.06	20.16	0.31	18.62	0.10	6.48	0.05	5.66	0.07	0.66	1.12	0.94	1.01
350	71.15	—0.05	19.85	—0.31	18.52	—0.09	6.53	+0.05	5.59	—0.08	0.67	1.14	—0.94	—1.02
351	71.10	—0.02	19.54	0.32	18.43	0.09	6.58	0.04	5.51	0.08	0.68	1.16	0.94	1.02
352	71.08	0.00	19.22	0.32	18.34	0.08	6.62	0.04	5.43	0.08	0.69	1.18	0.95	1.03
353	71.08	+0.02	18.90	0.34	18.26	0.07	6.66	0.04	5.35	0.08	0.71	1.20	0.95	1.04
354	71.10	0.05	18.56	0.34	18.19	0.05	6.70	0.03	5.27	0.09	0.72	1.22	0.95	1.05
355	71.15	+0.06	18.22	—0.34	18.14	—0.04	6.73	+0.03	5.18	—0.09	0.74	1.23	—0.95	—1.06
356	71.21	0.08	17.88	0.35	18.10	0.03	6.76	0.03	5.09	0.09	0.76	1.25	0.96	1.06
357	71.29	0.11	17.53	0.36	18.07	—0.02	6.79	0.02	5.00	0.09	0.78	1.26	0.97	1.06
358	71.40	0.12	17.17	0.37	18.05	0.00	6.81	0.02	4.91	0.10	0.80	1.27	0.98	1.07
359	71.52	0.15	16.80	0.37	18.05	+0.01	6.83	0.01	4.81	0.10	0.82	1.28	0.99	1.07
360	71.67		16.43		18.06		6.84		4.71		0.84	1.29	—0.99	—1.07

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
360	71.67	+0.17	16.43	—0.38	18.06	+0.02	6.84	0.00	4.71	—0.10	0.84	1.29	—0.99	—1.07
361	71.84	0.18	16.05	0.38	18.08	0.04	6.84	0.00	4.61	0.10	0.87	1.30	1.00	1.07
362	72.02	0.22	15.67	0.39	18.12	0.05	6.84	0.00	4.51	0.10	0.89	1.31	1.01	1.07
363	72.24	0.24	15.28	0.39	18.17	0.07	6.84	—0.01	4.41	0.10	0.91	1.31	1.02	1.07
364	72.48	0.25	14.89	0.39	18.24	0.08	6.83	0.02	4.31	0.10	0.94	1.32	1.03	1.07
365	72.73	+0.27	14.50	—0.39	18.32	+0.10	6.81	—0.02	4.21	—0.10	0.96	1.32	—1.03	—1.07
366	73.00	0.30	14.11	0.40	18.42	0.11	6.79	0.03	4.11	0.10	0.98	1.32	1.03	1.06
367	73.30	0.32	13.71	0.40	18.53	0.13	6.76	0.03	4.01	0.11	1.01	1.32	1.04	1.06
368	73.62	0.34	13.31	0.40	18.66	0.15	6.73	0.05	3.90	0.10	1.03	1.31	1.05	1.05
369	73.96	0.37	12.91	0.40	18.81	0.16	6.68	0.05	3.80	0.10	1.06	1.31	1.06	1.05
370	74.33	+0.38	12.51	—0.41	18.97	+0.18	6.63	—0.05	3.70	—0.10	1.08	1.30	—1.07	—1.05
371	74.71	0.41	12.10	0.40	19.15	0.20	6.58	0.06	3.60	0.09	1.10	1.29	1.07	1.05
372	75.12	0.42	11.70	0.40	19.35	0.22	6.52	0.06	3.51	0.10	1.12	1.28	1.07	1.04
373	75.54	0.45	11.30	0.40	19.57	0.23	6.46	0.07	3.41	0.09	1.14	1.27	1.08	1.04
374	75.99	0.46	10.90	0.40	19.80	0.24	6.39	0.07	3.32	0.09	1.16	1.25	1.08	1.03
375	76.45	+0.49	10.50	—0.40	20.04	+0.26	6.32	—0.08	3.23	—0.08	1.18	1.23	—1.08	—1.02
376	76.94	0.50	10.10	0.39	20.30	0.28	6.24	0.09	3.15	0.08	1.20	1.22	1.08	1.00
377	77.44	0.52	9.71	0.39	20.58	0.31	6.15	0.09	3.07	0.08	1.22	1.20	1.08	0.99
378	77.96	0.55	9.32	0.39	20.89	0.32	6.06	0.10	2.99	0.07	1.23	1.19	1.08	0.98
379	78.51	0.57	8.93	0.39	21.21	0.34	5.96	0.11	2.92	0.07	1.25	1.16	1.08	0.97
380	79.08	+0.58	8.54	—0.38	21.55	+0.36	5.85	—0.11	2.85	—0.07	1.26	1.13	—1.08	—0.96
381	79.66	0.61	8.16	0.37	21.91	0.37	5.74	0.11	2.78	0.06	1.27	1.11	1.07	0.96
382	80.27	0.63	7.79	0.36	22.28	0.39	5.63	0.12	2.72	0.05	1.28	1.09	1.07	0.95
383	80.90	0.64	7.43	0.36	22.67	0.41	5.51	0.12	2.67	0.05	1.28	1.06	1.07	0.94
384	81.54	0.65	7.07	0.34	23.08	0.43	5.39	0.12	2.62	0.04	1.29	1.04	1.06	0.93
385	82.19	+0.68	6.73	—0.34	23.51	+0.44	5.27	—0.13	2.58	—0.03	1.29	1.01	—1.06	—0.92
386	82.87	0.69	6.39	0.33	23.95	0.46	5.14	0.13	2.55	0.02	1.29	0.98	1.06	0.92
387	83.56	0.71	6.06	0.32	24.41	0.48	5.01	0.14	2.53	0.02	1.29	0.96	1.05	0.92
388	84.27	0.74	5.74	0.32	24.89	0.49	4.87	0.14	2.51	—0.02	1.28	0.93	1.04	0.91
389	85.01	0.74	5.42	0.30	25.38	0.51	4.73	0.14	2.49	0.00	1.28	0.91	1.03	0.91
390	85.75	+0.76	5.12	—0.28	25.89	+0.53	4.59	—0.14	2.49	0.00	1.27	0.88	—1.02	—0.90
391	86.51	0.78	4.84	0.28	26.42	0.54	4.45	0.14	2.49	+0.01	1.26	0.86	1.01	0.90
392	87.29	0.79	4.56	0.26	26.96	0.55	4.31	0.15	2.50	0.02	1.25	0.83	1.00	0.90
393	88 08	0.81	4.30	0.24	27.51	0.57	4.16	0.14	2.52	0.03	1.23	0.81	0.99	0.90
394	88.89	0.82	4.06	0.23	28.08	0.59	4.02	0.15	2.55	0.03	1.22	0.78	0.98	0.90
395	89.71	+0.83	3.83	—0.22	28.67	+0.60	3.87	—0.14	2.58	+0.05	1.20	0.76	—0.97	—0.90
396	90.54	0.85	3.61	0.20	29.27	0.61	3.73	0.15	2.63	0.05	1.17	0.74	0.95	0.91
397	91.39	0.86	3.41	0.19	29.88	0.62	3.58	0.14	2.68	0.06	1.15	0.72	0.94	0.91
398	92.25	0.88	3.22	0.16	30.50	0.64	3.44	0.14	2.74	0.07	1.13	0.70	0.93	0.92
399	93.13	0.89	3.06	0.15	31.14	0.65	3.30	0.14	2.81	0.07	1.10	0.69	0.92	0.92
400	94.02	+0.90	2.91	—0.14	31.79	+0.66	3.16	—0.14	2.88	+0.08	1.08	0.68	—0.91	—0.93
401	94.92	0.91	2.77	0.12	32.45	0.66	3.02	0.13	2.96	0.10	1.05	0.67	0.91	0.93
402	95.83	0.92	2.65	0.09	33.11	0.68	2.89	0.13	3.06	0.10	1.02	0.66	0.90	0.94
403	96.75	0.93	2.56	0.08	33.79	0.69	2.76	0.12	3.16	0.11	0.99	0.65	0.90	0.95
404	97.68	0.94	2 48	0.05	34.48	0.69	2.64	0.12	3.27	0.12	0.96	0.65	0.89	0.96
405	98.62	+0.95	2.43	—0.04	35.17	+0.70	2.52	—0.11	3.39	+0.12	0.93	0.64	—0.89	—0.97
406	99.57	0.96	2.39	—0.02	35.87	0.71	2.41	0.10	3.51	0.13	0.90	0.64	0.89	0.99
407	100.53	0.97	2.37	0.00	36.58	0.72	2.31	0.11	3.64	0.14	0.87	0.65	0.89	1.01
408	101.50	0.97	2.37	+0.02	37.30	0.72	2.20	0.10	3.78	0.15	0.84	0.65	0.90	1.02
409	102.47	0.99	2.39	0.05	38.02	0.72	2.10	0.09	3.93	0.15	0.80	0.66	0.90	1.03
410	103.46	+0.99	2.44	+0.08	38.74	+0.73	2.01	—0.08	4.08	+0.15	0.77	0.67	—0.90	—1.04
411	104.45	0.99	2.52	0.09	39.47	0.72	1.93	0.07	4.23	0.16	0.74	0.69	0.91	1.05
412	105.44	1.00	2.61	0.11	40.19	0.73	1.86	0.07	4.39	0.17	0.72	0.70	0.91	1.07
413	106.44	1.00	2.72	0.13	40.92	0.74	1.79	0.06	4.56	0.17	0.69	0.72	0.92	1.08
414	107.44	1.00	2.85	0.15	41.66	0.73	1.73	0.04	4.73	0.18	0.66	0.74	0.92	1.09
415	108.44	+1.01	3.00	+0.18	42.39	+0.73	1.69	—0.04	4.91	+0.18	0.63	0.77	—0.92	—1.10
416	109.45	1.01	3.18	0.20	43.12	0.72	1.65	0.03	5.09	0.18	0.61	0.79	0.93	1.10
417	110.46	1.01	3.38	0.22	43.84	0.73	1.62	0.02	5.27	0.19	0.59	0.82	0.94	1.10
418	111.47	1.02	3.60	0.25	44.57	0.72	1.60	0.01	5.46	0.19	0.56	0.85	0.96	1.11
419	112.49	1.02	3.85	0.27	45.29	0.72	1.59	0.00	5.65	0.19	0.55	0.88	0.97	1.12
420	113.51		4.12		46.01		1.59		5.84		0.53	0.92	—0.98	—1.12

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
420	113.51	+1.02	4.12	+0.30	46.01	+0.70	1.59	+0.01	5.84	+0.20	0.53	0.92	—0.98	—1.12
421	114.53	1.02	4.42	0.31	46.71	0.70	1.60	0.03	6.04	0.19	0.52	0.95	0.99	1.12
422	115.55	1.02	4.73	0.34	47.41	0.69	1.63	0.02	6.23	0.19	0.51	0.99	1.01	1.11
423	116.57	1.02	5.07	0.35	48.10	0.69	1.65	0.04	6.42	0.19	0.50	1.02	1.02	1.11
424	117.59	1.00	5.42	0.38	48.79	0.69	1.69	0.06	6.61	0.19	0.49	1.06	1.03	1.11
425	118.59	+1.00	5.80	+0.41	49.48	+0.67	1.75	+0.06	6.80	+0.18	0.49	1.10	—1.04	—1.11
426	119.59	1.00	6.21	0.42	50.15	0.65	1.81	0.07	6.98	0.19	0.49	1.14	1.06	1.10
427	120.59	1.01	6.63	0.44	50.80	0.64	1.88	0.08	7.17	0.18	0.49	1.18	1.08	1.10
428	121.60	0.99	7.07	0.47	51.44	0.62	1.96	0.10	7.35	0.17	0.50	1.22	1.09	1.10
429	122.59	0.99	7.54	0.49	52.06	0.61	2.06	0.10	7.52	0.18	0.51	1.26	1.10	1.09
430	123.58	+0.99	8.03	+0.51	52.67	+0.60	2.16	+0.10	7.70	+0.17	0.53	1.30	—1.11	—1.09
431	124.57	0.98	8.54	0.54	53.27	0.58	2.26	0.12	7.87	0.16	0.54	1.34	1.11	1.08
432	125.55	0.97	9.08	0.54	53.85	0.56	2.38	0.14	8.03	0.16	0.56	1.38	1.11	1.07
433	126.52	0.96	9.62	0.56	54.41	0.54	2.52	0.14	8.19	0.15	0.58	1.41	1.12	1.06
434	127.48	0.96	10.18	0.58	54.95	0.53	2.66	0.15	8.34	0.14	0.61	1.45	1.12	1.05
435	128.44	+0.94	10.76	+0.61	55.48	+0.51	2.81	+0.15	8.48	+0.14	0.64	1.48	—1.13	—1.04
436	129.38	0.93	11.37	0.62	55.99	0.49	2.96	0.17	8.62	0.12	0.66	1.51	1.13	1.02
437	130.31	0.93	11.99	0.63	56.48	0.46	3.13	0.17	8.74	0.12	0.70	1.54	1.13	1.00
438	131.24	0.91	12.62	0.64	56.94	0.44	3.30	0.17	8.86	0.11	0.73	1.57	1.13	0.99
439	132.15	0.91	13.26	0.67	57.38	0.42	3.47	0.18	8.97	0.10	0.77	1.59	1.13	0.98
440	133.06	+0.89	13.93	+0.68	57.80	+0.39	3.65	+0.19	9.07	+0.09	0.82	1.61	—1.13	—0.97
441	133.95	0.88	14.61	0.70	58.19	0.37	3.84	0.20	9.16	0.08	0.86	1.63	1.12	0.95
442	134.83	0.87	15.31	0.70	58.56	0.34	4.04	0.20	9.24	0.07	0.90	1.64	1.12	0.93
443	135.70	0.85	16.01	0.72	58.90	0.32	4.24	0.20	9.31	0.06	0.95	1.65	1.12	0.92
444	136.55	0.84	16.73	0.72	59.22	0.29	4.44	0.21	9.37	0.05	0.99	1.66	1.11	0.91
445	137.39	+0.82	17.45	+0.74	59.51	+0.27	4.65	+0.21	9.42	+0.03	1.04	1.67	—1.11	—0.90
446	138.21	0.81	18.19	0.74	59.78	0.24	4.86	0.21	9.45	0.03	1.09	1.67	1.09	0.90
447	139.02	0.79	18.93	0.75	60.02	0.20	5.07	0.21	9.48	+0.01	1.13	1.67	1.07	0.89
448	139.81	0.78	19.68	0.77	60.22	0.18	5.28	0.21	9.49	0.00	1.18	1.66	1.06	0.88
449	140.59	0.77	20.45	0.77	60.40	0.15	5.49	0.22	9.49	—0.01	1.23	1.65	1.05	0.87
450	141.36	+0.75	21.22	+0.77	60.55	+0.12	5.71	+0.21	9.48	—0.03	1.28	1.64	—1.04	—0.86
451	142.11	0.72	21.99	0.78	60.67	0.09	5.92	0.21	9.45	0.03	1.32	1.62	1.02	0.86
452	142.83	0.71	22.77	0.78	60.76	0.06	6.13	0.21	9.42	0.04	1.37	1.60	1.00	0.86
453	143.54	0.69	23.55	0.78	60.82	+0.03	6.34	0.21	9.38	0.06	1.41	1.58	0.99	0.86
454	144.23	0.68	24.33	0.78	60.85	0.00	6.55	0.20	9.32	0.07	1.45	1.55	0.98	0.86
455	144.91	+0.65	25.11	+0.78	60.85	—0.03	6.75	+0.21	9.25	—0.08	1.49	1.52	—0.97	—0.86
456	145.56	0.63	25.89	0.78	60.82	0.06	6.96	0.19	9.17	0.10	1.53	1.49	0.95	0.87
457	146.19	0.62	26.67	0.78	60.76	0.10	7.15	0.20	9.07	0.10	1.57	1.45	0.93	0.87
458	146.81	0.59	27.45	0.77	60.66	0.12	7.35	0.18	8.97	0.11	1.60	1.41	0 91	0.88
459	147.40	0.58	28.22	0.77	60.54	0.16	7.53	0.17	8.86	0.12	1.63	1.37	0.90	0.88
460	147.98	+0.56	28.99	+0.76	60.38	—0.18	7.70	+0.17	8.74	—0.14	1.66	1.33	—0.89	—0.89
461	148.54	0.53	29.75	0.76	60.20	0.22	7.87	0.17	8.60	0.14	1.68	1.29	0.88	0.90
462	149.07	0.52	30.51	0.75	59.98	0.25	8.04	0.16	8.46	0.15	1.70	1.24	0.87	0.91
463	149.59	0.49	31.26	0.74	59.73	0.28	8.20	0.15	8.31	0.17	1.72	1.19	0.86	0.92
464	150.08	0.46	32.00	0.73	59.45	0.31	8.35	0.14	8.14	0.17	1.73	1.14	0.85	0.93
465	150.54	+0.44	32.73	+0.72	59.14	—0.33	8.49	+0.13	7.97	—0.18	1.74	1.09	—0.85	—0.94
466	150.98	0.42	33.45	0.71	58.81	0.37	8.62	0.12	7.79	0.18	1.75	1.03	0.85	0.96
467	151.40	0.40	34.16	0.69	58.44	0.39	8.74	0.11	7.61	0.19	1.75	0.98	0.85	0.98
468	151.80	0.38	34.85	0.67	58.05	0.43	8.85	0.10	7.42	0.20	1.75	0.93	0.85	1.00
469	152.18	0.35	35.52	0.66	57.62	0.45	8.95	0.08	7.22	0.21	1.75	0.87	0.85	1.02
470	152.53	+0.34	36.18	+0.65	57.17	—0.48	9.03	+0.08	7.01	—0.20	1.73	0.82	—0.85	—1.03
471	152.87	0.31	36.83	0.63	56.69	0.51	9.11	0.06	6.81	0.21	1.72	0.77	0.86	1.05
472	153.18	0.28	37.46	0.61	56.18	0.54	9.17	0.05	6.60	0.22	1.70	0.72	0.86	1.07
473	153.46	0.26	38.07	0.60	55.64	0.55	9.22	0.05	6.38	0.22	1.68	0.67	0.87	1.09
474	153.72	0.23	38.67	0.57	55.09	0.58	9.27	0.03	6.16	0.23	1.66	0.62	0.87	1.10
475	153.95	+0.21	39.24	+0.55	54.51	—0.60	9.30	+0.01	5.93	—0.22	1.63	0.58	—0.88	—1.11
476	154.16	0.18	39.79	0.53	53.91	0.63	9.31	+0.01	5.71	0.23	1.60	0.54	0.90	1.12
477	153.34	0.17	40.32	0.50	53.28	0.66	9.32	—0.01	5.48	0.23	1.56	0.50	0.91	1.13
478	154.51	0.15	40.82	0.49	52.62	0.68	9.31	0.02	5.25	0.22	1.53	0.46	0.92	1.14
479	154.66	0.11	41.31	0.46	51.94	0.69	9.29	0.04	5.03	0.21	1.48	0.42	0.93	1.15
480	154.77		41.77		51.25		9.25		4.82		1.44	0.39	—0.94	—1.16

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	— ″
480	154.77		41.77		51.25		9.25		4.82		1.44	0.39	0.94	—1.16
481	154.86	+0.09	42.20	+0.43	50.53	—0.72	9.20	—0.05	4.60	—0.22	1.39	0.36	0.96	1.16
482	154.93	0.07	42.61	0.41	49.80	0.73	9.15	0.05	4.39	0.21	1.35	0.34	0.98	1.16
483	154.98	0.05	42.99	0.38	49.04	0.76	9.08	0.07	4.17	0.22	1.30	0.31	1.00	1.16
484	154.99	+0.01	43.34	0.35	48.28	0.76	9.00	0.08	3.96	0.21	1.25	0.30	1.01	1.16
485	154.98	—0.01	43.67	0.33	47.49	0.79	8.90	0.10	3.75	0.21	1.20	0.28	—1.02	—1.16
486	154.95	—0.03	43.98	+0.31	46.69	—0.80	8.80	—0.10	3.54	—0.21	1 15	0.27	1.04	1.15
487	154.90	0.05	44.25	0.27	45.88	0.81	8.69	0.11	3.34	0.20	1 10	0.26	1.06	1.15
488	154.82	0.08	44.49	0.24	45.05	0.83	8.57	0.12	3.15	0.19	1.04	0.26	1.08	1.15
489	154.72	0.10	44.71	0.22	44.21	0.84	8.44	0.13	2.98	0.17	0.99	0.26	1.09	1.14
490	154.59	0.13	44.90	0.19	43.37	0.84	8.30	0.14	2.81	0.17	0.94	0.27	—1.10	—1.14
491	154.45	—0.14	45.05	+0.15	42.51	—0.86	8.14	—0.16	2.64	—0.17	0.88	0.28	1.11	1.13
492	154.28	0.17	45.18	0.13	41.64	0.87	7.98	0.16	2.49	0.15	0.83	0.29	1.12	1.12
493	154.08	0.20	45.28	0.10	40.77	0.87	7.81	0.17	2.34	0.15	0.78	0.30	1.13	1.11
494	153.87	0.21	45.35	0.07	39.90	0.87	7.64	0.17	2.20	0.14	0.74	0.32	1.14	1.10
495	153.63	0.24	45.38	0.03	39.01	0.89	7.45	0.19	2.07	0.13	0.69	0.35	—1.15	—1.09
496	153.37	—0.26	45.39	+0.01	38.12	—0.89	7.26	—0.19	1.95	—0.12	0.65	0.38	1.15	1.07
497	153.09	0.28	45.37	—0.02	37.24	0.88	7.06	0.20	1.84	0.11	0.60	0.41	1.15	1.05
498	152.79	0.30	45.32	0.05	36.35	0.89	6.86	0.20	1.74	0.10	0.56	0.44	1.16	1.03
499	152.46	0.33	45.23	0.09	35.46	0.89	6.66	0.20	1.66	0.08	0.53	0.47	1.16	1.01
500	152.11	0.35	45.12	0.11	34.57	0.89	6.45	0.21	1.59	0.07	0.49	0.51	—1.17	—0.99
501	151.74	—0.37	44.98	—0.14	33.68	—0.89	6.23	—0.22	1.54	—0.05	0.46	0.55	1.17	0.97
502	151.36	0.38	44.81	0.17	32.80	0.88	6.02	0.21	1.49	0.05	0.43	0.59	1.16	0.95
503	150.95	0.41	44.60	0.21	31.92	0.88	5.80	0.22	1.45	0.04	0.41	0.64	1.16	0.93
504	150.52	0.43	44.38	0.22	31.04	0.88	5.59	0.21	1.42	0.03	0.39	0.68	1.15	0.91
505	150.08	0.44	44.12	0.26	30.18	0.86	5.37	0.22	1.40	—0.02	0.37	0.73	—1.15	—0.90
506	149.61	—0.47	43.84	—0.28	29.32	—0.86	5.15	—0.22	1.40	0.00	0.36	0.78	1.13	0.89
507	148.13	0.48	43.52	0.32	28.46	0.86	4.93	0.22	1.40	0.00	0.35	0.83	1.11	0.87
508	148.63	0.50	43.18	0.34	27.62	0.84	4.72	0.21	1.43	+0.03	0.34	0.87	1.10	0.86
509	148.11	0.52	42.82	0.36	26.80	0.82	4.51	0.21	1.46	0.03	0.34	0.92	1.09	0.85
510	147.58	0.53	42.42	0.40	25.98	0.82	4.30	0.21	1.51	0.05	0.34	0.97	—1.08	—0.84
511	147.03	—0.55	42.00	—0.42	25.17	—0.81	4.09	—0.21	1.57	+0.06	0.34	1.02	1 06	0.84
512	146.46	0.57	41.56	0.44	24.38	0.79	3.89	0.20	1.64	0.07	0.35	1.06	1.04	0.84
513	145.87	0.59	41.09	0.47	23.61	0.77	3.69	0.20	1.72	0.08	0.36	1.11	1.02	0.83
514	145.27	0.60	40.60	0.49	22.85	0.76	3.50	0.19	1.81	0.09	0.37	1.16	1.01	0.83
515	144.66	0.61	40.09	0.51	22.11	0.74	3.32	0.18	1.90	0.09	0.39	1.20	—1.00	—0.82
516	144.04	—0.62	39.55	—0.54	21.38	—0.73	3.14	—0.18	2.01	+0.11	0.41	1.24	0.98	0.82
517	143.40	0.64	38.99	0.56	20.68	0.70	2.98	0.16	2.12	0.11	0.43	1.28	0.96	0.82
518	142.74	0.66	38.41	0.58	19.99	0.69	2.82	0.16	2.25	0.13	0.46	1.32	0.94	0.83
519	142.07	0.67	37.81	0.60	19.32	0.67	2.66	0.16	2.39	0.14	0.49	1.35	0.92	0.83
520	141.39	0.68	37.19	0.62	18.68	0.64	2.51	0.15	2.53	0.14	0.52	1.39	—0.90	—0.84
521	140.70	—0.69	36.56	—0.63	18.05	—0.63	2.38	—0.13	2.68	+0.15	0.55	1.42	0.98	0.85
522	140.00	0.70	35.91	0.65	17.45	0.60	2.25	0.13	2.84	0.16	0.58	1.44	0.97	0.87
523	139.29	0.71	35.25	0.66	16.87	0.58	2.13	0.12	3.00	0.16	0.62	1.47	0.96	0.88
524	138.57	0.72	34.57	0.68	16.32	0.55	2.03	0.10	3.16	0.16	0.66	1.49	0.95	0.89
525	137.84	0.73	33.87	0.70	15.79	0.53	1.94	0.09	3.33	0.17	0.70	1.51	—0.94	—0.90
526	137.10	—0.74	33.15	—0.72	15.28	—0.51	1.86	—0.08	3.51	+0.18	0.74	1.53	0.94	0.91
527	136.35	0.75	32.43	0.72	14.79	0.49	1.78	0.08	3.69	0.18	0.78	1.54	0.93	0.93
528	135.59	0.76	31.70	0.73	14.34	0.45	1.72	0.06	3.87	0.18	0.82	1.55	0.93	0.95
529	134.83	0.76	30.96	0.74	13.91	0.43	1.67	0.05	4.06	0.19	0.86	1.55	0.92	0.97
530	134.07	0.76	30.22	0.74	13.51	0.40	1.63	0.04	4.25	0.19	0.90	1.56	—0.91	—0.99
531	133.30	—0.77	29.46	—0.76	13.13	—0.38	1.60	—0.03	4.44	+0.19	0.94	1.55	0.91	1.01
532	132.52	0.78	28.69	0.77	12.78	0.35	1.58	0.02	4.63	0.19	0.98	1.55	0.92	1.03
533	131.73	0.79	27.92	0.77	12.46	0.32	1.57	—0.01	4.82	0.19	1.02	1.54	0.92	1.05
534	130.94	0.79	27.15	0.77	12.16	0.30	1.58	+0.01	5.00	0.18	1.06	1.54	0.93	1.07
535	130.15	0.79	26.37	0.78	11.90	0.26	1.60	0.02	5.18	0.18	1.09	1.53	—0.93	—1.09
536	129.35	—0.80	25.58	—0.79	11.66	—0.24	1.62	+0.02	5.36	+0.18	1.13	1.51	0.95	1.10
537	128.55	0.80	24.81	0.77	11.45	0.21	1.65	0.03	5.54	0.18	1.17	1.50	0.96	1.12
538	127.75	0.80	24.03	0.78	11.26	0.19	1.70	0.05	5.72	0.18	1.20	1.48	0.97	1.13
539	126.95	0.80	23.25	0.78	11.10	0.16	1.76	0.06	5.90	0.18	1.23	1.46	0.98	1.14
540	126.15	0.80	22.48	0.77	10.98	0.12	1.82	0.06	6.07	0.17	1.26	1.43	—0.99	—1.15

TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
540	126.15		22.48		10.98		1.82		6.07		1.26	1.43	—0.89	—1.15
541	125.36	—0.79	21.70	—0.78	10.88	—0.10	1.90	+0.08	6.23	+0.16	1.28	1.41	0.91	1.15
542	124.56	0.80	20.93	0.77	10.80	0.08	1.98	0.08	6.39	0.16	1.30	1.38	0.93	1 16
543	123.76	0.80	20.16	0.77	10.75	0.05	2.08	0.10	6.54	0.15	1.33	1.36	0.95	1.16
544	122.96	0.80	19.40	0.76	10.73	—0.02	2.18	0.10	6.69	0.15	1.35	1.33	0.96	1.17
545	122.17	0.79	18.65	0.75	10.74	+0.01	2.29	0.11	6.83	0.14	1.37	1.30	—0.97	—1.18
546	121.37	—0.80	17.90	—0.75	10.77	+0.03	2.40	+0.11	6.96	+0.13	1.38	1.26	0.99	1.18
547	120.58	0.79	17.17	0.73	10.83	0.06	2.52	0.12	7.08	0.12	1.40	1.23	1.02	1.18
548	119.80	0.78	16.44	0.73	10.92	0.09	2.65	0.13	7.20	0.12	1.41	1.20	1.04	1.18
549	119.03	0.77	15.73	0.71	11.02	0.10	2.79	0.14	7.31	0.11	1.41	1.17	1.06	1.18
550	118.26	0.77	15.03	0.70	11.15	0.13	2.93	0.14	7.41	0.10	1.42	1.14	—1.08	—1.18
551	117.50	—0.76	14.34	—0.69	11.31	+0.16	3.08	+0.15	7.51	+0.10	1.42	1.11	1.10	1.17
552	116.74	0.76	13.66	0.68	11.49	0.18	3.23	0.15	7.59	0.08	1.42	1.08	1.12	1.16
553	115.99	0.75	12.99	0.67	11.69	0.20	3.38	0.15	7.67	0.08	1.41	1.04	1.13	1.15
554	115.24	0.75	12.35	0.64	11.91	0.22	3.54	0.16	7.73	0.06	1.41	1.01	1.14	1.14
555	114.50	0.74	11.71	0.64	12.16	0.25	3.70	0.16	7.79	0.06	1.41	0.98	—1.15	—1.13
556	113.77	—0.73	11.09	—0.62	12.43	+0.27	3.86	+0.16	7.83	+0.04	1 40	0.95	1.15	1.11
557	113.05	0.72	10.48	0.61	12.71	0.28	4.02	0.16	7.87	0.04	1.39	0.92	1.16	1.09
558	112.35	0.70	9.89	0.59	13.02	0.31	4.19	0.17	7.90	0.03	1.38	0.90	1.17	1.07
559	111.65	0.70	9.32	0.57	13.35	0.33	4.35	0.16	7.92	0.02	1.36	0.87	1.18	1.05
560	110.96	0.69	8.76	0.56	13.69	0.34	4.51	0.16	7.93	0.01	1.34	0.85	—1.19	—1.03
561	110.29	—0.67	8.23	—0.53	14.04	+0.35	4.67	+0.16	7.94	+0.01	1.32	0.83	1.19	1.01
562	109.63	0.66	7.71	0.52	14.42	0.38	4.84	0.17	7.93	—0.01	1.30	0.81	1.19	0.99
563	108.98	0.65	7.21	0.50	14.81	0.39	5.00	0.16	7.92	0.01	1.28	0.79	1.19	0.97
564	108.33	0.65	6.73	0.48	15.22	0.41	5.16	0.16	7.90	0.02	1.26	0.77	1.19	0.95
565	107.70	0.63	6.26	0.47	15.64	0.42	5.32	0.16	7.87	0.03	1.24	0.76	—1.19	—0.93
566	107.08	—0.62	5.82	—0.44	16.07	+0.43	5.47	+0.15	7.83	—0.04	1.22	0.74	1.17	0.91
567	106.48	0.60	5.39	0.43	16.52	0.45	5.62	0.15	7.78	0.05	1.20	0.73	1.16	0.89
568	105.88	0.60	4.98	0.41	16.98	0.46	5.77	0.15	7.73	0.05	1.17	0.72	1.15	0.87
569	105.30	0.58	4.60	0.38	17.45	0.47	5.91	0.14	7.67	0.06	1.15	0.72	1.14	0.86
570	104.74	0.56	4.23	0.37	17.93	0.48	6.05	0.14	7.60	0.07	1.12	0.71	—1.13	—0.85
571	104.19	—0.55	3.87	—0.36	18.42	+0.49	6.19	+0.14	7.52	—0.08	1.09	0.71	1.11	0.85
572	103.65	0.54	3.54	0.33	18.92	0.50	6.32	0.13	7.44	0.08	1.07	0.71	1.09	0.84
573	103.13	0.52	3.22	0.32	19.42	0.50	6.44	0.12	7.35	0.09	1.04	0.71	1.07	0.83
574	102.62	0.51	2.93	0.29	19.93	0.51	6.56	0.12	7.26	0.09	1.02	0.71	1.05	0.82
575	102.12	0.50	2.65	0.28	20.45	0.52	6.68	0.12	7.17	0.09	1.00	0.71	—1.04	—0.81
576	101.63	—0.49	2.39	—0.26	20.97	+0.52	6.79	+0.11	7.06	—0.11	0.98	0.72	1.02	0.81
577	101.15	0.48	2.15	0.24	21.49	0.52	6.89	0.10	6.95	0.11	0.95	0.72	1.00	0.81
578	100.69	0.46	1.92	0.23	22.03	0.54	6.98	0.09	6.84	0.11	0.93	0.73	0.98	0.80
579	100.25	0.43	1.71	0.21	22.58	0.55	7.07	0.09	6.73	0.11	0.91	0.74	0.96	0.80
580	99.81	0.44	1.53	0.18	23.12	0.54	7.16	0.09	6.61	0.12	0.89	0.76	—0.94	—0.80
581	99.39	—0.42	1.35	—0.18	23.66	+0.54	7.24	+0.08	6.48	—0.13	0.87	0.77	0.92	0.81
582	98.98	0.41	1.19	0.16	24.21	0.55	7.30	0.06	6.35	0.13	0.85	0.78	0.90	0.82
583	98.57	0.41	1.06	0.13	24.77	0.56	7.36	0.06	6.23	0.12	0.83	0.80	0.88	0.83
584	98.18	0.39	0.93	0.13	25.33	0.56	7.42	0.06	6.10	0.13	0.82	0.82	0.87	0.84
585	97.80	0.38	0.82	0.11	25 88	0.55	7.47	0.05	5.97	0.13	0.81	0.84	—0.86	—0.85
586	97.43	—0.37	0.73	—0.09	26.44	+0.56	7.52	+0.05	5.83	—0.14	0.80	0.86	0.86	0.87
587	97.07	0.36	0.65	0.08	27.00	0.56	7.55	0.03	5.69	0.14	0.79	0.87	0.85	0.89
588	96.72	0.35	0.58	0.07	27.56	0.56	7.58	0.03	5.56	0.13	0.78	0.90	0.84	0.91
589	96.38	0.34	0.53	0.05	28.12	0.56	7.61	0.03	5.42	0.14	0.77	0.92	0.83	0.93
590	96.04	0.34	0.50	0.03	28.68	0.56	7.62	0.01	5.28	0.14	0.76	0.94	—0.82	—0.94
591	95.72	—0.32	0.47	—0.03	29.24	+0.56	7.63	+0.01	5.14	—0.14	0.76	0.96	0.82	0.96
592	95.40	0.32	0.46	—0.01	29.79	0.55	7.63	0.00	5.01	0.13	0.76	0.98	0.82	0.98
593	95.08	0.32	0.47	+0.01	30.35	0.56	7.63	0.00	4.87	0.14	0.76	1.00	0.81	1.00
594	94.77	0.31	0.49	0.02	30.91	0.56	7.62	—0.01	4.73	0.14	0.76	1.02	0.81	1.02
595	94.46	0.31	0.52	0.03	31.46	0.55	7.60	0.02	4.60	0.13	0.76	1.04	—0.81	—1.04
596	94.15	—0.31	0.57	+0.05	32.01	+0.55	7.58	—0.02	4.47	—0.13	0 77	1.06	0.82	1.06
597	93.85	0.30	0.63	0.06	32.56	0.55	7.55	0.03	4.34	0.13	0.78	1.08	0.83	1.08
598	93.55	0.30	0.70	0.07	33.11	0.55	7.51	0.04	4.21	0.13	0.78	1.10	0.84	1.10
599	93.26	0.29	0.78	0.08	33.67	0.56	7.46	0.05	4.08	0.13	0.79	1.13	0.85	1.12
600	92.96	0.30	0.87	0.09	34.22	0.55	7.42	0.04	3.95	0.13	0.80	1.15	—0.85	—1.13

TABLE X, Arg. 3.—*Continued.*

Arg.	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)	($\rho.s.2$)	($\rho.c.2$)	Arg.	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)	($\rho.s.2$)	($\rho.c.2$)
0	524	180	89	49	58	60	746	17	244	33	10
1	525	178	90	50	58	61	747	17	249	32	11
2	525	176	90	50	57	62	748	16	255	30	11
3	526	174	91	51	57	63	749	16	260	29	11
4	527	172	91	52	56	64	749	16	266	27	11
5	528	170	92	53	56	65	750	16	271	26	12
6	529	168	92	54	55	66	750	16	277	24	13
7	531	165	92	55	55	67	750	17	282	23	13
8	533	163	93	55	54	68	750	17	287	22	14
9	535	161	93	56	53	69	749	18	293	20	14
10	538	158	94	57	53	70	748	19	298	19	15
11	540	156	94	57	52	71	747	21	303	18	16
12	543	154	95	58	51	72	746	22	308	17	17
13	546	151	96	59	50	73	744	24	313	16	18
14	550	149	96	59	49	74	742	26	318	14	19
15	553	146	97	60	48	75	740	29	323	13	20
16	557	144	98	60	47	76	737	31	327	12	21
17	560	141	99	61	46	77	734	34	332	11	22
18	564	138	100	61	45	78	731	37	336	10	23
19	568	135	101	62	44	79	728	40	340	09	24
20	573	132	102	62	44	80	724	43	344	09	26
21	577	129	103	62	42	81	721	47	348	08	27
22	582	126	105	62	41	82	717	51	352	07	28
23	586	123	106	63	40	83	713	55	355	07	30
24	591	120	108	63	39	84	708	59	359	06	31
25	596	117	109	63	38	85	703	63	362	06	33
26	601	113	111	63	37	86	698	68	364	05	34
27	606	110	113	63	36	87	693	72	367	05	36
28	611	107	115	63	34	88	688	77	369	05	37
29	617	103	117	63	33	89	682	82	372	05	39
30	622	100	119	62	32	90	676	87	373	05	41
31	627	96	121	62	31	91	670	92	375	05	42
32	633	93	123	62	30	92	663	97	376	05	44
33	638	90	126	62	29	93	657	102	378	05	45
34	643	86	129	61	28	94	650	108	378	06	47
35	648	82	132	61	27	95	643	114	379	06	48
36	654	79	135	60	26	96	636	119	379	06	50
37	659	76	139	60	24	97	629	125	379	07	51
38	664	72	142	59	23	98	622	130	379	08	52
39	669	69	145	58	22	99	614	136	379	08	54
40	674	65	149	58	21	100	606	141	378	09	55
41	680	62	152	57	20	101	598	147	377	10	56
42	685	58	156	56	19	102	590	153	376	11	57
43	689	55	160	55	18	103	582	159	374	12	58
44	694	52	164	54	18	104	574	164	373	13	60
45	698	49	168	53	17	105	565	170	371	14	61
46	703	46	173	52	16	106	557	176	368	15	62
47	707	43	177	51	15	107	548	181	366	16	62
48	711	40	182	50	14	108	539	186	363	17	63
49	715	37	186	49	14	109	530	192	360	19	64
50	719	35	191	47	13	110	521	197	357	20	64
51	722	32	196	46	13	111	512	202	353	21	65
52	726	30	201	45	12	112	503	207	350	23	65
53	729	28	206	43	12	113	494	212	346	24	66
54	732	26	211	42	11	114	484	217	342	25	66
55	735	24	216	40	11	115	475	222	338	27	66
56	737	22	222	39	11	116	465	227	333	28	67
57	740	21	227	38	11	117	456	231	329	30	67
58	742	19	233	36	11	118	446	235	324	31	67
59	744	18	238	35	10	119	436	239	319	32	67
60	746	17	244	33	10	120	427	243	314	34	66

TABLE X, Arg. 3.—*Continued.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)
120	427	243	314	34	66	180	11	172	66	26	33
121	417	247	309	35	66	181	10	168	66	25	33
122	407	250	303	37	66	182	9	165	67	25	34
123	397	254	298	38	65	183	9	162	67	24	35
124	387	257	292	39	65	184	8	159	68	24	36
125	378	260	286	41	64	185	8	156	69	24	37
126	368	263	280	42	63	186	8	153	71	24	38
127	359	266	274	43	63	187	8	150	72	23	39
128	349	268	268	44	62	188	9	147	74	23	40
129	339	270	262	45	61	189	9	144	75	23	41
130	330	272	256	46	60	190	10	141	77	23	42
131	320	274	250	47	59	191	11	138	79	24	43
132	310	275	244	48	58	192	13	136	81	24	44
133	301	277	238	49	57	193	14	133	84	24	45
134	292	278	232	50	56	194	16	131	86	24	46
135	282	279	225	50	55	195	18	129	89	25	47
136	273	279	219	51	54	196	21	127	91	25	48
137	263	280	213	52	52	197	23	125	94	26	49
138	254	280	207	52	51	198	26	123	97	26	50
139	245	280	201	52	50	199	29	121	99	27	51
140	236	280	195	53	49	200	32	120	102	28	52
141	227	280	189	53	47	201	36	118	105	28	52
142	218	280	183	53	46	202	39	117	108	29	53
143	209	279	177	53	45	203	43	116	112	30	54
144	201	278	171	53	44	204	47	114	115	31	54
145	193	277	165	53	42	205	51	113	118	32	55
146	184	275	160	53	41	206	55	112	122	33	55
147	176	274	154	52	40	207	60	112	125	34	56
148	168	272	149	52	39	208	65	111	129	35	56
149	160	271	144	52	38	209	69	110	132	36	57
150	152	269	139	51	36	210	75	110	135	37	57
151	145	267	134	51	35	211	80	110	139	38	57
152	137	265	129	50	35	212	85	110	143	39	57
153	130	263	124	49	34	213	91	110	146	40	57
154	123	260	120	49	33	214	96	110	150	41	57
155	116	257	115	48	32	215	102	110	153	42	57
156	109	255	111	47	31	216	108	110	157	43	57
157	103	252	107	46	30	217	114	110	160	44	57
158	96	249	103	45	30	218	120	111	164	45	57
159	90	246	100	45	29	219	126	112	168	46	57
160	84	243	96	44	29	220	133	113	171	47	57
161	78	240	93	43	28	221	139	114	175	48	56
162	73	236	90	42	28	222	145	115	178	49	56
163	67	233	87	41	27	223	152	116	182	50	55
164	62	229	84	40	27	224	159	117	185	50	55
165	57	226	81	39	27	225	166	118	188	51	54
166	53	222	79	38	27	226	173	119	192	52	53
167	48	219	77	36	27	227	180	120	195	53	52
168	44	215	75	35	27	228	187	122	198	54	52
169	40	212	73	34	27	229	194	123	201	54	51
170	36	208	71	33	27	230	201	125	204	55	51
171	32	204	70	32	27	231	208	127	207	55	50
172	29	201	68	31	28	232	216	128	210	56	49
173	26	197	67	30	28	233	223	130	213	56	49
174	23	193	67	29	28	234	230	132	216	57	48
175	21	190	66	28	29	235	238	134	218	57	47
176	18	186	66	28	30	236	245	136	221	57	46
177	16	182	65	27	30	237	253	138	224	58	45
178	14	179	65	27	31	238	260	140	226	58	45
179	12	175	65	26	32	239	268	142	228	58	44
180	11	172	66	26	33	240	275	144	231	58	43

TABLE X, Arg. 3.—*Continued.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)
240	275	144	231	58	43	300	562	208	250	42	32
241	283	146	233	58	42	301	562	208	250	42	32
242	290	148	235	58	41	302	562	208	250	42	32
243	298	150	237	58	40	303	561	208	250	42	32
244	305	152	239	58	40	304	560	208	250	41	32
245	313	154	240	57	39	305	559	208	250	41	31
246	320	156	242	57	38	306	558	208	250	41	31
247	328	159	244	57	38	307	557	208	250	41	31
248	335	161	245	57	37	308	555	208	251	41	31
249	342	163	246	57	36	309	553	208	251	40	30
250	350	165	248	56	36	310	551	208	251	40	30
251	357	167	249	56	35	311	549	209	251	40	30
252	364	169	250	55	34	312	547	209	251	39	30
253	371	171	251	55	34	313	544	209	251	39	29
254	378	173	252	54	33	314	542	210	251	39	29
255	385	175	253	54	33	315	539	210	251	38	29
256	392	177	254	54	33	316	536	210	252	38	29
257	399	179	254	53	32	317	532	211	252	38	28
258	406	180	255	52	32	318	529	211	252	37	28
259	412	182	256	52	32	319	525	212	252	37	28
260	419	184	256	51	31	320	521	213	252	36	28
261	426	185	256	51	21	321	517	213	252	36	28
262	432	187	257	50	31	322	513	214	252	35	28
263	438	189	257	50	31	323	509	215	252	34	28
264	444	190	257	50	31	324	504	216	252	34	28
265	450	192	257	49	31	325	500	217	252	33	28
266	456	193	257	49	31	326	495	218	252	33	28
267	462	194	257	48	31	327	490	219	252	32	28
268	468	196	257	48	31	328	485	221	251	31	28
269	473	197	257	47	31	329	480	222	251	31	28
270	478	198	257	47	31	330	474	223	251	30	28
271	484	199	257	46	31	331	469	224	250	30	28
272	489	200	257	46	31	332	463	225	249	29	29
273	494	201	257	45	31	333	457	227	249	29	29
274	499	202	256	45	31	334	451	228	248	28	29
275	503	203	256	45	31	335	446	229	248	27	30
276	509	203	256	45	31	336	439	231	247	27	30
277	512	204	256	44	31	337	433	233	246	26	31
278	517	205	255	44	31	338	427	234	245	25	31
279	521	205	255	44	32	339	421	236	244	25	31
280	524	206	255	44	32	340	414	238	243	24	32
281	528	206	254	43	32	341	407	239	242	24	33
282	531	207	254	43	32	342	401	241	241	24	33
283	535	207	253	43	32	343	394	243	240	23	34
284	538	207	253	43	32	344	387	244	238	23	35
285	541	208	253	43	32	345	380	246	236	23	35
286	544	208	252	43	33	346	373	248	235	22	36
287	546	208	252	43	33	347	366	249	233	22	37
288	549	208	252	43	33	348	359	251	231	22	38
289	551	208	251	42	33	349	352	253	229	22	38
290	553	208	251	42	33	350	344	255	227	21	39
291	555	208	251	42	33	351	337	256	225	21	40
292	556	208	251	42	33	352	330	258	223	21	41
293	558	208	251	42	33	353	323	260	221	21	42
294	559	208	251	42	33	354	315	261	218	21	42
295	560	208	250	42	33	355	308	263	216	22	43
296	561	208	250	42	33	356	300	264	213	22	44
297	562	208	250	42	32	357	293	266	211	22	45
298	562	208	250	42	32	358	285	267	208	22	46
299	562	208	250	42	32	359	278	269	205	23	47
300	562	208	250	42	32	360	270	270	203	23	47

TABLE X, Arg. 3.—*Continued.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)
360	270	270	203	23	47	420	14	183	69	44	32
361	263	272	200	23	48	421	16	180	70	44	32
362	255	273	197	24	49	422	18	176	72	43	32
363	248	274	194	25	50	423	20	173	74	42	32
364	241	275	190	25	50	424	22	170	76	41	32
365	233	276	187	26	51	425	24	166	78	41	32
366	226	277	184	26	52	426	27	163	81	40	32
367	219	278	181	27	52	427	30	160	83	39	32
368	211	278	177	28	53	428	33	157	86	38	33
369	204	279	174	29	53	429	36	153	89	38	33
370	197	280	171	29	54	430	40	150	92	37	33
371	190	280	167	30	54	431	44	147	95	36	34
372	183	281	164	31	54	432	48	144	99	35	34
373	176	281	160	32	55	433	53	142	103	35	35
374	169	281	157	33	55	434	57	139	106	34	36
375	162	281	153	34	55	435	62	136	111	34	36
376	155	281	150	35	55	436	67	134	115	33	37
377	149	281	146	36	55	437	72	132	119	33	38
378	142	281	142	37	55	438	77	130	124	32	39
379	136	281	139	38	55	439	83	128	128	32	40
380	129	280	135	39	55	440	89	126	133	32	40
381	123	280	132	40	55	441	95	124	138	32	41
382	117	279	128	41	55	442	102	122	143	31	42
383	111	278	125	42	55	443	108	121	148	31	43
384	105	278	122	42	54	444	115	120	153	31	44
385	99	277	118	43	54	445	121	118	159	31	45
386	94	276	115	44	54	446	128	118	164	31	46
387	89	274	111	45	53	447	135	117	170	31	47
388	83	273	108	46	53	448	142	116	176	32	48
389	78	272	105	47	52	449	150	116	181	32	49
390	73	270	102	47	52	450	157	115	187	32	50
391	68	268	99	48	51	451	165	115	193	33	51
392	63	266	97	48	50	452	173	115	199	33	52
393	59	265	94	49	50	453	181	116	205	34	53
394	54	263	91	49	49	454	189	117	211	34	54
395	50	261	88	50	48	455	197	117	217	35	55
396	46	258	86	50	47	456	206	118	223	36	56
397	42	256	83	51	47	457	215	120	229	36	57
398	39	254	81	51	46	458	223	121	236	37	58
399	35	251	79	51	45	459	232	123	242	38	58
400	32	248	77	52	45	460	241	125	248	39	59
401	29	246	75	52	44	461	250	127	254	40	60
402	27	243	73	52	43	462	259	129	260	41	60
403	24	240	72	52	42	463	268	131	265	42	61
404	22	237	70	52	41	464	278	134	271	43	61
405	19	234	69	52	40	465	287	137	277	44	62
406	17	231	68	52	39	466	296	140	283	45	62
407	15	228	67	51	38	467	306	143	288	47	62
408	14	225	66	51	38	468	315	147	294	48	62
409	13	221	65	51	37	469	324	150	299	49	62
410	12	218	64	50	36	470	334	154	305	51	62
411	11	215	64	50	36	471	343	158	310	52	62
412	11	211	64	49	35	472	353	162	315	53	62
413	10	208	64	49	34	473	362	167	320	54	62
414	10	204	64	48	34	474	372	171	325	55	62
415	10	201	64	48	33	475	382	176	329	57	62
416	10	197	65	47	33	476	391	181	334	58	61
417	11	194	66	46	33	477	401	186	338	59	61
418	12	190	67	46	32	478	411	191	342	60	60
419	13	187	68	45	32	479	420	196	346	61	60
420	14	183	69	44	32	480	430	202	349	62	59

TABLE X, Arg. 3.—*Concluded.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)
480	430	202	349	62	59	540	741	381	186	29	19
481	439	207	353	63	58	541	739	378	181	28	20
482	449	213	356	64	57	542	737	375	176	27	21
483	458	218	359	65	56	543	735	372	171	26	22
484	468	223	361	66	55	544	732	368	167	25	23
485	477	230	364	67	54	545	730	365	162	24	24
486	486	236	366	68	53	546	727	361	158	24	25
487	496	242	368	69	52	547	724	357	153	23	26
488	505	248	370	69	51	548	721	353	149	22	27
489	514	254	371	70	50	549	717	350	145	22	28
490	523	260	373	70	49	550	714	346	141	21	29
491	532	266	374	71	48	551	710	342	137	21	30
492	540	272	374	71	46	552	706	338	133	21	31
493	549	278	375	72	45	553	702	333	130	20	32
494	558	284	375	72	44	554	698	329	126	20	33
495	566	289	375	72	42	555	693	325	123	20	34
496	574	295	374	72	41	556	689	321	120	20	35
497	583	301	374	72	39	557	684	316	117	19	36
498	591	307	373	72	38	558	679	312	114	19	38
499	599	313	872	72	37	559	674	308	112	19	39
500	606	318	371	72	35	560	669	303	109	20	40
501	614	324	369	71	34	561	664	299	107	20	41
502	621	329	367	71	33	562	659	295	105	20	42
503	629	334	365	71	31	563	654	290	103	20	43
504	636	339	363	70	30	564	649	286	101	20	44
505	643	344	360	70	29	565	644	282	99	20	45
506	649	349	357	69	28	566	638	278	97	21	46
507	656	353	354	68	27	567	633	274	96	21	47
508	662	358	351	68	25	568	628	270	94	22	48
509	668	362	347	67	24	569	623	266	93	22	49
510	674	366	344	66	23	570	618	262	92	23	50
511	680	370	340	65	22	571	613	258	91	23	50
512	686	374	336	64	21	572	608	255	90	24	51
513	691	377	332	63	20	573	602	251	90	25	52
514	696	380	327	62	19	574	598	247	89	25	53
515	701	383	323	61	18	575	593	244	88	26	54
516	706	386	318	59	18	576	588	240	88	27	54
517	710	388	313	58	17	577	583	237	87	28	55
518	714	390	308	57	16	578	578	234	87	28	56
519	718	392	303	56	16	579	574	231	87	29	56
520	721	394	298	54	15	580	570	228	86	30	57
521	725	396	293	53	15	581	565	225	86	31	57
522	728	397	287	52	14	582	561	222	86	32	58
523	731	398	282	50	14	583	557	219	86	32	58
524	733	399	276	49	14	584	554	216	86	33	58
525	736	400	270	48	14	585	551	214	86	34	58
526	738	400	265	46	14	586	547	211	86	35	59
527	740	400	259	45	14	587	544	209	86	36	59
528	742	400	253	44	14	588	541	206	86	37	59
529	743	399	247	42	14	589	539	204	86	38	59
530	744	399	242	41	14	590	536	202	87	39	60
531	745	398	236	40	14	591	534	199	87	40	60
532	745	397	230	38	14	592	532	197	87	41	60
533	746	396	225	37	15	593	530	195	88	42	60
534	746	394	219	36	15	594	528	193	88	43	60
535	745	392	213	34	16	595	527	190	88	44	59
536	745	390	208	33	17	596	526	188	88	45	59
537	744	388	202	32	17	597	525	186	88	46	59
538	743	386	197	31	18	598	525	184	89	47	59
539	742	383	192	30	19	599	524	182	89	48	58
540	741	381	186	29	19	600	524	180	89	49	58

TAB.	XI		XII.			XIII.			XIV.			XV.			XVI.		
ARG	4.		5.			6.			7.			8.			9.		
	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
0	0.67	0.13	1.11	0.03	0.21	0.06	0.13	0.18	0.10	0.18	0.19	0.05	0.17	0.14	0.13	0.10	0.13
10	0.64	0.13	1.12	0.03	0.22	0.06	0.14	0.18	0.10	0.18	0.19	0.09	0.15	0.19	0.12	0.10	0.12
20	0.61	0.14	1.12	0.03	0.24	0.07	0.15	0.18	0.10	0.18	0.19	0.15	0.13	0.25	0.12	0.09	0.11
30	0.58	0.16	1.11	0.03	0.26	0.07	0.16	0.17	0.10	0.18	0.19	0.22	0.11	0.31	0.11	0.09	0.10
40	0.54	0.17	1.10	0.03	0.28	0.08	0.16	0.16	0.10	0.18	0.19	0.30	0.10	0.37	0.11	0.09	0.09
50	0.50	0.19	1.08	0.04	0.30	0.08	0.17	0.16	0.10	0.18	0.19	0.39	0.10	0.44	0.10	0.08	0.08
60	0.47	0.21	1.06	0.04	0.31	0.08	0.17	0.15	0.10	0.18	0.19	0.50	0.11	0.52	0.09	0.08	0.07
70	0.43	0.24	1.04	0.05	0.32	0.09	0.18	0.14	0.09	0.17	0.18	0.61	0.13	0.59	0.09	0.08	0.06
80	0.39	0.26	1.00	0.06	0.34	0.09	0.18	0.13	0.09	0.17	0.18	0.74	0.15	0.67	0.08	0.07	0.05
90	0.35	0.29	0.97	0.07	0.35	0.09	0.18	0.12	0.09	0.16	0.17	0.87	0.17	0.75	0.08	0.07	0.04
100	0.31	0.31	0.93	0.08	0.36	0.09	0.18	0.12	0.08	0.16	0.17	1.00	0.21	0.83	0.07	0.07	0.03
110	0.27	0.34	0.88	0.09	0.37	0.09	0.19	0.11	0.08	0.15	0.16	1.14	0.25	0.91	0.06	0.07	0.02
120	0.23	0.37	0.84	0.11	0.37	0.09	0.19	0.10	0.07	0.14	0.15	1.28	0.29	0.98	0.06	0.06	0.02
130	0.20	0.40	0.79	0.12	0.38	0.09	0.18	0.09	0.07	0.14	0.14	1.42	0.34	1.06	0 06	0.06	0.01
140	0.17	0.43	0.74	0.14	0.38	0.09	0.18	0.08	0.06	0.13	0.13	1.56	0.40	1.13	0.05	0.06	0.01
150	0.14	0.46	0.68	0.16	0.38	0.09	0.18	0.07	0.06	0.12	0.12	1.70	0.45	1.19	0.05	0.06	0.01
160	0.11	0.49	0.63	0.18	0.38	0.09	0.18	0.06	0.05	0.11	0.11	1.83	0.52	1.25	0.04	0.06	0.00
170	0.09	0.51	0.57	0.19	0.37	0.08	0.17	0.05	0.05	0.10	0.10	1.96	0.58	1.31	0.04	0.06	0.00
180	0.07	0.54	0.52	0.21	0.37	0.08	0.17	0.04	0.04	0.10	0.09	2.09	0.65	1.36	0.04	0.06	0.00
190	0.05	0.56	0.46	0.23	0.36	0.08	0.16	0.04	0.04	0.09	0.08	2.20	0.72	1.40	0.04	0.07	0.00
200	0.04	0.59	0.41	0.25	0.35	0.08	0.16	0.04	0.03	0.08	0.07	2.31	0.78	1.43	0.04	0.07	0.00
210	0.03	0.61	0.36	0.26	0.34	0.07	0.15	0.02	0.03	0.07	0.06	2.40	0.85	1.46	0.04	0.07	0.00
220	0.02	0.63	0.32	0.28	0.32	0.07	0.14	0.02	0.02	0.0[illegible]	0.06	2.48	0.92	1.48	0.04	0.07	0.01
230	0.02	0.64	0.27	0.30	0.31	0.07	0.13	0.02	0.02	0.06	0.05	2.55	0.98	1.50	0.04	0.08	0.01
240	0.02	0.66	0.23	0.31	0.30	0.06	0.12	0.01	0.02	0.05	0.04	2.61	1.05	1.50	0.04	0.08	0.02
250	0.03	0.67	0.20	0.32	0.23	0.06	0.11	0.01	0.01	0.04	0.03	2.65	1.10	1.50	0.05	0.08	0.02
260	0.04	0.67	0.16	0.34	0.26	0.06	0.11	0.01	0.01	0.04	0.03	2.68	1.16	1.48	0.05	0.09	0.03
270	0.06	0.68	0.14	0.35	0.24	0.05	0.10	0.01	0.01	0.03	0.02	2.70	1.21	1.46	0.06	0.09	0.04
280	0.08	0.68	0.12	0.35	0.22	0.05	0.09	0.01	0.00	0.03	0.02	2.70	1.25	1.43	0.06	0.09	0.05
290	0.10	0.68	0.10	0.36	0.21	0.04	0.08	0.01	0.00	0.02	0.01	2.68	1.29	1.40	0.06	0.10	0.06
300	0.13	0.67	0 08	0.37	0.19	0.04	0.07	0.02	0.00	0.02	0.01	2.65	1.33	1.36	0.07	0.10	0.07
310	0.16	0.67	0.08	0.37	0.18	0.04	0.06	0.02	0.00	0.02	0.01	2.61	1.35	1.31	0.08	0.10	0.08
320	0.19	0.66	0.08	0.37	0.16	0.03	0.05	0.02	0.00	0.02	0.01	2.55	1.37	1.25	0.08	0.11	0.09
330	0.22	0.64	0.09	0.37	0.14	0.03	0.04	0.03	0.00	0.02	0.01	2.48	1.39	1.19	0.09	0.11	0.10
340	0.26	0.63	0.10	0.37	0.12	0.02	0.04	0.04	0.00	0.02	0.01	2.40	1.40	1.13	0.09	0.11	0.11
350	0.30	0.61	0.12	0.36	0.10	0.02	0.03	0.04	0.00	0.02	0.01	2.31	1.40	1.06	0.10	0.12	0.12
360	0.33	0.59	0.14	0.36	0.09	0.01	0.03	0.05	0.00	0.02	0.02	2.20	1.39	0.98	0.11	0.12	0.13
370	0.37	0.56	0.16	0.35	0.08	0.01	0.02	0.06	0.01	0.03	0.02	2.09	1.37	0.91	0.11	0.12	0.14
380	0.41	0.54	0.20	0.34	0.06	0.01	0.02	0.07	0.01	0.03	0.03	1.96	1.35	0.83	0.12	0.13	0.15
390	0.45	0.51	0.23	0.33	0.05	0.01	0.02	0.08	0.01	0.04	0.03	1.83	1.32	0.75	0.12	0.13	0.16
400	0.49	0.49	0.27	0.32	0.04	0.01	0.02	0.08	0.02	0.04	0.04	1.70	1.29	0.67	0.13	0.13	0.17
410	0.53	0.46	0.32	0.31	0.03	0.01	0.01	0.09	0.02	0.05	0.05	1.56	1.25	0.59	0.13	0.13	0.18
420	0.57	0.43	0.36	0.29	0.03	0.01	0.01	0.10	0.03	0.06	0.06	1.42	1.21	0.52	0.14	0.14	0.18
430	0.60	0.40	0.41	0.28	0.02	0.01	0.02	0.11	0.03	0.06	0.06	1.28	1.16	0.44	0.14	0.14	0.19
440	0.63	0.37	0.46	0.26	0.02	0.01	0.02	0.12	0.04	0.07	0.07	1.14	1.10	0.37	0.15	0.14	0.19
450	0.66	0.34	0.52	0.24	0.02	0.01	0.02	0.13	0.04	0.08	0.08	1.00	1.05	0.31	0.15	0.14	0.19
460	0.69	0.31	0.57	0.22	0.02	0.01	0.02	0.14	0.05	0.09	0.09	0.87	0.98	0.25	0.16	0.14	0.20
470	0.71	0.29	0.63	0.21	0.03	0.02	0.03	0.15	0.05	0.10	0.10	0.74	0.92	0.19	0.16	0.14	0.20
480	0.73	0.26	0.68	0.19	0.03	0.02	0.03	0.16	0.06	0.10	0.11	0.61	0.85	0.14	0.16	0.14	0.20
490	0.75	0.24	0.74	0.17	0.04	0.02	0.04	0.16	0.07	0.11	0.12	0.50	0.78	0.10	0.16	0.13	0.20
500	0.76	0.21	0.79	0.15	0.05	0.02	0.04	0.17	0.07	0.12	0.13	0.39	0.72	0.07	0.16	0.13	0.20
510	0.77	0.19	0.84	0.14	0.06	0.03	0.05	0.18	0.07	0.13	0.14	0.30	0.65	0.04	0.16	0.13	0.20
520	0.78	0.17	0.88	0.12	0.08	0.03	0.06	0.18	0.08	0.14	0.14	0.22	0.58	0.02	0.16	0.13	0.19
530	0.78	0.16	0.93	0.10	0.09	0.03	0.07	0.18	0.08	0.14	0.15	0.15	0.52	0.00	0.16	0.12	0.19
540	0.78	0.14	0.97	0.09	0.10	0.04	0.08	0.19	0.08	0.15	0.16	0.09	0.45	0.00	0.16	0.12	0.18
550	0.77	0.13	1.00	0.08	0.12	0.04	0.09	0.19	0.09	0.16	0.17	0.05	0.40	0.00	0.15	0.12	0.18
560	0.76	0.13	1.04	0.07	0.14	0.04	0.09	0.19	0.09	0.16	0.17	0.02	0.34	0.02	0.15	0.11	0.17
570	0.74	0.12	1.06	0.06	0.16	0.05	0.10	0.19	0.09	0.17	0.18	0.00	0.29	0.04	0.15	0.11	0.16
580	0.72	0.12	1.08	0.05	0.18	0.05	0.11	0.19	0.10	0.17	0.18	0.00	0.25	0.07	0.14	0.11	0.15
590	0.70	0.12	1.10	0.04	0.19	0.06	0.12	0.19	0.10	0.18	0.19	0.02	0.21	0.10	0.14	0.10	0.14
600	0.67	0.13	1.11	0.03	0.21	0.06	0.13	0.18	0.10	0.18	0.19	0.05	0.17	0.14	0.13	0.10	0.13

TABLE XVII *a*.

Year.	(*v.s.*1)	(*v.c.*1)	Year.	(*v.s.*1)	(*v.c.*1)
	″	″		″	″
1800	—240.33	—162.11	1850	—183.57	—189.69
1801	239.19	162.61	1851	182.44	190.29
1802	238.06	163.12	1852	181.30	190.89
1803	236.92	163.63	1853	180.17	191.50
1804	235.79	164.14	1854	179.04	192.10
1805	—234.65	—164.65	1855	—177.91	—192.71
1806	233.51	165.16	1856	176.78	193.32
1807	232.38	165.68	1857	175.65	193.93
1808	231.24	166.20	1858	174.51	194.54
1809	230.11	166.72	1859	173.38	195.15
1810	—228.97	—167.24	1860	—172.25	—195.77
1811	227.83	167.76	1861	171.12	196.39
1812	226.70	168.29	1862	169.99	197.01
1813	225.56	168.82	1863	168.86	197.63
1814	224.43	169.35	1864	167.72	198.25
1815	—223.29	—169.88	1865	—166.59	—198.88
1816	222.15	170.41	1866	165.46	199.51
1817	221.02	170.95	1867	164.33	200.14
1818	219.88	171.49	1868	163.20	200.77
1819	218.75	172.03	1869	162.07	201.40
1820	—217.61	—172.57	1870	—160.94	—202.04
1821	216.47	173.11	1871	159.81	202.68
1822	215.34	173.66	1872	158.68	203.32
1823	214.20	174.21	1873	157.55	203.96
1824	213.07	174.76	1874	156.42	204.60
1825	—211.93	—175.31	1875	—155.29	—205.24
1826	210.80	175.86	1876	154.16	205.89
1827	209.66	176.42	1877	153.03	206.54
1828	208.53	176.97	1878	151.91	207.19
1829	207.39	177.53	1879	150.78	207.84
1830	—206.26	—178.09	1880	—149.65	—208.49
1831	205.13	178.65	1881	148.52	209.15
1832	203.99	179.22	1882	147.39	209.81
1833	202.86	179.78	1883	146.27	210.47
1834	201.72	180.35	1884	145.14	211.13
1835	—200.59	—180.92	1885	—144.01	—211.79
1836	199.45	181.49	1886	142.89	212.45
1837	198.32	182.06	1887	141.76	213.12
1838	197.18	182.64	1888	140.64	213.79
1839	196.05	183.22	1889	139.51	214.46
1840	—194.91	—183.80	1890	—138.39	—215.13
1841	193.78	184.38	1891	137.27	215.80
1842	192.64	184.96	1892	136.14	216.48
1843	191.51	185.55	1893	135.02	217.17
1844	190.37	186.13	1894	133.90	217.84
1845	—189.24	—186.72	1895	—132.77	—218.52
1846	188.10	187.31	1896	131.65	219.20
1847	186.97	187.90	1897	130.53	219.88
1848	185.84	188.50	1898	129.40	220.57
1849	184.70	189.09	1899	128.28	221.26
1850	—183.57	—189.69	1900	—127.16	—221.95

TABLE XVII *b*.

Year.	(*v.s.*1)		(*v.c.*1)	
	″		″	
1500	—560.44		— 96.24	
1510	550.85	+ 9.59	95.81	+0.43
1520	541.17	9.68	95.54	0.27
1530	531.40	9.77	95.44	+0.10
1540	521.54	9.86	95.51	—0.07
1550	—511.59	9.95	— 95.75	0.24
1560	501.55	+10.04	96.16	—0.41
1570	491.42	10.13	96.75	0.59
1580	481.20	10.22	97.52	0.77
1590	470.90	10.30	98.47	0.95
1600	—460.52	10.38	— 99.60	1.13
1610	450.06	+10.46	100.93	—1.33
1620	439.53	10.53	102.44	1.51
1630	428.92	10.61	104.14	1.70
1640	418.24	10.68	106.02	1.88
1650	—407.49	10.75	—108.09	2.07
1660	396.67	+10.82	110.35	—2.26
1670	385.79	10.88	112.80	2.45
1680	374.86	10.93	115.45	2.65
1690	363.87	10.99	118.28	2.83
1700	—352.83	11.04	—121.30	3.02
1710	341.74	+11.09	124.52	—3.22
1720	330.61	11.13	127.93	3.41
1730	319.43	11.18	131.53	3.60
1740	308.22	11.21	135.32	3.79
1750	—296.97	11.25	—139.30	3.98
1760	285.69	+11.28	143.48	—4.18
1770	274.38	11.31	147.85	4.37
1780	263.04	11.34	152.42	4.57
1790	251.69	11.35	157.17	4.75
1800	—240.33	11.36	—162.11	4.94
1810	228.97	+11.36	167.24	—5.13
1820	217.61	11.36	172.57	5.33
1830	206.26	11.35	178.09	5.52
1840	194.91	11.35	183.80	5.71
1850	—183.57	11.34	—189.69	5.89
1860	172.25	+11.32	195.77	—6.08
1870	160.94	11.31	202.04	6.27
1880	149.65	11.29	208.49	6.45
1890	138.39	11.26	215.13	6.64
1900	—127.16	11.23	—221.95	6.82
1910	115.98	+11.18	228.94	—6.99
1920	104.84	11.14	236.11	7.17
1930	93.75	11.09	243.46	7.35
1940	82.72	11.03	251.00	7.54
1950	— 71.74	10.98	—258.72	7.72
1960	60.82	+10.92	266.61	—7.89
1970	49.97	10.85	274.68	8.07
1980	39.19	10.78	282.93	8.25
1990	28.49	10.70	291.35	8.42
2000	— 17.88	10.61	—299.95	8.60

Note.—The values of (*v.s.*1) and (*v.c.*1) must be taken from only *one* of the two tables XVII *a* and XVII *b*.

TABLE XVII *b*.—*Concluded.*

Year.	(v.s.2)		(v.c.2)		(v.s.3)	(v.c.3)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	"		"		"	"							
1500	—152.85	+0.42	—124.83	—0.11	—10.39	—6.70	+1379	— 891	+2361	—382	—158	— 96	— 80
1510	152.43	0.43	124.94	0.12	10.35	6.70	1369	885	2258	382	165	96	80
1520	152.00	0.43	125.06	0.13	10.31	6.70	1359	881	2154	382	172	96	81
1530	151.57	0.44	125.19	0.15	10.28	6.69	1349	878	2047	381	179	96	81
1540	151.13	0.44	125.34	0.16	10.24	6.69	1338	877	1944	381	187	96	82
1550	—150.69	+0.45	—125.50	—0.17	—10.20	—0.69	+1328	— 878	+1838	—381	—195	— 96	— 82
1560	150.24	0.46	125.67	0.18	10.16	6.69	1318	881	1731	381	203	96	83
1570	149.78	0.46	125.85	0.19	10.13	6.69	1307	886	1623	381	210	96	83
1580	149.32	0.46	126.04	0.20	10.09	6.69	1297	893	1513	382	218	96	84
1590	148.86	0.47	126.24	0.20	10.05	6.70	1286	901	1403	382	226	96	84
1600	—148.39	+0.47	—126.44	—0.22	—10.02	—6.70	+1275	— 911	+1292	—383	—234	— 96	— 85
1610	147.92	0.48	126.66	0.23	9.98	6.70	1264	923	1180	384	242	96	86
1620	147.44	0.48	126.89	0.24	9.94	6.71	1253	937	1067	385	250	96	86
1630	146.96	0.48	127.13	0.25	9.91	6.71	1241	954	954	386	258	96	87
1640	146.48	0.49	127.38	0.26	9.87	6.72	1230	972	840	388	266	96	87
1650	—145.99	+0.49	—127.64	—0.27	— 9.83	—6.73	+1219	— 992	+ 725	—389	—274	— 96	— 88
1660	145.50	0.50	127.91	0.28	9.79	6 74	1208	1014	610	391	282	96	89
1670	145.00	0.50	128.19	0.30	9.75	6.75	1197	1038	495	392	290	96	89
1680	144.50	0.50	128.49	0.31	9.71	6.76	1185	1064	379	394	298	96	90
1690	144.00	0.51	128.80	0.32	9.67	6.77	1174	1092	262	396	306	96	90
1700	—143.49	+0.51	—129.12	—0.33	— 9.63	—6.78	+1163	—1123	+ 145	—398	—314	— 97	— 91
1710	142 98	0.51	129.45	0.34	9.59	6.79	1152	1155	+ 27	400	323	97	92
1720	142.47	0.51	129 79	0.35	9.55	6.80	1141	1189	— 92	403	331	97	92
1730	141.96	0.52	130.14	0.37	9.51	6.81	1130	1226	212	405	339	97	93
1740	141.44	0.51	130.51	0.37	9.47	6.83	1119	1264	333	408	348	98	93
1750	—140.93	+0.52	—130.88	—0.39	— 9.43	—6.84	+1108	—1304	— 454	—411	—356	— 98	— 94
1760	140.41	0.53	131.27	0.40	9.39	6.85	1097	1346	575	414	365	98	95
1770	139.88	0.52	131.67	0.41	9.35	6.87	1086	1391	696	417	373	98	95
1780	139.36	0.53	132.08	0.42	9.31	6.89	1074	1437	817	420	382	99	96
1790	138.83	0.52	132.50	0.43	9.27	6.90	1063	1486	938	424	390	99	96
1800	—138.31	+0.52	—132.93	—0.44	— 9.23	—6.92	+1052	—1536	—1059	—427	—399	— 99	— 97
1810	137.79	0.53	133.37	0.45	9.19	6.93	1041	1588	1180	431	408	99	98
1820	137.26	0.52	133.82	0.46	9.14	6.95	1030	1642	1301	435	416	99	98
1830	136.74	0.53	134.28	0.48	9.10	6.96	1019	1699	1422	439	425	100	99
1840	136.21	0.52	134.76	0.49	9.06	6.98	1008	1758	1544	443	433	100	99
1850	—135.69	+0.52	—135.25	—0.50	— 9.02	—7.00	+ 997	—1819	—1665	—447	—442	—100	—100
1860	135.17	0.52	135.75	0.51	8.98	7.02	986	1881	1786	451	450	100	101
1870	134.65	0.52	136.26	0.52	8.94	7.04	975	1945	1906	456	459	101	101
1880	134.13	0.52	136.78	0.53	8.90	7.07	964	2012	2027	461	467	101	102
1890	133.61	0.52	137.31	0.54	8.86	7.09	953	2081	2147	466	476	101	102
1900	—133.09	+0.52	—137.85	—0.55	— 8.82	—7.12	+ 942	—2151	—2267	—471	—484	—102	—103
1910	132.57	0.51	138.40	0.56	8.78	7.15	931	2223	2386	476	492	102	104
1920	132.06	0.51	138.96	0.57	8.74	7.17	921	2297	2505	481	500	103	104
1930	131.55	0.51	139.53	0.58	8.70	7.20	910	2372	2623	486	509	103	105
1940	131.04	0.50	140.11	0.59	8.66	7.22	900	2449	2741	492	517	103	105
1950	—130.54	+0.50	—140.70	—0.61	— 8.62	—7.25	+ 889	—2529	—2859	—497	—525	—104	—106
1960	130.04	0.49	141.31	0.62	8.58	7.28	879	2611	2976	503	533	104	106
1970	129.55	0.49	141.93	0.63	8.54	7.31	868	2695	3092	509	541	105	107
1980	129.06	0.49	142.56	0.64	8.50	7.34	858	2780	3207	515	550	105	107
1990	128.57	0.48	143.20	0.65	8.46	7.37	848	2867	3321	521	558	106	108
2000	—128.09		—143.85		— 8.42	—7.40	+ 838	—2956	—3434	—527	—566	—106	—108

TABLE XVIII.—Reduction to the Ecliptic. Argument *u*.

u	*u*	*R*		*u*	*u*	*R*		*u*	*u*	*R*		*u*	*u*	*R*	
°	°	″		°	°	″		°	°	″		°	°	″	
0	180	10.00	−0.33	45	225	0.63	+0.01	90	270	10.00	+0.33	135	315	19.37	−0.01
1	181	9.67	0.32	46	226	0.64	0.01	91	271	10.33	0.32	136	316	19.36	0.01
2	182	9.35	0.33	47	227	0.65	0.03	92	272	10.65	0.33	137	317	19.35	0.03
3	183	9.02	0.32	48	228	0.68	0.04	93	273	10.98	0.32	138	318	19.32	0.04
4	184	8.70	0.33	49	229	0.72	0.05	94	274	11.30	0.33	139	319	19.28	0.05
5	185	8.37	−0.32	50	230	0.77	+0.07	95	275	11.63	+0.32	140	320	19.23	−0.07
6	186	8.05	0.32	51	231	0.84	0.07	96	276	11.95	0.32	141	321	19.16	0.07
7	187	7.73	0.31	52	232	0.91	0.08	97	277	12.27	0.31	142	322	19.09	0.08
8	188	7.42	0.32	53	233	0.99	0.10	98	278	12.58	0.32	143	323	19.01	0.10
9	189	7.10	0.30	54	234	1.09	0.11	99	279	12.90	0.30	144	324	18.91	0.11
10	190	6.80	−0.31	55	235	1.20	+0.11	100	280	13.20	+0.31	145	325	18.80	−0.11
11	191	6.49	0.30	56	236	1.31	0.13	101	281	13.51	0.30	146	326	18.69	0.13
12	192	6.19	0.30	57	237	1.44	0.14	102	282	13.81	0.30	147	327	18.56	0.14
13	193	5.89	0.29	58	238	1.58	0.15	103	283	14.11	0.29	148	328	18.42	0.15
14	194	5.60	0.28	59	239	1.73	0.15	104	284	14.40	0.28	149	329	18.27	0.15
15	195	5.32	−0.28	60	240	1.88	+0.17	105	285	14.68	+0.28	150	330	18.12	−0.17
16	196	5.04	0.28	61	241	2.05	0.18	106	286	14.96	0.28	151	331	17.95	0.18
17	197	4.76	0.27	62	242	2.23	0.19	107	287	15.24	0.27	152	332	17.77	0.19
18	198	4.49	0.26	63	243	2.42	0.20	108	288	15.51	0.26	153	333	17.58	0.20
19	199	4.23	0.25	64	244	2.62	0.20	109	289	15.77	0.25	154	334	17.38	0.20
20	200	3.98	−0.25	65	245	2.82	+0.22	110	290	16.02	+0.25	155	335	17.18	−0.22
21	201	3.73	0.24	66	246	3.04	0.22	111	291	16.27	0.24	156	336	16.96	0.22
22	202	3.49	0.23	67	247	3.26	0.23	112	292	16.51	0.23	157	337	16.74	0.23
23	203	3.26	0.22	68	248	3.49	0.24	113	293	16.74	0.22	158	338	16.51	0.24
24	204	3.04	0.22	69	249	3.73	0.25	114	294	16.96	0.22	159	339	16.27	0.25
25	205	2.82	−0.20	70	250	3.98	+0.25	115	295	17.18	+0.20	160	340	16.02	−0.25
26	206	2.62	0.20	71	251	4.23	0.26	116	296	17.38	0.20	161	341	15.77	0.26
27	207	2.42	0.19	72	252	4.49	0.27	117	297	17.58	0.19	162	342	15.51	0.27
28	208	2.23	0.18	73	253	4.76	0.28	118	298	17.77	0.18	163	343	15.24	0.28
29	209	2.05	0.17	74	254	5.04	0.28	119	299	17.95	0.17	164	344	14.96	0.28
30	210	1.88	−0.15	75	255	5.32	+0.28	120	300	18.12	+0.15	165	345	14.68	−0.28
31	211	1.73	0.15	76	256	5.60	0.29	121	301	18.27	0.15	166	346	14.40	0.29
32	212	1.58	0.14	77	257	5.89	0.30	122	302	19.42	0.14	167	347	14.11	0.30
33	213	1.44	0.13	78	258	6.19	0.30	123	303	18.56	0.13	168	348	13.81	0.30
34	214	1.31	0.11	79	259	6.49	0.31	124	304	18.69	0.11	169	349	13.51	0.31
35	215	1.20	−0.11	80	260	6.80	+0.30	125	305	18.80	+0.11	170	350	13.20	−0.30
36	216	1.09	0.10	81	261	7.10	0.32	126	306	18.91	0.10	171	351	12.90	0.32
37	217	0 99	0.08	82	262	7.42	0.31	127	307	19.01	0.08	172	352	12.58	0.31
38	218	0.91	0.07	83	263	7.73	0.32	128	308	19.09	0.07	173	353	12.27	0.32
39	219	0.84	0.07	84	264	8.05	0.32	129	309	19.16	0.07	174	354	11.95	0.32
40	220	0.77	−0.05	85	265	8.37	+0.33	130	310	19.23	+0.05	175	355	11.63	−0.33
41	221	0.72	0.04	86	266	8.70	0.32	131	311	19.28	0.04	176	356	11.30	0.32
42	222	0.68	0.03	87	267	9.02	0.33	132	312	19.32	0.03	177	357	10.98	0.33
43	223	0.65	0.01	88	268	9.35	0.32	133	313	19.35	0.01	178	358	10.65	0.32
44	224	0.64	0.01	89	269	9.67	0.33	134	314	19.36	0.01	179	359	10.33	0.33
45	225	0.63		90	270	10.00		135	315	19.37		180	360	10.00	

TABLE XIX.—Principal Term of the Latitude. Argument *u*.

u	β			*u*	β			*u*	β		
180°	′ ″		360°	190°	′ ″		350°	200°	′ ″		340°
0°	0 0.00	8.09	180°	10°	8 2.82	7.96	170°	20°	15 50.98	7.60	160°
10′	0 8.09	8.09	50′	10′	8 10.78	7.96	50′	10′	15 58.58	7.59	50′
20	0 16.18	8.09	40	20	8 18.74	7.95	40	20	16 6.17	7.58	40
30	0 24.27	8.08	30	30	8 26.69	7.95	30	30	16 13.75	7.57	30
40	0 32.35	8.09	20	40	8 34.64	7.95	20	40	16 21.32	7.56	20
50	0 40.44	8.09	10	50	8 42.59	7.95	10	50	16 28.88	7.55	10
1°	0 48.53	8.09	179°	11°	8 50.54	7.94	169°	21°	16 36.43	7.55	159°
10′	0 56.62	8.08	50′	10′	8 58.48	7.93	50′	10′	16 43.98	7.54	50′
20	1 4.70	8.09	40	20	9 6.41	7.93	40	20	16 51.52	7.53	40
30	1 12.79	8.08	30	30	9 14.34	7.92	30	30	16 59.05	7.52	30
40	1 20.87	8.09	20	40	9 22.26	7.92	20	40	17 6.57	7.51	20
50	1 28.96	8.08	10	50	9 30.18	7.91	10	50	17 14.08	7.50	10
2°	1 37.04	8.08	178°	12°	9 38.09	7.90	168°	22°	17 21.58	7.49	158°
10′	1 45.12	8.08	50′	10′	9 45.99	7.90	50′	10′	17 29.07	7.48	50′
20	1 53.20	8.08	40	20	9 53.89	7.90	40	20	17 36.55	7.48	40
30	2 1.28	8.08	30	30	10 1.79	7.89	30	30	17 44.03	7.47	30
40	2 9.36	8.08	20	40	10 9.68	7.89	20	40	17 51.50	7.46	20
50	2 17.44	8.08	10	50	10 17.57	7.89	10	50	17 58.96	7.45	10
3°	2 25.52	8.08	177°	13°	10 25.46	7.88	167°	23°	18 6.41	7.44	157°
10′	2 33.60	8.07	50′	10′	10 33.34	7.87	50′	10′	18 13.85	7.43	50′
20	2 41.67	8.08	40	20	10 41.21	7.87	40	20	18 21.28	7.42	40
30	2 49.75	8.07	30	30	10 49.08	7.86	30	30	18 28.70	7.41	30
40	2 57.82	8.07	20	40	10 56.94	7.86	20	40	18 36.11	7.41	20
50	3 5.89	8.07	10	50	11 4.80	7.85	10	50	18 43.52	7.40	10
4°	3 13.96	8.07	176°	14°	11 12.65	7.85	166°	24°	18 50.92	7.39	156°
10′	3 22.03	8.06	50′	10′	11 20.50	7.84	50′	10′	18 58.31	7.37	50′
20	3 30.09	8.07	40	20	11 28.34	7.83	40	20	19 5.68	7.37	40
30	3 38.16	8.06	30	30	11 36.17	7.83	30	30	19 13.05	7.35	30
40	3 46.22	8.06	20	40	11 44.00	7.82	20	40	19 20.40	7.35	20
50	3 54.28	8.06	10	50	11 51.82	7.81	10	50	19 27.75	7.33	10
5°	4 2.34	8.05	175°	15°	11 59.63	7.80	165°	25°	19 35.08	7.33	155°
10′	4 10.39	8.06	50′	10′	12 7.43	7.81	50′	10′	19 42.41	7.31	50′
20	4 18.45	8.05	40	20	12 15.24	7.80	40	20	19 49.72	7.30	40
30	4 26.50	8.06	30	30	12 23.04	7.79	30	30	19 57.02	7.30	30
40	4 34.56	8.05	20	40	12 30.83	7.78	20	40	20 4.32	7.29	20
50	4 42.61	8.04	10	50	12 38.61	7.79	10	50	20 11.61	7.27	10
6°	4 50.65	8.04	174°	16°	12 46.40	7.78	164°	26°	20 18.88	7.26	154°
10′	4 58.69	8.04	50′	10′	12 54.18	7.77	50′	10′	20 26.14	7.26	50′
20	5 6.73	8.03	40	20	13 1.95	7.75	40	20	20 33.40	7.24	40
30	5 14.76	8.03	30	30	13 9.70	7.75	30	30	20 40.64	7.24	30
40	5 22.79	8.03	20	40	13 17.45	7.74	20	40	20 47.88	7.22	20
50	5 30.82	8.03	10	50	13 25.19	7.74	10	50	20 55.10	7.21	10
7°	5 38.85	8.03	173°	17°	13 32.93	7.73	163°	27°	21 2.31	7.20	153°
10′	5 46.88	8.02	50′	10′	13 40.66	7.73	50′	10′	21 9.51	7.19	50′
20	5 54.90	8.02	40	20	13 48.39	7.72	40	20	21 16.70	7.18	40
30	6 2.92	8.02	30	30	13 56.11	7.71	30	30	21 23.88	7.17	30
40	6 10.94	8.01	20	40	14 3.82	7.70	20	40	21 31.05	7.16	20
50	6 18.95	8.01	10	50	14 11.52	7.69	10	50	21 38.21	7.14	10
8°	6 26.96	8.01	172°	18°	14 19.21	7.69	162°	28°	21 45.35	7.14	152°
10′	6 34.97	8.00	50′	10′	14 26.90	7.68	50′	10′	21 52.49	7.12	50′
20	6 42.97	8.00	40	20	14 34.58	7.67	40	20	21 59.61	7.12	40
30	6 50.97	8.00	30	30	14 42.25	7.67	30	30	22 6.73	7.10	30
40	6 58.97	7.99	20	40	14 49.92	7.66	20	40	22 13.83	7.09	20
50	7 6.96	8.00	10	50	14 57.58	7.65	10	50	22 20.92	7.08	10
9°	7 14.96	7.99	171°	19°	15 5.23	7.65	161°	29°	22 28.00	7.07	151°
10′	7 22.95	7.98	50′	10′	15 12.88	7.64	50′	10′	22 35.07	7.05	50′
20	7 30.93	7.98	40	20	15 20.52	7.63	40	20	22 42.12	7.05	40
30	7 38.91	7.97	30	30	15 28.15	7.62	30	30	22 49.17	7.03	30
40	7 46.88	7.97	20	40	15 35.77	7.61	20	40	22 56.20	7.03	20
50	7 54.85	7.97	10	50	15 43.38	7.60	10	50	23 3.23	7.01	10
10°	8 2.82		170°	20°	15 50.98		160°	30°	23 10.24		150°
190°	β		350° *u*	200°	β		340° *u*	210°	β		330° *u*

TABLE XIX, Arg. *u*.—*Continued.*

u	β			*u*	β			*u*	β		
210°	′ ″		330°	220°	′ ″		320°	230°	′ ″		310°
30°	23 10.24	7.00	150°	40°	29 47.27	6.19	140°	50°	35 29.99	5.19	130°
10′	23 17.24	6.99	50′	10′	29 53.46	6.17	50′	10′	35 35.18	5.17	50′
20	23 24.23	6.97	40	20	29 59.63	6.16	40	20	35 40.35	5.15	40
30	23 31.20	6.97	30	30	30 5.79	6.14	30	30	35 45.50	5.13	30
40	23 38.17	6.95	20	40	30 11.93	6.13	20	40	35 50.63	5.12	20
50	23 45.12	6.94	10	50	30 13.06	6.11	10	50	35 55.75	5.10	10
31°	23 52.06	6.93	149°	41°	30 24.17	6.10	139°	51°	36 0.85	5.08	129°
10′	23 58.99	6.91	50′	10′	30 30.27	6.08	50′	10′	36 5.93	5.07	50′
20	24 5.90	6.90	40	20	30 36.35	6.06	40	20	36 11.00	5.05	40
30	24 12.80	6.89	30	30	30 42.41	6.05	30	30	36 16.05	5.02	30
40	24 19.69	6.88	20	40	30 48.46	6.03	20	40	36 21.07	5.01	20
50	24 26.57	6.86	10	50	30 54.49	6.02	10	50	36 26.08	4.99	10
32°	24 33.43	6.85	148°	42°	31 0.51	6.00	138°	52°	36 31.07	4.97	128°
10′	24 40.28	6.84	50′	10′	31 6.51	5.99	50′	10′	36 36.04	4.95	50′
20	24 47.12	6.83	40	20	31 12.50	5.97	40	20	36 40.99	4.94	40
30	24 53.95	6.82	30	30	31 18.47	5.96	30	30	36 45.93	4.91	30
40	25 0.77	6.80	20	40	31 24.43	5.94	20	40	36 50.84	4.89	20
50	25 7.57	6.79	10	50	31 30.37	5.92	10	50	36 55.73	4.88	10
33°	25 14.36	6.78	147°	43°	31 36.29	5.91	137°	53°	37 0.61	4.86	127°
10′	25 21.14	6.76	50′	10′	31 42.20	5.89	50′	10′	37 5.47	4.84	50′
20	25 27.90	6.75	40	20	31 48.09	5.87	40	20	37 10.31	4.82	40
30	25 34.65	6.74	30	30	31 53.96	5.86	30	30	37 15.13	4.80	30
40	25 41.39	6.72	20	40	31 59.82	5.84	20	40	37 19.93	4.79	20
50	25 48.11	6.71	10	50	32 5.66	5.83	10	50	37 24.72	4.76	10
34°	25 54.82	6.70	146°	44°	32 11.49	5.81	136°	54°	37 29.48	4.74	126°
10′	26 1.52	6.68	50′	10′	32 17.30	5.79	50′	10′	37 34.22	4.73	50′
20	26 8.20	6.68	40	20	32 23.09	5.78	40	20	37 38.95	4.71	40
30	26 14.88	6.66	30	30	32 28.87	5.76	30	30	37 43.66	4.68	30
40	26 21.54	6.65	20	40	32 34.63	5.75	20	40	37 48.34	4.67	20
50	26 28.19	6.63	10	50	32 40.38	5.73	10	50	37 53.01	4.65	10
35°	26 34.82	6.62	145°	45°	32 46.11	5.71	135°	55°	37 57.66	4.63	125°
10′	26 41.44	6.60	50′	10′	32 51.82	5.69	50′	10′	38 2.29	4.61	50′
20	26 48.04	6.59	40	20	32 57.51	5.68	40	20	38 6.90	4.59	40
30	26 54.63	6.58	30	30	33 3.19	5.66	30	30	38 11.49	4.58	30
40	27 1.21	6.56	20	40	33 8.85	5.64	20	40	38 16.07	4.55	20
50	27 7.77	6.55	10	50	33 14.49	5.63	10	50	38 20.62	4.53	10
36°	27 14.32	6.54	144°	46°	33 20.12	5.61	134°	56°	38 25.15	4.51	124°
10′	27 20.86	6.52	50′	10′	33 25.73	5.59	50′	10′	38 29.66	4.50	50′
20	27 27.38	6.51	40	20	33 31.32	5.58	40	20	38 34.16	4.47	40
30	27 33.89	6.50	30	30	33 36.90	5.56	30	30	38 38.63	4.45	30
40	27 40.39	6.48	20	40	33 42.46	5.54	20	40	38 43.08	4.44	20
50	27 46.87	6.47	10	50	33 48.00	5.53	10	50	38 47.52	4.41	10
37°	27 53.34	6.45	143°	47°	33 53.53	5.51	133°	57°	38 51.93	4.40	123°
10′	27 59.79	6.44	50′	10′	33 59.04	5.49	50′	10′	38 56.33	4.37	50′
20	28 6.23	6.42	40	20	34 4.53	5.47	40	20	39 0.70	4.36	40
30	28 12.65	6.41	30	30	34 10.00	5.46	30	30	39 5.06	4.34	30
40	28 19.06	6.40	20	40	34 15.46	5.44	20	40	39 9.40	4.31	20
50	28 25.46	6.38	10	50	34 20.90	5.42	10	50	39 13.71	4.30	10
38°	28 31.84	6.37	142°	48°	34 26.32	5.40	132°	58°	39 18.01	4.28	122°
10′	28 38.21	6.35	50′	10′	34 31.72	5.39	50′	10′	39 22.29	4.25	50′
20	28 44.56	6.33	40	20	34 37.11	5.37	40	20	39 26.54	4.24	40
30	28 50.89	6.33	30	30	34 42.48	5.35	30	30	39 30.78	4.21	30
40	28 57.22	6.31	20	40	34 47.83	5.33	20	40	39 34.99	4.20	20
50	29 3.53	6.29	10	50	34 53.16	5.31	10	50	39 39.19	4.17	10
39°	29 9.82	6.28	141°	49°	34 58.47	5.30	131°	59°	39 43.36	4.16	121°
10′	29 16.10	6.26	50′	10′	35 3.77	5.28	50′	10′	39 47.52	4.13	50′
20	29 22.36	6.25	40	20	35 9.05	5.26	40	20	39 51.65	4.12	40
30	29 28.61	6.24	30	30	35 14.31	5.24	30	30	39 55.77	4.10	30
40	29 34.85	6.22	20	40	35 19.55	5.23	20	40	39 59.87	4.07	20
50	29 41.07	6.20	10	50	35 24.78	5.21	10	50	40 3.94	4.06	10
40°	29 47.27		140°	50°	35 29.99		130°	60°	40 8.00		120°
220°	β		320° *u*	230°	β		310° *u*	240°	β		300° *u*

TABLE XIX, Arg. *u*.—*Continued.*

u	β			*u*	β			*u*	β		
240°	′ ″		300°	250°	′ ″		290°	260°	′ ″		280°
60°	40 8.00	4.03	120°	70°	43 32.84	2.76	110°	80°	45 38.29	1.39	100°
10′	40 12.03	4.01	50′	10′	43 35.60	2.73	50′	10′	45 39.68	1.37	50′
20	40 16.04	4.00	40	20	43 38.33	2.71	40	20	45 41.05	1.35	40
30	40 20.04	3.97	30	30	43 41.04	2.69	30	30	45 42.40	1.33	30
40	40 24.01	3.95	20	40	43 43.73	2.67	20	40	45 43.73	1.30	20
50	40 27.96	3.93	10	50	43 46.40	2.64	10	50	45 45.03	1.27	10
61°	40 31.89	3.91	119°	71°	43 49.04	2.62	109°	81°	45 46.30	1.26	99°
10′	40 35.80	3.89	50′	10′	43 51.66	2.60	50′	10′	45 47.56	1.23	50′
20	40 39.69	3.87	40	20	43 54.26	2.58	40	20	45 48.79	1.21	40
30	40 43.56	3.85	30	30	43 56.84	2.56	30	30	45 50.00	1.18	30
40	40 47.41	3.83	20	40	43 59.40	2.53	20	40	45 51.18	1.16	20
50	40 51.24	3.81	10	50	44 1.93	2.51	10	50	45 52.34	1.14	10
62°	40 55.05	3.79	118°	72°	44 4.44	2.49	108°	82°	45 53.48	1.11	98°
10′	40 58.84	3.76	50′	10′	44 6.93	2.46	50′	10′	45 54.59	1.09	50′
20	41 2.60	3.75	40	20	44 9.39	2.44	40	20	45 55.68	1.07	40
30	41 6.35	3.72	30	30	44 11.83	2.42	30	30	45 56.75	1.04	30
40	41 10.07	3.71	20	40	44 14.25	2.40	20	40	45 57.79	1.02	20
50	41 13.78	3.68	10	50	44 16.65	2.38	10	50	45 58.81	0.99	10
63°	41 17.46	3.67	117°	73°	44 19.03	2.35	107°	83°	45 59.80	0.98	97°
10′	41 21.13	3.64	50′	10′	44 21.38	2.33	50′	10′	46 0.78	0.95	50′
20	41 24.77	3.62	40	20	44 23.71	2.31	40	20	46 1.73	0.93	40
30	41 28.39	3.60	30	30	44 26.02	2.28	30	30	46 2.66	0.90	30
40	41 31.99	3.57	20	40	44 28.30	2.27	20	40	46 3.56	0.88	20
50	41 35.56	3.56	10	50	44 30.57	2.25	10	50	46 4.44	0.86	10
64°	41 39.12	3.53	116°	74°	44 32.82	2.22	106°	84°	46 5.30	0.84	96°
10′	41 42.65	3.52	50′	10′	44 35.04	2.19	50′	10′	46 6.14	0.81	50′
20	41 46.17	3.49	40	20	44 37.23	2.17	40	20	46 6.95	0.79	40
30	41 49.66	3.47	30	30	44 39.40	2.15	30	30	46 7.74	0.76	30
40	41 53.13	3.45	20	40	44 41.55	2.13	20	40	46 8.50	0.74	20
50	41 56.58	3.43	10	50	44 43.68	2.11	10	50	46 9.24	0.72	10
65°	42 0.01	3.41	115°	75°	44 45.79	2.08	105°	85°	46 9.96	0.69	95°
10′	42 3.42	3.38	50′	10′	44 47.87	2.06	50′	10′	46 10.65	0.67	50′
20	42 6.80	3.37	40	20	44 49.93	2.04	40	20	46 11.32	0.65	40
30	42 10.17	3.34	30	30	44 51.97	2.01	30	30	46 11.97	0.62	30
40	42 13.51	3.32	20	40	44 53.98	1.99	20	40	46 12.59	0.60	20
50	42 16.83	3.30	10	50	44 55.97	1.97	10	50	46 13.19	0.57	10
66°	42 20.13	3.28	114°	76°	44 57.94	1.95	104°	86°	46 13.76	0.55	94°
10′	42 23.41	3.26	50′	10′	44 59.89	1.92	50′	10′	46 14.31	0.53	50′
20	42 26.67	3.24	40	20	45 1.81	1.90	40	20	46 14.84	0.51	40
30	42 29.91	3.21	30	30	45 3.71	1.88	30	30	46 15.35	0.48	30
40	42 33.12	3.20	20	40	45 5.59	1.85	20	40	46 15.83	0.46	20
50	42 36.32	3.17	10	50	45 7.44	1.83	10	50	46 16.29	0.43	10
67°	42 39.49	3.15	113°	77°	45 9.27	1.81	103°	87°	46 16.72	0.41	93°
10′	42 42.64	3.13	50′	10′	45 11.08	1.78	50′	10′	46 17.13	0.39	50′
20	42 45.77	3.10	40	20	45 12.86	1.76	40	20	46 17.52	0.37	40
30	42 48.87	3.09	30	30	45 14.62	1.74	30	30	46 17.89	0.34	30
40	42 51.96	3.06	20	40	45 16.36	1.72	20	40	46 18.23	0.32	20
50	42 55.02	3.04	10	50	45 18.08	1.69	10	50	46 18.55	0.29	10
68°	42 58.06	3.02	112°	78°	45 19.77	1.67	102°	88°	46 18.84	0.27	92°
10′	43 1.08	3.00	50′	10′	45 21.44	1.65	50′	10′	46 19.11	0.25	50′
20	43 4.08	2.97	40	20	45 23.09	1.62	40	20	46 19.36	0.23	40
30	43 7.05	2.96	30	30	45 24.71	1.60	30	30	46 19.59	0.20	30
40	43 10.01	2.93	20	40	45 26.31	1.58	20	40	46 19.79	0.18	20
50	43 12.94	2.91	10	50	45 27.89	1.56	10	50	46 19.97	0.15	10
69°	43 15.85	2.89	111°	79°	45 29.45	1.53	101°	89°	46 20.12	0.13	91°
10′	43 18.74	2.86	50′	10′	45 30.98	1.51	50′	10′	46 20.25	0.11	50′
20	43 21.60	2.85	40	20	45 32.49	1.48	40	20	46 20.36	0.08	40
30	43 24.45	2.82	30	30	45 33.97	1.46	30	30	46 20.44	0.06	30
40	43 27.27	2.79	20	40	45 35.43	1.44	20	40	46 20.50	0.03	20
50	43 30.06	2.78	10	50	45 36.87	1.42	10	50	46 20.53	0.01	10
70°	43 32.84		110°	80°	45 38.29		100°	90°	46 20.54		90°
250°	β		290° *u*	260°	β		280° *u*	270°	β		270° *u*

Table	XX.		XXI.					XXII.				
Arg.	1		2					3				
	(b.s.1)	(b.c.1)	(b.c.0)	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)	(b.c.0)	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)
°	″	″	″	″	″	″	″	″	″	″	″	″
0	1.20	1.12	0.04	5.99	5.42	0.21	0.17	0.06	1.58	1.34	0.07	0.17
10	1.15	1.17	0.03	6.13	5.23	0.19	0.18	0.06	1.63	1.14	0.08	0.23
20	1.10	1.21	0.03	6.25	5.04	0.18	0.18	0.05	1.67	0.93	0.11	0.29
30	1.04	1.25	0.02	6.34	4.84	0.17	0.18	0.04	1.68	0.72	0.15	0.32
40	0.98	1.28	0.02	6.41	4.62	0.16	0.18	0.03	1.67	0.51	0.21	0.33
50	0.92	1.31	0.02	6.45	4.40	0.15	0.18	0.03	1.60	0.33	0.27	0.32
60	0.86	1.33	0.02	6.45	4.18	0.15	0.18	0.02	1.48	0.18	0.32	0.28
70	0.79	1.34	0.02	6.40	3.94	0.16	0.18	0.01	1.32	0.09	0.35	0.23
80	0.72	1.35	0.02	6.32	3.69	0.16	0.17	0.00	1.13	0.07	0.36	0.18
90	0.66	1.35	0.03	6.19	3.43	0.17	0.16	0.00	0.94	0.13	0.35	0.13
100	0.59	1.34	0.03	6.02	3.15	0.17	0.15	0.01	0.77	0.25	0.32	0.09
110	0.52	1.32	0.04	5.81	2.87	0.18	0.14	0.02	0.66	0.41	0.29	0.07
120	0.46	1.30	0.04	5.56	2.57	0.19	0.12	0.04	0.62	0.60	0.25	0.06
130	0.40	1.27	0.04	5.29	2.26	0.20	0.11	0.06	0.64	0.77	0.22	0.06
140	0.34	1.24	0.05	4.99	1.96	0.20	0.10	0.08	0.72	0.91	0.20	0.07
150	0.28	1.20	0.05	4.68	1.66	0.20	0.09	0.09	0.83	1.00	0.18	0.08
160	0.23	1.15	0.05	4.36	1.38	0.20	0.08	0.10	0.95	1.04	0.17	0.09
170	0.19	1.10	0.06	4.03	1.11	0.20	0.08	0.11	1.04	1.04	0.17	0.10
180	0.15	1.04	0.06	3.69	0.86	0.19	0.07	0.11	1.11	1.01	0.16	0.10
190	0.12	0.98	0.06	3.36	0.65	0.17	0.07	0.11	1.14	0.97	0.14	0.11
200	0.09	0.92	0.06	3.03	0.56	0.16	0.08	0.11	1.14	0.93	0.13	0.12
210	0.07	0.86	0.06	2.71	0.32	0.14	0.08	0.11	1.11	0.91	0.11	0.13
220	0.06	0.79	0.06	2.39	0.20	0.12	0.09	0.10	1.07	0.91	0.10	0.15
230	0.05	0.72	0.06	2.08	0.12	0.11	0.11	0.10	1.02	0.92	0.10	0.18
240	0.05	0.66	0.07	1.77	0.08	0.09	0.13	0.09	0.98	0.96	0.10	0.21
250	0.06	0.59	0.07	1.48	0.08	0.07	0.15	0.09	0.96	1.00	0.12	0.23
260	0.08	0.52	0.07	1.21	0.10	0.05	0.17	0.08	0.95	1.05	0.14	0.25
270	0.10	0.46	0.08	1.05	0.16	0.03	0.20	0.08	0.94	1.09	0.16	0.26
280	0.13	0.40	0.08	0.72	0.26	0.02	0.22	0.07	0.95	1.13	0.19	0.26
290	0.16	0.34	0.09	0.52	0.38	0.01	0.25	0.06	0.95	1.16	0.22	0.25
300	0.20	0.28	0.10	0.35	0.54	0.01	0.27	0.06	0.94	1.18	0.23	0.23
310	0.25	0.23	0.11	0.22	0.73	0.01	0.29	0.06	0.93	1.20	0.24	0.21
320	0.30	0.19	0.12	0.12	0.94	0.02	0.31	0.07	0.91	1.22	0.23	0.19
330	0.36	0.15	0.13	0.06	1.18	0.03	0.33	0.08	0.88	1.25	0.22	0.18
340	0.42	0.12	0.14	0.04	1.44	0.05	0.35	0.09	0.86	1.28	0.20	0.17
350	0.48	0.09	0.15	0.06	1.72	0.07	0.36	0.09	0.84	1.34	0.18	0.17
360	0.54	0.07	0.16	0.12	2.01	0.10	0.37	0.10	0.85	1.40	0.16	0.18
370	0.61	0.06	0.17	0.21	2.31	0.13	0.37	0.11	0.88	1.46	0.15	0.19
380	0.68	0.05	0.18	0.34	2.62	0.16	0.37	0.12	0.92	1.51	0.14	0.20
390	0.74	0.05	0.19	0.50	2.92	0.19	0.36	0.12	0.98	1.54	0.13	0.21
400	0.81	0.06	0.20	0.70	3.24	0.23	0.35	0.11	1.03	1.54	0.13	0.22
410	0.88	0.08	0.20	0.92	3.54	0.26	0.34	0.10	1.07	1.50	0.12	0.23
420	0.94	0.10	0.21	1.17	3.84	0.30	0.33	0.08	1.06	1.44	0.12	0.25
430	1.00	0.13	0.21	1.45	4.14	0.33	0.31	0.06	1.01	1.37	0.12	0.27
440	1.06	0.16	0.21	1.75	4.42	0.36	0.29	0.04	0.91	1.30	0.12	0.29
450	1.12	0.20	0.21	2.06	4.70	0.38	0.27	0.03	0.77	1.28	0.12	0.32
460	1.17	0.25	0.21	2 39	4.96	0.40	0.25	0.02	0.61	1.31	0.14	0.35
470	1.21	0.30	0.20	2.72	5.20	0.41	0.23	0.01	0.47	1.40	0.17	0.38
480	1.25	0.36	0.19	3.06	5.42	0.42	0.21	0.01	0.36	1.56	0.21	0.39
490	1.28	0.42	0.18	3.39	5.62	0.42	0.20	0.01	0.32	1.76	0.27	0.39
500	1.31	0.48	0.17	3.71	5.79	0.42	0.18	0.01	0.36	1.96	0.32	0.37
510	1.33	0.54	0.16	4.02	5.92	0.41	0.17	0.01	0.46	2.15	0.37	0.33
520	1.34	0.61	0.14	4.31	6.01	0.39	0.16	0.02	0.62	2.21	0.40	0.27
530	1.35	0.68	0.13	4.58	6.06	0.38	0.16	0.02	0.81	2.35	0.40	5.20
540	1.35	0.74	0.11	4.84	6.08	0.35	0.15	0.03	1.00	2.34	0.38	0.14
550	1.34	0.81	0.10	5.07	6.04	0.33	0.15	0.03	1.18	2.26	0.33	0.08
560	1.32	0.88	0.09	5.28	5.98	0.30	0.15	0.04	1.31	2.11	0.27	0.05
570	1.30	0.94	0.07	5.48	5.88	0.28	0.16	0.04	1.41	1.94	0.20	0.04
580	1.27	1.00	0.06	5.67	5.75	0.26	0.16	0.05	1.48	1.74	0.14	0.07
590	1.24	1.06	0.05	5.84	5.59	0.23	0.17	0.06	1.53	1.54	0.10	0.11
600	1.20	1.12	0.04	5.99	5.42	0.21	0.17	0.06	1.58	1.34	0.07	0.17

TABLE XXIII.

Year.	(b.c.0)	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)	Year.	(b.c.0)	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)
	"	"	"	"	"		"	"	"	"	"
1300	+0.66	—3.85	—12.74	—0.34	—0.97	1800	+0.15	—4.60	—6.00	—0.44	—0.46
1310	0.65	3.81	12.65	0.34	0.96	1810	0.14	4.68	5.80	0.45	0.45
1320	0.64	3.78	12.56	0.34	0.95	1820	0.13	4.75	5.61	0.45	0.44
1330	0.62	3.74	12.46	0.34	0.94	1830	0.12	4.83	5.41	0.45	0.43
1340	0.61	3.70	12.37	0.34	0.93	1840	0.11	4.92	5.21	0.46	0.42
1350	+0.60	—3.66	—12.27	—0.34	—0.92	1850	+0.10	—5.00	—5.00	—0.46	—0.41
1360	0.59	3.63	12.17	0.34	0.90	1860	0.09	5.09	4.79	0.47	0.40
1370	0.58	3.60	12.07	0.34	0.89	1870	0.08	5.18	4.58	0.47	0.39
1380	0.57	3.57	11.97	0.34	0.88	1880	0.07	5.27	4.36	0.48	0.38
1390	0.56	3.54	11.86	0.34	0.87	1890	0.06	5.36	4.14	0.48	0.37
1400	+0.55	—3.51	—11.76	—0.35	—0.86	1900	+0.05	—5.45	—3.92	—0.49	—0.36
1410	0.54	3.48	11.65	0.35	0.85	1910	0.04	5.55	3.69	0.49	0.35
1420	0.53	3.46	11.54	0.35	0.84	1920	0.03	5.64	3.46	0.50	0.34
1430	0.52	3.44	11.43	0.35	0.83	1930	0.02	5.74	3.22	0.50	0.33
1440	0.51	3.43	11.32	0.35	0.82	1940	+0.01	5.84	2.97	0.50	0.32
1450	+0.50	—3.42	—11.21	—0.35	—0.80	1950	0.00	—5.94	—2.73	—0.51	—0.30
1460	0.49	3.41	11.10	0.35	0.79	1960	—0.01	6.04	2.48	0.51	0.29
1470	0.48	3.40	10.98	0.36	0.78	1970	0.02	6.14	2.24	0.52	0.28
1480	0.46	3.40	10.86	0.36	0.77	1980	0.04	6.25	1.99	0.52	0.27
1490	0.45	3.40	10.74	0.36	0.76	1990	0.05	6.36	1.73	0.52	0.26
1500	+0.44	—3.40	—10.62	—0.36	—0.75	2000	—0.06	—6.47	—1.47	—0.53	—0.25
1510	0.43	3.40	10.50	0.36	0.74	2010	0.07	6.58	1.21	0.53	0.24
1520	0.42	3.41	10.38	0.36	0.73	2020	0.08	6.69	0.94	0.54	0.23
1530	0.41	3.42	10.26	0.37	0.72	2030	0.09	6.80	0.66	0.54	0.22
1540	0.40	3.43	10.14	0.37	0.71	2040	0.10	6.92	0.38	0.54	0.20
1550	+0.39	—3.45	—10.01	—0.37	—0.70	2050	—0.11	—7.03	—0.10	—0.55	—0.19
1560	0.38	3.47	9.88	0.37	0.70	2060	0.12	7.14	+0.18	0.55	0.18
1570	0.37	3.49	9.75	0.37	0.69	2070	0.13	7.26	0.47	0.56	0.17
1580	0.36	3.51	9.61	0.37	0.68	2080	0.15	7.37	0.76	0.56	0.16
1590	0.35	3.53	9.47	0.38	0.67	2090	0.16	7.48	1.05	0.56	0.15
1600	+0.34	—3.56	— 9.33	—0.38	—0.66	2100	—0.17	—7.60	+1.35	—0.57	—0.14
1610	0.33	3.59	9.19	0.38	0.65	2110	0.18	7.72	1.65	0.57	0.13
1620	0.32	3.62	9.05	0.39	0.64	2120	0.19	7.84	1.96	0.57	0.12
1630	0.32	3.65	8.90	0.39	0.63	2130	0.21	7.97	2.27	0.58	0.10
1640	0.31	3.69	8.75	0.39	0.62	2140	0.22	8.09	2.58	0.58	0.09
1650	+0.30	—3.73	— 8.60	—0.39	—0.61	2150	—0.23	—8.21	+2.90	—0.58	—0.08
1660	0.29	3.77	8.44	0.40	0.60	2160	0.24	8.33	3.22	0.59	0.07
1670	0.28	3.81	8.29	0.40	0.59	2170	0.25	8.44	3.54	0.59	0.06
1680	0.27	3.85	8.13	0.40	0.58	2180	0.27	8.56	3.87	0.59	0.05
1690	0.26	3.90	7.96	0.41	0.57	2190	0.28	8.68	4.20	0.60	0.03
1700	+0.25	—3.95	— 7.79	—0.41	—0.56	2200	—0.29	—8.80	+4.54	—0.60	—0.02
1710	0.24	4.00	7.63	0.41	0.55	2210	0.30	8.92	4.88	0.60	—0.01
1720	0.23	4.06	7.46	0.42	0.54	2220	0.31	9.04	5.23	0.61	0.00
1730	0.22	4.12	7.29	0.42	0.53	2230	0.33	9.16	5.58	0.61	+0.01
1740	0.21	4.18	7.12	0.42	0.52	2240	0.34	9.28	5.94	0.61	0.03
1750	+0.20	—4.25	— 6.95	—0.43	—0.51	2250	—0.35	—9.40	+6.29	—0.62	+0.04
1760	0.19	4.32	6.77	0.43	0.50	2260	0.36	9.52	6.65	0.62	0.05
1770	0.18	4.38	6.58	0.43	0.49	2270	0.38	9.64	7.01	0.63	0.07
1780	0.17	4.45	6.39	0.44	0.48	2280	0.39	9.76	7.37	0.63	0.08
1790	0.16	4.53	6.20	0.44	0.47	2290	0.40	9.88	7.74	0.63	0.10
1800	+0.15	—4.60	— 6.00	—0.44	—0.46	2300	—0.42	—10.00	+8.10	—0.64	+0.11

TABLE FOR FORMING THE PRODUCTS OF GIVEN NUMBERS BY THE SINE OR COSINE OF A GIVEN ANGLE.

THIS table is formed for the especial purpose of facilitating the formation of the products $(v.s.3) \sin 3g$, $(v.c.3) \cos 3g$, etc., $(\rho.s.1) \sin g$, $(\rho.c.1) \cos g$, for entire degrees of g. It is so arranged that the required products can be taken out at sight. Supposing the number to be given in seconds and decimal fractions of a second, we first seek the given angle at the top or bottom of the page, and then enter one of the first nine lines of the table with the fraction part of the second, interpolating for the hundredths. We then add the result mentally to the number corresponding to the entire seconds. The algebraic signs at the sides of the angles are those of the sines or cosines corresponding to the angle and to the column above or below. If the number does not exceed 3″ we can enter the table as if it were ten times greater, and remove the decimal point one place to the left in the result.

For example, to find the value of

$$21''.67 \sin 280^\circ + 2''.25 \cos 280^\circ$$

we find the angle 280° at the bottom of a pair of columns, the right hand one being the sine column. Entering this column with 0.67 as the argument, we find 0.66. Entering with 2.1, we find 20.68, to which adding 0.66, we have 21″.34 as the sine product. Entering the other column with 22.5, and moving the decimal point, we find 0″.39 for the cosine product. Noticing the algebraic signs on each side of 280°, we find the result to be $-21''.34 + 0''.39 = -20''.95$.

	+ 1°+ +179 — —181 — —359 +		+ 2°+ +178 — —182 — —358 +		+ 3°+ +177 — —183 — —357 +		+ 4°+ +176 — —184 — —356 +		+ 5°+ +175 — —185 — —355 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.00	0.10	0.00	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.1
0.2	0.00	0.20	0.01	0.20	0.01	0.20	0.01	0.20	0.02	0.20	0.2
0.3	0.01	0.30	0.01	0.30	0.02	0.30	0.02	0.30	0.03	0.30	0.3
0.4	0.01	0.40	0.01	0.40	0.02	0.40	0.03	0.40	0.03	0.40	0.4
0.5	0.01	0.50	0.02	0.50	0.03	0.50	0.03	0.50	0.04	0.50	0.5
0.6	0.01	0.60	0.02	0.60	0.03	0.60	0.04	0.60	0.05	0.60	0.6
0.7	0.01	0.70	0.02	0.70	0.04	0.70	0.05	0.70	0.06	0.70	0.7
0.8	0.01	0.80	0.03	0.80	0.04	0.80	0.06	0.80	0.07	0.80	0.8
0.9	0.02	0.90	0.03	0.90	0.05	0.90	0.06	0.90	0.08	0.90	0.9
1.0	0.02	1.00	0.03	1.00	0.05	1.00	0.07	1.00	0.09	1.00	1.0
2.0	0.03	2.00	0.07	2.00	0.10	2.00	0.14	2.00	0.17	1.99	2.0
3.0	0.05	3.00	0.10	3.00	0.16	3.00	0.21	2.99	0.26	2.99	3.0
4.0	0.07	4.00	0.14	4.00	0.21	3.99	0.28	3.99	0.35	3.98	4.0
5.0	0.09	5.00	0.17	5.00	0.26	4.99	0.35	4.99	0.44	4.98	5.0
6.0	0.10	6.00	0.21	6.00	0.31	5.99	0.42	5.99	0.52	5.98	6.0
7.0	0.12	7.00	0.24	7.00	0.37	6.99	0.49	6.98	0.61	6.97	7.0
8.0	0.14	8.00	0.28	8.00	0.42	7.99	0.56	7.98	0.70	7.97	8.0
9.0	0.16	9.00	0.31	8.99	0.47	8.99	0.63	8.98	0.78	8.97	9.0
10.0	0.17	10.00	0.35	9.99	0.52	9.99	0.70	9.98	0.87	9.96	10.0
11.0	0.19	11.00	0.38	10.99	0.58	10.98	0.77	10.97	0.96	10.96	11.0
12.0	0.21	12.00	0.42	11.99	0.63	11.98	0.84	11.97	1.05	11.95	12.0
13.0	0.23	13.00	0.45	12.99	0.68	12.98	0.91	12.97	1.13	12.95	13.0
14.0	0.24	14.00	0.49	13.99	0.73	13.98	0.98	13.97	1.22	13.95	14.0
15.0	0.26	15.00	0.52	14.99	0.79	14.98	1.05	14.96	1.31	14.94	15.0
16.0	0.28	16.00	0.56	15.99	0.84	15.98	1.12	15.96	1.39	15.94	16.0
17.0	0.30	17.00	0.59	16.99	0.89	16.98	1.19	16.96	1.48	16.94	17.0
18.0	0.31	18.00	0.63	17.99	0.94	17.98	1.26	17.96	1.57	17.93	18.0
19.0	0.33	19.00	0.66	18.99	0.99	18.97	1.33	18.95	1.66	18.93	19.0
20.0	0.35	20.00	0.70	19.99	1.05	19.97	1.40	19.95	1.74	19.92	20.0
21.0	0.37	21.00	0.73	20.99	1.10	20.97	1.46	20.95	1.83	20.92	21.0
22.0	0.38	22.00	0.77	21.99	1.15	21.97	1.53	21.95	1.92	21.92	22.0
23.0	0.40	23.00	0.80	22.99	1.20	22.97	1.60	22.94	2.00	22.91	23.0
24.0	0.42	24.00	0.84	23.99	1.26	23.97	1.67	23.94	2.09	23.91	24.0
25.0	0.44	25.00	0.87	24.98	1.31	24.97	1.74	24.94	2.18	24.90	25.0
26.0	0.45	26.00	0.91	25.98	1.36	25.96	1.81	25.94	2.27	25.90	26.0
27.0	0.47	27.00	0.94	26.98	1.41	26.96	1.88	26.93	2.35	26.90	27.0
28.0	0.49	28.00	0.98	27.98	1.47	27.96	1.95	27.93	2.44	27.89	28.0
29.0	0.51	29.00	1.01	28.98	1.52	28.96	2.02	28.93	2.53	28.89	29.0
30.0	0.52	30.00	1.05	29.98	1.57	29.96	2.09	29.93	2.61	29.89	20.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+271°— —269 — — 91 + + 89 +		+272 — —268 — — 92 + + 88 +		+273 — —267 — — 93 + + 87 +		+274 — —266 — — 94 + + 86 +		+275 — —265 — — 95 + + 85 +		

	+ 6° + + 174 — — 186 — — 354 +		+ 7° + + 173 — — 187 — — 353 +		+ 8° + + 172 — — 188 — — 352 +		+ 9° + + 171 — — 189 — — 351 +		+ 10° + + 170 — — 190 — — 350 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.01	0.10	0.01	0.10	0.01	0.10	0.02	0.10	0.02	0.10	0.1
0.2	0.02	0.20	0.02	0.20	0.03	0.20	0.03	0.20	0.03	0.20	0.2
0.3	0.03	0.30	0.04	0.30	0.04	0.30	0.05	0.30	0.05	0.30	0.3
0.4	0.04	0.40	0.05	0.40	0.06	0.40	0.06	0.40	0.07	0.39	0.4
0.5	0.05	0.50	0.06	0.50	0.07	0.50	0.08	0.49	0.09	0.49	0.5
0.6	0.06	0.60	0.07	0.60	0.08	0.59	0.09	0.59	0.10	0.59	0.6
0.7	0.07	0.70	0.09	0.69	0.10	0.69	0.11	0.69	0.12	0.69	0.7
0.8	0.08	0.80	0.10	0.79	0.11	0.79	0.13	0.79	0.14	0.79	0.8
0.9	0.09	0.90	0.11	0.89	0.13	0.89	0.14	0.89	0.16	0.89	0.9
1.0	0.10	0.99	0.12	0.99	0.14	0.99	0.16	0.99	0.17	0.98	1.0
2.0	0.21	1.99	0.24	1.99	0.28	1.98	0.31	1.98	0.35	1.97	2.0
3.0	0.31	2.98	0.37	2.98	0.42	2.97	0.47	2.96	0.52	2.95	3.0
4.0	0.42	3.98	0.49	3.97	0.56	3.96	0.63	3.95	0.69	3.94	4.0
5.0	0.52	4.97	0.61	4.96	0.70	4.95	0.78	4.94	0.87	4.92	5.0
6.0	0.63	5.97	0.73	5.96	0.84	5.94	0.94	5.93	1.04	5.91	6.0
7.0	0.73	6.96	0.85	6.95	0.97	6.93	1.10	6.91	1.22	6.89	7.0
8.0	0.84	7.96	0.97	7.94	1.11	7.92	1.25	7.90	1.39	7.88	8.0
9.0	0.94	8.95	1.10	8.93	1.25	8.91	1.41	8.89	1.56	8.86	9.0
10.0	1.05	9.95	1.22	9.93	1.39	9.90	1.56	9.88	1.74	9.85	10.0
11.0	1.15	10.94	1.34	10.92	1.53	10.89	1.72	10.86	1.91	10.83	11.0
12.0	1.25	11.93	1.46	11.91	1.67	11.88	1.88	11.85	2.08	11.82	12.0
13.0	1.36	12.93	1.58	12.90	1.81	12.87	2.03	12.84	2.26	12.80	13.0
14.0	1.46	13.92	1.71	13.90	1.95	13.86	2.19	13.83	2.43	13.79	14.0
15.0	1.57	14.92	1.83	14.89	2.09	14.85	2.35	14.82	2.60	14.77	15.0
16.0	1.67	15.91	1.95	15.88	2.23	15.84	2.50	15.80	2.78	15.76	16.0
17.0	1.78	16.91	2.07	16.87	2.37	16.83	2.66	16.79	2.95	16.74	17.0
18.0	1.88	17.90	2.19	17.87	2.51	17.82	2.82	17.78	3.13	17.73	18.0
19.0	1.99	18.90	2.32	18.86	2.64	18.82	2.97	18.77	3.30	18.71	19.0
20.0	2.09	19.89	2.44	19.85	2.78	19.81	3.13	19.75	3.47	19.70	20.0
21.0	2.20	20.88	2.56	20.84	2.92	20.80	3.29	20.74	3.65	20.68	21.0
22.0	2.30	21.88	2.68	21.84	3.06	21.79	3.44	21.73	3.82	21.67	22.0
23.0	2.40	22.87	2.80	22.83	3.20	22.78	3.60	22.72	3.99	22.65	23.0
24.0	2.51	23.87	2.92	23.82	3.34	23.77	3.75	23.70	4.17	23.64	24.0
25.0	2.61	24.86	3.05	24.81	3.48	24.76	3.91	24.69	4.34	24.62	25.0
26.0	2.72	25.86	3.17	25.81	3.62	25.75	4.07	25.68	4.51	25.61	26.0
27.0	2.82	26.85	3.29	26.80	3.76	26.74	4.22	26.67	4.69	26.59	27.0
28.0	2.93	27.85	3.41	27.79	3.90	27.73	4.38	27.66	4.86	27.57	28.0
29.0	3.03	28.84	3.53	28.78	4.04	28.72	4.54	28.64	5.04	28.56	29.0
30.0	3.14	29.84	3.66	29.78	4.18	29.71	4.69	29.63	5.21	29.54	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 276 — — 264 — — 96 + + 84 +		+ 277 — — 263 — — 97 + + 83 +		+ 278 — — 262 — — 93 + + 82 +		+ 279 — — 261 — — 99 + + 81 +		+ 280 — — 260 — — 100 + + 80 +		

	+ 11°+ + 169 — — 191 — — 349 +		+ 12°+ + 168 — — 192 — — 348 +		+ 13°+ + 167 — — 193 — — 347 +		+ 14°+ + 166 — — 194 — — 346 +		+ 15°+ + 165 — — 195 — — 345 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.03	0.10	0.1
0.2	0.04	0.20	0.04	0.20	0.04	0.19	0.05	0.19	0.05	0.19	0.2
0.3	0.06	0.29	0.06	0.29	0.07	0.29	0.07	0.29	0.08	0.29	0.3
0.4	0.08	0.39	0.08	0.39	0.09	0.39	0.10	0.39	0.10	0.39	0.4
0.5	0.10	0.49	0.10	0.49	0.11	0.49	0.12	0.49	0.13	0.48	0.5
0.6	0.11	0.59	0.12	0.59	0.13	0.58	0.15	0.58	0.16	0.58	0.6
0.7	0.13	0.69	0.15	0.68	0.16	0.68	0.17	0.68	0.18	0.68	0.7
0.8	0.15	0.79	0.17	0.78	0.18	0.78	0.19	0.78	0.21	0.77	0.8
0.9	0.17	0.88	0.19	0.88	0.20	0.88	0.22	0.87	0.23	0.87	0.9
1.0	0.19	0.98	0.21	0.98	0.22	0.97	0.24	0.97	0.26	0.97	1.0
2.0	0.38	1.96	0.42	1.96	0.45	1.95	0.48	1.94	0.52	1.93	2.0
3.0	0.57	2.94	0.62	2.93	0.67	2.92	0.73	2.91	0.78	2.90	3.0
4.0	0.76	3.93	0.83	3.91	0.90	3.90	0.97	3.88	1.04	3.86	4.0
5.0	0.95	4.91	1.04	4.89	1.12	4.87	1.21	4.85	1.29	4.83	5.0
6.0	1.14	5.89	1.25	5.87	1.35	5.85	1.45	5.82	1.55	5.80	6.0
7.0	1.34	6.87	1.46	6.85	1.57	6.82	1.69	6.79	1.81	6.76	7.0
8.0	1.53	7.85	1.66	7.83	1.80	7.79	1.94	7.76	2.07	7.73	8.0
9.0	1.72	8.83	1.87	8.80	2.02	8.77	2.18	8.73	2.33	8.69	9.0
10.0	1.91	9.82	2.08	9.78	2.25	9.74	2.42	9.70	2.59	9.66	10.0
11.0	2.10	10.80	2.29	10.76	2.47	10.72	2.66	10.67	2.85	10.63	11.0
12.0	2.29	11.78	2.49	11.74	2.70	11.69	2.90	11.64	3.11	11.59	12.0
13.0	2.48	12.76	2.70	12.72	2.92	12.67	3.14	12.61	3.36	12.56	13.0
14.0	2.67	13.74	2.91	13.69	3.15	13.64	3.39	13.58	3.62	13.52	14.0
15.0	2.86	14.72	3.12	14.67	3.37	14.62	3.63	14.55	3.88	14.49	15.0
16.0	3.05	15.71	3.33	15.65	3.60	15.59	3.87	15.52	4.14	15.45	16.0
17.0	3.24	16.69	3.53	16.63	3.82	16.56	4.11	16.50	4.40	16.42	17.0
18.0	3.43	17.67	3.74	17.61	4.05	17.54	4.35	17.47	4.66	17.39	18.0
19.0	3.63	18.65	3.95	18.58	4.27	18.51	4.60	18.44	4.92	18.35	19.0
20.0	3.82	19.63	4.16	19.56	4.50	19.49	4.84	19.41	5.18	19.32	20.0
21.0	4.01	20.61	4.37	20.54	4.72	20.46	5.08	20.38	5.44	20.28	21.0
22.0	4.20	21.60	4.57	21.52	4.95	21.44	5.32	21.35	5.69	21.25	22.0
23.0	4.39	22.58	4.78	22.50	5.17	22.41	5.56	22.32	5.95	22.22	23.0
24.0	4.58	23.56	4.99	23.48	5.40	23.38	5.81	23.29	6.21	23.18	24.0
25.0	4.77	24.54	5.20	24.45	5.62	24.36	6.05	24.26	6.47	24.15	25.0
26.0	4.96	25.52	5.41	25.43	5.85	25.33	6.29	25.23	6.73	25.11	26.0
27.0	5.15	26.50	5.61	26.41	6.07	26.31	6.53	26.20	6.99	26.08	27.0
28.0	5.34	27.49	5.82	27.39	6.30	27.28	6.77	27.17	7.25	27.05	28.0
29.0	5.53	28.47	6.03	28.37	6.52	28.26	7.02	28.14	7.51	28.01	29.0
30.0	5.72	29.45	6.24	29.34	6.75	29.23	7.26	29.11	7.76	28.98	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 281 — — 259 — — 101 + + 79 +		+ 282 — — 258 — — 102 + + 78 +		+ 283 — — 257 — — 103 + + 77 +		+ 284 — — 256 — — 104 + + 76 +		+ 285 — — 255 — — 105 + + 75 +		

	+ 16° + + 164 − − 196 − − 344 +		+ 17° + + 163 − − 197 − − 343 +		+ 18° + + 162 − − 198 − − 342 +		+ 19° + + 161 − − 199 − − 341 +		+ 20° + + 160 − − 200 − − 340 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.03	0.10	0.03	0.10	0.03	0.10	0.03	0.09	0.03	0.09	0.1
0.2	0.06	0.19	0.06	0.19	0.06	0 19	0.07	0.19	0.07	0.19	0.2
0.3	0.08	0.29	0.09	0.29	0.09	0.29	0.10	0.28	0.10	0.28	0.3
0.4	0.11	0.38	0.12	0.38	0.12	0.38	0.13	0.38	0.14	0.38	0.4
0.5	0.14	0.48	0.15	0.48	0.15	0.48	0.16	0.47	0.17	0.47	0.5
0.6	0.17	0.58	0.18	0.57	0.19	0.57	0.20	0.57	0.21	0.56	0.6
0.7	0.19	0.67	0.20	0.67	0.22	0.67	0.23	0.66	0.24	0.66	0.7
0.8	0.22	0.77	0.23	0.77	0.25	0.76	0.26	0.76	0.27	0.75	0.8
0.9	0.25	0.87	0.26	0.86	0.28	0.86	0.29	0.85	0.31	0.85	0.9
1.0	0.28	0.96	0.29	0.96	0.31	0.95	0.33	0.95	0.34	0.94	1.0
2.0	0.55	1.92	0.58	1.91	0.62	1.90	0.65	1.89	0.68	1.88	2.0
3.0	0.83	2.88	0.88	2.87	0.93	2.85	0.98	2.84	1.03	2.82	3.0
4.0	1.10	3.85	1.17	3.83	1.24	3.80	1.30	3.78	1.37	3.76	4.0
5.0	1.38	4.81	1.46	4.78	1.55	4.76	1.63	4.73	1.71	4.70	5.0
6.0	1.65	5.77	1.75	5.74	1.85	5.71	1.95	5.67	2.05	5.64	6.0
7.0	1.93	6.73	2.05	6.69	2.16	6.66	2.28	6.62	2.39	6.58	7.0
8.0	2.21	7.69	2.34	7.65	2.47	7.61	2.60	7.56	2.74	7.52	8.0
9.0	2.48	8.65	2.63	8.61	2.78	8.56	2.93	8.51	3.08	8.46	9.0
10.0	2.76	9.61	2.92	9.56	3.09	9.51	3.26	9.46	3.42	9.40	10.0
11.0	3.03	10.57	3.22	10.52	3.40	10.46	3.58	10.40	3.76	10.34	11.0
12.0	3.31	11.54	3.51	11.48	3.71	11.41	3.91	11.35	4.10	11.28	12.0
13.0	3.58	12.50	3.80	12.43	4.02	12.36	4.23	12.29	4.45	12.22	13.0
14.0	3.86	13.46	4.09	13.39	4.33	13.31	4.56	13.24	4.79	13.16	14.0
15.0	4.13	14.42	4.39	14.34	4.64	14.27	4.88	14.18	5.13	14.10	15.0
16.0	4.41	15.38	4.68	15.30	4.94	15.22	5.21	15.13	5.47	15.04	16.0
17.0	4.69	16.34	4.97	16.26	5.25	16.17	5.53	16.07	5.81	15.97	17.0
18.0	4.96	17.30	5.26	17.21	5.56	17.12	5.86	17.02	6.16	16.91	18.0
19.0	5.24	18.26	5.56	18.17	5.87	18.07	6.19	17.96	6.50	17.85	19.0
20.0	5.51	19.23	5.85	19.13	6.18	19.02	6.51	18.91	6.84	18.79	20.0
21.0	5.79	20.19	6.14	20.08	6.49	19.97	6.84	19.86	7.18	19.73	21.0
22.0	6.06	21.15	6.43	21.04	6.80	20.92	7.16	20.80	7.52	20.67	22.0
23.0	6.34	22.11	6.72	22.00	7.11	21.87	7.49	21.75	7.87	21.61	23.0
24.0	6.62	23.07	7.02	22.95	7.42	22.83	7.81	22.69	8.21	22.55	24.0
25.0	6.89	24.03	7.31	23.91	7.73	23.78	8.14	23.64	8.55	23.49	25 0
26.0	7.17	24.99	7.60	24.86	8.03	24.73	8.46	24.58	8.89	24.43	26.0
27.0	7.44	25.95	7.89	25.82	8.34	25.68	8.79	25.53	9.23	25.37	27.0
28.0	7.72	26.92	8.19	26.78	8.65	26.63	9.12	26.47	9.58	26.31	28.0
29.0	7.99	27.88	8.48	27.73	8.96	27.58	9.44	27.42	9.92	27.25	29.0
30.0	8.27	28.84	8.77	28.69	9.27	28.53	9.77	28.37	10.26	28.19	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 286 − − 254 − − 106 + + 74 +		+ 287 − − 253 − − 107 + + 73 +		+ 288 − − 252 − − 108 + + 72 +		+ 289 − − 251 − − 109 + + 71 +		+ 290 − − 250 − − 110 + + 70 +		

	+ 21° + + 159 — — 201 — — 339 +		+ 22° + + 158 — — 202 — — 338 +		+ 23° + + 157 — — 203 — — 337 +		+ 24° + + 156 — — 204 — — 336 +		+ 25° + + 155 — — 205 — — 335 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.04	0.09	0.1
0.2	0.07	0.19	0.07	0.19	0.08	0.18	0.08	0.18	0.08	0.18	0.2
0.3	0.11	0.28	0.11	0.28	0.12	0.28	0.12	0.27	0.13	0.27	0.3
0.4	0.14	0.37	0.15	0.37	0.16	0.37	0.16	0.37	0.17	0.36	0.4
0.5	0.18	0.47	0.19	0.46	0.20	0.46	0.20	0.46	0.21	0.45	0.5
0.6	0.22	0.56	0.22	0.56	0.23	0.55	0.24	0.55	0.25	0.54	0.6
0.7	0.25	0.65	0.26	0.65	0.27	0.64	0.28	0.64	0.30	0.63	0.7
0.8	0.29	0.75	0.30	0.74	0.31	0.74	0.33	0.73	0.34	0.73	0.8
0.9	0.32	0.84	0.34	0.83	0.35	0.83	0.37	0.82	0.38	0.82	0.9
1.0	0.36	0.93	0.37	0.93	0.39	0.92	0.41	0.91	0.42	0.91	1.0
2.0	0.72	1.87	0.75	1.85	0.78	1.84	0.81	1.83	0.85	1.81	2.0
3.0	1.08	2.80	1.12	2.78	1.17	2.76	1.22	2.74	1.27	2.72	3.0
4.0	1.43	3.73	1.50	3.71	1.56	3.68	1.63	3.65	1.69	3.63	4.0
5.0	1.79	4.67	1.87	4.64	1.95	4.60	2.03	4.57	2.11	4.53	5.0
6.0	2.15	5.60	2.25	5.56	2.34	5.52	2.44	5.48	2.54	5.44	6.0
7.0	2.51	6.54	2.62	6.49	2.74	6.44	2.85	6.39	2.96	6.34	7.0
8.0	2.87	7.47	3.00	7.42	3.13	7.36	3.25	7.31	3.38	7.25	8.0
9.0	3.23	8.40	3.37	8.34	3.52	8.28	3.66	8.22	3.80	8.16	9.0
10.0	3.58	9.34	3.75	9.27	3.91	9.21	4.07	9.14	4.23	9.06	10.0
11.0	3.94	10.27	4.12	10.20	4.30	10.13	4.47	10.05	4.65	9.97	11.0
12.0	4.30	11.20	4.50	11.13	4.69	11.05	4.88	10.96	5.07	10.88	12.0
13.0	4.66	12.14	4.87	12.05	5.08	11.97	5.29	11.88	5.49	11.78	13.0
14.0	5.02	13.07	5.24	12.98	5.47	12.89	5.69	12.79	5.92	12.69	14.0
15.0	5.38	14.00	5.62	13.91	5.86	13.81	6.10	13.70	6.34	13.59	15.0
16.0	5.73	14.94	5.99	14.83	6.25	14.73	6.51	14.62	6.76	14.50	16.0
17.0	6.09	15.87	6.37	15.76	6.64	15.65	6.91	15.53	7.18	15.41	17.0
18.0	6.45	16.80	6.74	16.69	7.03	16.57	7.32	16.44	7.61	16.31	18.0
19.0	6.81	17.74	7.12	17.62	7.42	17.49	7.73	17.36	8.03	17.22	19.0
20.0	7.17	18.67	7.49	18.54	7.81	18.41	8.13	18.27	8.45	18.13	20.0
21.0	7.53	19.61	7.87	19.47	8.21	19.33	8.54	19.18	8.87	19.03	21.0
22.0	7.88	20.54	8.24	20.40	8.60	20.25	8.95	20.10	9.30	19.94	22.0
23.0	8.24	21.47	8.62	21.33	8.99	21.17	9.35	21.01	9.72	20.85	23.0
24.0	8.60	22.41	8.99	22.25	9.38	22.09	9.76	21.93	10.14	21.75	24.0
25.0	8.96	23.34	9.37	23.18	9.77	23.01	10.17	22.84	10.57	22.66	25.0
26.0	9.32	24.27	9.74	24.11	10.16	23.93	10.58	23.75	10.99	23.56	26.0
27.0	9.68	25.21	10.11	25.03	10.55	24.85	10.98	24.67	11.41	24.47	27.0
28.0	10.03	26.14	10.49	25.96	10.94	25.77	11.39	25.58	11.83	25.38	28.0
29.0	10.39	27.07	10.86	26.89	11.33	26.69	11.80	26.49	12.26	26.28	29.0
30.0	10.75	28.01	11.24	27.82	11.72	27.62	12.20	27.41	12.68	27.19	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 291 — — 249 — — 111 + + 69 +		+ 292 — — 248 — — 112 + + 68 +		+ 293 — — 247 — — 113 + + 67 +		+ 294 — — 246 — — 114 + + 66 +		+ 295 — — 245 — — 115 + + 65 +		

	+ 26° + + 154 − − 206 − − 334 +		+ 27° + + 153 − − 207 − − 333 +		+ 28° + + 152 − − 208 − − 332 +		+ 29° + + 151 − − 209 − − 331 +		+ 30° + + 150 − − 210 − − 330 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.04	0.09	0.05	0.09	0.05	0.09	0.05	0.09	0.05	0.09	0.1
0.2	0.09	0.18	0.09	0.18	0.09	0.18	0.10	0.17	0.10	0.17	0.2
0.3	0.13	0.27	0.14	0.27	0.14	0.26	0.15	0.26	0.15	0.26	0.3
0.4	0.18	0.36	0.18	0.36	0.19	0.35	0.19	0.35	0.20	0.35	0.4
0.5	0.22	0.45	0.23	0.45	0.23	0.44	0.24	0.44	0.25	0.43	0.5
0.6	0.26	0.54	0.27	0.53	0.28	0.53	0.29	0.52	0.30	0.52	0.6
0.7	0.31	0.63	0.32	0.62	0.33	0.62	0.34	0.61	0.35	0.61	0.7
0.8	0.35	0.72	0.36	0.71	0.38	0.71	0.39	0.70	0.40	0.69	0.8
0.9	0.39	0.81	0.41	0.80	0.42	0.79	0.44	0.79	0.45	0.78	0.9
1.0	0.44	0.90	0.45	0.89	0.47	0.88	0.48	0.87	0.50	0.87	1.0
2.0	0.88	1.80	0.91	1.78	0.94	1.77	0.97	1.75	1.00	1.73	2.0
3.0	1.32	2.70	1.36	2.67	1.41	2.65	1.45	2.62	1.50	2.60	3.0
4.0	1.75	3.60	1.82	3.56	1.88	3.53	1.94	3.50	2.00	3.46	4.0
5.0	2.19	4.49	2.27	4.46	2.35	4.41	2.42	4.37	2.50	4.33	5.0
6.0	2.63	5.39	2.72	5.35	2.82	5.30	2.91	5.25	3.00	5.20	6.0
7.0	3.07	6.29	3.18	6.24	3.29	6.18	3.39	6.12	3.50	6.06	7.0
8.0	3.51	7.19	3 63	7.13	3.76	7.06	3.88	7.00	4.00	6.93	8.0
9.0	3.95	8.09	4.09	8.02	4.23	7.95	4.36	7.87	4.50	7.79	9.0
10.0	4.38	8.99	4.54	8.91	4.69	8.83	4.85	8.75	5.00	8.66	10.0
11.0	4.82	9.89	4.99	9.80	5.16	9.71	5.33	9.62	5.50	9.53	11.0
12.0	5.26	10.79	5.45	10.69	5.63	10.60	5.82	10.50	6.00	10.39	12.0
13.0	5.70	11.68	5.90	11.58	6.10	11.48	6.30	11.37	6.50	11.26	13.0
14.0	6.14	12.58	6.36	12.47	6.57	12.36	6.79	12.24	7.00	12.12	14.0
15.0	6.58	13.48	6.81	13.37	7.04	13.24	7.27	13.12	7.50	12.99	15.0
16.0	7.01	14.38	7.26	14.26	7.51	14.13	7.76	13.99	8.00	13.86	16.0
17.0	7.45	15.28	7.72	15.15	7.98	15.01	8.24	14.87	8.50	14.72	17.0
18.0	7.89	16.18	8.17	16.04	8.45	15.89	8.73	15.74	9.00	15.59	18.0
19.0	8.33	17.08	8.63	16.93	8.92	16.78	9.21	16.62	9.50	16.45	19.0
20.0	8.77	17.98	9.08	17.82	9.39	17.66	9.70	17.49	10.00	17.32	20.0
21.0	9.21	18.87	9.53	18.71	9.86	18.54	10.18	18.37	10.50	18.19	21.0
22.0	9.64	19.77	9.99	19.60	10.33	19.42	10.67	19.24	11.00	19.05	22.0
23.0	10.08	20.67	10.44	20.49	10.80	20.31	11.15	20.12	11.50	19.92	23.0
24.0	10.52	21.57	10.90	21.38	11.27	21.19	11.64	20.99	12.00	20.78	24.0
25.0	10.96	22.47	11.35	22.28	11.74	22.07	12.12	21.87	12.50	21.65	25 0
26.0	11.40	23.37	11.80	23.17	12.21	22.96	12.61	22.74	13.00	22.52	26.0
27.0	11.84	24.27	12.26	24.06	12.68	23.84	13.09	23.61	13.50	23.38	27.0
28.0	12.27	25.17	12.71	24.95	13.15	24.72	13.57	24.49	14.00	24.25	28.0
29.0	12.71	26.07	13.17	25.84	13.61	25.61	14.06	25.36	14.50	25.11	29.0
30.0	13.15	26.96	13.62	26.73	14.08	26.49	14.54	26.24	15.00	25.98	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 296 − − 244 − − 116 + + 64 +		+ 297 − − 243 − − 117 + + 63 +		+ 298 − − 242 − − 118 + + 62 +		+ 299 − − 241 − − 119 + + 61 +		+ 300 − − 240 − − 120 + + 60 +		

	+ 31°+ +149 — — 211 — —329 +		+ 32°+ +148 — —212 — —328 +		+ 33°+ +147 — —213 — —327 +		+ 34°+ +146 — —214 — — 326 +		+ 35°+ +145 — —215 — —325 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.05	0.09	0.05	0.08	0.05	0.08	0.06	0.08	0.06	0.08	0.1
0.2	0.10	0.17	0.11	0.17	0.11	0.17	0.11	0.17	0.11	0.16	0.2
0.3	0.15	0.26	0.16	0.25	0.16	0.25	0.17	0 25	0.17	0.25	0.3
0.4	0.21	0.34	0.21	0.34	0.22	0.34	0.22	0.33	0.23	0.33	0.4
0.5	0.26	0.43	0.26	0.42	0.27	0.42	0.28	0.41	0.29	0.41	0.5
0.6	0.31	0.51	0.32	0.51	0.33	0.50	0.34	0.50	0.34	0.49	0.6
0.7	0.36	0.60	0.37	0.59	0.38	0.59	0.39	0.58	0.40	0.57	0.7
0.8	0.41	0.69	0.42	0.68	0.44	0.67	0.45	0.66	0.46	0.66	0.8
0.9	0.46	0.77	0.48	0.76	0.49	0.75	0.50	0.75	0.52	0.74	0.9
1.0	0.52	0.86	0.53	0.85	0.54	0.84	0.56	0.83	0.57	0.82	1.0
2.0	1.03	1.71	1.06	1.70	1.09	1.68	1.12	1.66	1.15	1.64	2.0
3.0	1.55	2.57	1.59	2.54	1.63	2.52	1.68	2.49	1.72	2.46	3.0
4.0	2.06	3.43	2.12	3.39	2.18	3.35	2.24	3.32	2.29	3.28	4.0
5.0	2.58	4.29	2.65	4.24	2.72	4.19	2.80	4.15	2.87	4.10	5.0
6.0	3.09	5.14	3.18	5.09	3.27	5.03	3.36	4.97	3.44	4.91	6.0
7.0	3.61	6.00	3.71	5.94	3.81	5.87	3.91	5.80	4.02	5.73	7.0
8.0	4.12	6.86	4.24	6.78	4.36	6.71	4.47	6.63	4.59	6.55	8.0
9.0	4.64	7.71	4.77	7.63	4.90	7.55	5 03	7.46	5.16	7.37	9.0
10.0	5.15	8.57	5.30	8.48	5.45	8.39	5.59	8.29	5.74	8.19	10.0
11.0	5.67	9.43	5.83	9.33	5.99	9.23	6.15	9.12	6.31	9.01	11.0
12.0	6.18	10.29	6.36	10.18	6.54	10.06	6.71	9.95	6.88	9.83	12.0
13.0	6.70	11.14	6.89	11.02	7.08	10.90	7.27	10.78	7.46	10.65	13.0
14.0	7.21	12.00	7.42	11.87	7.62	11.74	7.83	11.61	8.03	11.47	14.0
15.0	7.73	12.86	7.95	12.72	8.17	12.58	8.39	12.44	8.60	12.29	15.0
16.0	8.24	13.71	8.48	13.57	8.71	13.42	8.95	13.26	9.18	13.11	16.0
17.0	8.76	14.57	9.01	14.42	9.26	14.26	9.51	14.09	9.75	13.93	17.0
18.0	9.27	15.43	9.54	15.26	9.80	15.10	10.07	14.92	10.32	14.74	18.0
19.0	9.79	16.29	10.07	16.11	10.35	15.93	10.62	15.75	10.90	15.56	19.0
20.0	10.30	17.14	10.60	16.96	10.89	16.77	11.18	16.58	11.47	16.38	20.0
21.0	10.82	18.00	11.13	17.81	11.44	17.61	11.74	17.41	12.05	17.20	21.0
22.0	11.33	18.86	11.66	18.66	11.98	18.45	12.30	18.24	12.62	18.02	22.0
23.0	11.85	19.71	12.19	19.51	12.53	19.29	12.86	19.07	13.19	18.84	23.0
24.0	12.36	20.57	12.72	20.35	13.07	20.13	13.42	19.90	13.77	19.66	24.0
25.0	12.88	21.43	13.25	21.20	13.62	20.97	13.98	20.73	14.34	20.48	25.0
26.0	13.39	22.29	13.78	22.05	14.16	21.81	14.54	21.55	14.91	21.30	26.0
27 0	13.91	23.14	14.31	22.90	14.71	22.64	15.10	22.38	15.49	22.12	27.0
28.0	14.42	24.00	14.84	23.75	15.25	23.48	15.66	23.21	16.06	22.94	28.0
29.0	14.94	24.86	15.37	24.59	15.79	24.32	16.22	24.04	16.63	23.76	29.0
30.0	15.45	25.71	15.90	25.44	16.34	25.16	16.78	24.87	17.21	24.57	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+301°— —239 — —121 + + 59 +		+302 — —238 — —122 + + 58 +		+303 — —237 — —123 + + 57 +		+304 — —236 — —124 + + 56 +		+305 — —235 — —125 + + 55 +		

	+ 36°+ +144 — — 216 — — 324 +		+ 37°+ + 143 — — 217 — — 323 +		+ 38°+ + 142 — — 218 — — 322 +		+ 39°+ + 141 — — 219 — — 321 +		+ 40°+ + 140 — — 220 — — 320 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.06	0.08	0.06	0.08	0.06	0.08	0.06	0.08	0.06	0.08	0.1
0.2	0.12	0.16	0.12	0.16	0.12	0.16	0.13	0.16	0.13	0.15	0.2
0.3	0.18	0.24	0.18	0.24	0.18	0.24	0.19	0.23	0.19	0.23	0.3
0.4	0.24	0.32	0.24	0.32	0.25	0.32	0.25	0.31	0.26	0.31	0.4
0.5	0.29	0.40	0.30	0.40	0.31	0.39	0.31	0.39	0.32	0.38	0.5
0.6	0.35	0.49	0.36	0.48	0.37	0.47	0.38	0.47	0.39	0.46	0.6
0.7	0.41	0.57	0.42	0.56	0.43	0 55	0.44	0.54	0.45	0.54	0.7
0.8	0.47	0.65	0.48	0.64	0.49	0.63	0.50	0.62	0.51	0.61	0.8
0.9	0.53	0.73	0.54	0.72	0.55	0.71	0.57	0.70	0.58	0.69	0.9
1.0	0.59	0.81	0.60	0.80	0.62	0.79	0.63	0.78	0.64	0.77	1.0
2.0	1.18	1.62	1.20	1.60	1.23	1.58	1.26	1.55	1.29	1.53	2.0
3.0	1.76	2.43	1.81	2.40	1.85	2.36	1.89	2.33	1.93	2.30	3.0
4.0	2.35	3.24	2.41	3.19	2.46	3.15	2.52	3.11	2.57	3.06	4.0
5.0	2.94	4.05	3.01	3.99	3.08	3.94	3.15	3.89	3.21	3.83	5.0
6.0	3.53	4.85	3.61	4.79	3.69	4.73	3.78	4.66	3.86	4.60	6.0
7.0	4.11	5.66	4.21	5.59	4.31	5.52	4.41	5.44	4.50	5.36	7.0
8.0	4.70	6.47	4.81	6.39	4.93	6.30	5.03	6.22	5.14	6.13	8.0
9.0	5.29	7.28	5.42	7.19	5.54	7.09	5.66	6.99	5.79	6.89	9.0
10.0	5.88	8.09	6.02	7.99	6.16	7.88	6.29	7.77	6.43	7.66	10.0
11.0	6.47	8.90	6.62	8.78	6.77	8.67	6.92	8.55	7.07	8.43	11.0
12.0	7.05	9.71	7.22	9.58	7.39	9.46	7.55	9.33	7.71	9.19	12.0
13.0	7.64	10.52	7.82	10.38	8.00	10.24	8.18	10.10	8.36	9.96	13.0
14.0	8.23	11.33	8.43	11.18	8.62	11.03	8.81	10.88	9.00	10.72	14.0
15.0	8.82	12.14	9.03	11.98	9.23	11.82	9.44	11.66	9.64	11.49	15.0
16.0	9.40	12.94	9.63	12.78	9.85	12.61	10.07	12.43	10.28	12.26	16.0
17.0	9.99	13.75	10.23	13.58	10.47	13.40	10.70	13.21	10.93	13.02	17.0
18.0	10.58	14.56	10.83	14.38	11.08	14.18	11.33	13.99	11.57	13.79	18.0
19.0	11.17	15.37	11.43	15.17	11.70	14.97	11.96	14.77	12.21	14.55	19.0
20.0	11.76	16.18	12.04	15.97	12.31	15.76	12.59	15.54	12.86	15.32	20.0
21.0	12.34	16.99	12.64	16.77	12.93	16.55	13.22	16.32	13.50	16.09	21.0
22.0	12.93	17.80	13.24	17.57	13.54	17.34	13.85	17.10	14.14	16.85	22.0
23.0	13.52	18.61	13.84	18.37	14.16	18.12	14.47	17.87	14.78	17.62	23.0
24.0	14.11	19.42	14.44	19.17	14.78	18.91	15.10	18.65	15.43	18.39	24.0
25.0	14.69	20.23	15.05	19.97	15.39	19.70	15.73	19.43	16.07	19.15	25.0
26.0	15.28	21.03	15.65	20.76	16.01	20.49	16.36	20.21	16.71	19.92	26.0
27.0	15.87	21.84	16.25	21.56	16.62	21.28	16.99	20.98	17.36	20.68	27.0
28.0	16.46	22.65	16.85	22.36	17.24	22.06	17.62	21.76	18.00	21.45	28.0
29.0	17.05	23.46	17.45	23.16	17.85	22.85	18.25	22.54	18.64	22.22	29.0
30.0	17.63	24.27	18.05	23.96	18.47	23.64	18.88	23.31	19.28	22.98	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 306 — — 234 — — 126 + + 54 +		+ 307 — — 233 — — 127 + + 53 +		+ 308 — — 232 — — 128 + + 52 +		+ 309 — — 231 — — 129 + + 51 +		+ 310 — — 230 — — 130 + + 50 +		

	+ 41°+ + 139 — — 221 — — 319 +		+ 42°+ + 138 — — 222 — — 318 +		+ 43°+ + 137 — — 223 — — 317 +		+ 44°+ + 136 — — 224 — — 316 +		+ 45°+ + 135 — — 225 — — 315 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.07	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.1
0.2	0.13	0.15	0.13	0.15	0.14	0.15	0.14	0.14	0.14	0.14	0.2
0.3	0.20	0.23	0.20	0.22	0.20	0.22	0.21	0.22	0.21	0.21	0.3
0.4	0.26	0.30	0.27	0.30	0.27	0.29	0.28	0.29	0.28	0.28	0.4
0.5	0.33	0.38	0.33	0.37	0.34	0.37	0.35	0.36	0.35	0.35	0.5
0.6	0.39	0.45	0.40	0.45	0.41	0.44	0.42	0.43	0.42	0.42	0.6
0.7	0.46	0.53	0.47	0.52	0.48	0.51	0.49	0.50	0.49	0.49	0.7
0.8	0.52	0.60	0.54	0.59	0.55	0.59	0.56	0.58	0.57	0.57	0.8
0.9	0.59	0.68	0.60	0.67	0.61	0.66	0.63	0.65	0.64	0.64	0.9
1.0	0.66	0.75	0.67	0.74	0.68	0.73	0.69	0.72	0.71	0.71	1.0
2.0	1.31	1.51	1.34	1.49	1.36	1.46	1.39	1.44	1.41	1.41	2.0
3.0	1.97	2.26	2.01	2.23	2.05	2.19	2.08	2.16	2.12	2.12	3.0
4.0	2.62	3.02	2.68	2.97	2.73	2.93	2.78	2.88	2.83	2.83	4.0
5.0	3.28	3.77	3.35	3.72	3.41	3.66	3.47	3.60	3.54	3.54	5.0
6.0	3.94	4.53	4.01	4.46	4.09	4.39	4.17	4.32	4.24	4.24	6.0
7.0	4.59	5.28	4.68	5.20	4.77	5.12	4.86	5.04	4.95	4.95	7.0
8.0	5.25	6.04	5.35	5.95	5.46	5.85	5.56	5.75	5.66	5.66	8.0
9.0	5.90	6.79	6.02	6.69	6.14	6.58	6.25	6.47	6.36	6.36	9.0
10.0	6.56	7.55	6.69	7.43	6.82	7.31	6.95	7.19	7.07	7.07	10.0
11.0	7.22	8.30	7.36	8.17	7.50	8.04	7.64	7.91	7.78	7.78	11.0
12.0	7.87	9.06	8.03	8.92	8.18	8.78	8.34	8.63	8.49	8.49	12.0
13.0	8.53	9.81	8.70	9.66	8.87	9.51	9.03	9.35	9.19	9.19	13.0
14.0	9.18	10.57	9.37	10.40	9.55	10.24	9.73	10.07	9.90	9.90	14.0
15.0	9.84	11.32	10.04	11.15	10.23	10.97	10.42	10.79	10.61	10.61	15.0
16.0	10.50	12.08	10.71	11.89	10.91	11.70	11.11	11.51	11.31	11.31	16.0
17.0	11.15	12.83	11.38	12.63	11.59	12.43	11.81	12.23	12.02	12.02	17.0
18.0	11.81	13.58	12.04	13.38	12.28	13.16	12.50	12.95	12.73	12.73	18.0
19.0	12.47	14.34	12.71	14.12	12.96	13.90	13.20	13.67	13.44	13.44	19.0
20.0	13.12	15.09	13.38	14.86	13.64	14.63	13.89	14.39	14.14	14.14	20.0
21.0	13.78	15.85	14.05	15.61	14.32	15.36	14.59	15.11	14.85	14.85	21.0
22.0	14.43	16.60	14.72	16.35	15.00	16.09	15.28	15.83	15.56	15.56	22.0
23.0	15.09	17.36	15.39	17.09	15.69	16.82	15.98	16.54	16.26	16.26	23.0
24.0	15.75	18.11	16.06	17.84	16.37	17.55	16.67	17.26	16.97	16.97	24.0
25.0	16.40	18.87	16.73	18.58	17.05	18.28	17.37	17.98	17.68	17.68	25.0
26.0	17.06	19.62	17.40	19.32	17.73	19.02	18.06	18.70	18.38	18.38	26.0
27.0	17.71	20.38	18.07	20.06	18.41	19.75	18.76	19.42	19.09	19.09	27.0
28.0	18.37	21.13	18.74	20.81	19.10	20.48	19.45	20.14	19.80	19.80	28.0
29.0	19.03	21.89	19.40	21.55	19.78	21.21	20.15	20.86	20.51	20.51	29.0
30.0	19.68	22.64	20.07	22.29	20.46	21.94	20.84	21.58	21.21	21.21	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 311 — — 229 — — 131 + + 49 +		+ 312 — — 228 — — 132 + + 48 +		+ 313 — — 227 — — 133 + + 47 +		+ 314 — — 226 — — 134 + + 46 +		+ 315 — — 225 — — 135 + + 45 +		